GGSRDN

Educational Services Private Limited

9^{th}, 10^{th}, NEET, JEE(Main/Advanced)

अभ्यास ही सबसे बड़ा गुरू है।

CLASS : XII (MATHS)

DPP

DAILY PRACTICE PROBLEM

DPP- 41 to 50

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 41

Total Marks : 27

Max. Time : 29 min.

Topics : **Sequence & Series, Application of Derivatives**

Type of Questions		M.M., Min.
Comprehension (no negative marking) Q.1 to Q.3	(3 marks, 3 min.)	[9, 9]
Single choice Objective (no negative marking) Q. 4,5	(3 marks, 3 min.)	[6, 6]
Fill in the Blanks (no negative marking) Q.6	(4 marks, 4 min.)	[4, 4]
Subjective Questions (no negative marking) Q.7,8	(4 marks, 5 min.)	[8, 10]

COMPREHENSION (Q. NO. 1 TO 3)

Consider $S_n = \frac{8}{5} + \frac{16}{65} + ... + \frac{8r}{4r^4+1}$

1. Sum of infinite terms of above series will be

(A) 0 (B) 1/2 (C) 2 (D) None of these

2. The value of S_{16} must be

(A) $\frac{80}{41}$ (B) $\frac{1088}{545}$ (C) $\frac{107}{245}$ (D) None of these

3. If $S_n = \frac{an^2+bn}{cn^3+dn^2+en+1}$, where a, b, c, d, e are independent of 'n', then

(A) a = 4, e = 2 (B) c = 0, d = 4 (C) b = 4, e = 4 (D) None of these

4. Tangent and normal to the curve y = 2 sinx + sin2x are drawn at p $\left(x = \frac{\pi}{3}\right)$. The area of the quadrilateral formed by the tangent, the normal and coordinate axes is.

(A) $\frac{\pi\sqrt{3}}{2}$ (B) $\frac{\pi}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) None of these

5. The point(s) of minimum of the function, $f(x) = 4x^3 - x\,|x-2|$, $x \in [0, 3]$ is :

(A) x = 0 (B) x = 1/3 (C) x = 1/2 (D) x = 2

6. The value of a for which the function $f(x) = (4a-3)(x + \log 5) + 2(a-7)\cot\frac{x}{2}\sin^2\frac{x}{2}$ does not posses critical points is _______.

7. Find the difference between the greatest and least values of the function,

$f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x.$

8. Find values of a and b such that $f(x) = \frac{a}{x} + bx$ has a minimum at point (1, 6).

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 42

Total Marks : 33

Max. Time : 37 min.

Topics : Solution of Triangle, Application of Derivatives, Method of Differentiation

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q. 1,2,3	(3 marks, 3 min.)	[9, 9]
Subjective Questions (no negative marking) Q.4,5,6,7	(4 marks, 5 min.)	[16, 20]
Match the Following (no negative marking) Q.8	(8 marks, 8 min.)	[8, 8]

1. In a ΔABC, if $\frac{s-a}{11} = \frac{s-b}{12} = \frac{s-c}{13}$, then $\tan^2 \frac{A}{2}$ is equal to

(A) $\frac{143}{342}$ (B) $\frac{13}{33}$ (C) $\frac{11}{39}$ (D) $\frac{12}{37}$

2. The two adjacent sides of a cyclic quadrilateral are 2 and 5 and the angle between them is 60°. If the third side is 3, remaining fourth side is.

(A) 2 (B) 3 (C) 4 (D) 5

3. If $y = \cos^{-1}\sqrt{\frac{\sqrt{1+x^2}+1}{2\sqrt{1+x^2}}}$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{1}{2(1+x^2)}, x \in R$ (B) $\frac{1}{2(1+x^2)}, x > 0$ (C) $\frac{-1}{2(1+x^2)}, x < 0$ (D) $\frac{1}{2(1+x^2)}, x < 0$

4. In a triangle ABC, if cos A + 2 cos B + cos C = 2. Prove that the sides of the triangle are in A.P.

5. If x and y are positive numbers and x + y = 1, then prove that $\left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right) \ge 9$

6. Prove following inequalities :

(i) $\frac{x}{1+x} < \ell n(1+x) < x$ for $x > 0$

(ii) $2x > 3\sin x - x\cos x$ for $0 < x < \pi/2$

7. Find the greatest & least value of $f(x) = \sin^{-1}\frac{x}{\sqrt{x^2+1}} - \ell n\, x$ in $\left[\frac{1}{\sqrt{3}}, \sqrt{3}\right]$.

8. If P(x) = $x^3 + px^2 + qx + 6$, then match the entries in column - I with column - II

Column - I	Column - II
(A) If P(x) is divisible by $x^2 + ax + b$ and $x^2 + bx + a$, (a, b, $\in$ R), a $\neq$ b, then P(x)	(p) have point of local maximum less than point of local minimum
(B) If $3q > p^2$, then P(x)	(q) is monotonic $\forall$ x $\in$ R
(C) If p and q are two consecutive natural numbers such that p > q, then P(x)	(r) has point of local maximum greater than point of local minimum
(D) If $Q(x) = P(x) - 2x^3 - 2qx$ and $p^2 > 3q$, then Q(x)	(s) possesses local maxima and local minima

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MATHEMATICS DPP
DAILY PRACTICE PROBLEMS

DPP No. 43

Total Marks : 32

Max. Time : 36 min.

Topics : Solution of Triangle, Application of Derivatives, Straight Line

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q. 1,2	(3 marks, 3 min.)	[6, 6]
Subjective Questions (no negative marking) Q.3,4,5,6,7,8	(4 marks, 5 min.)	[26, 30]

1. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A **false** statement among the following is

(A) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$.
(B) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$.

(C) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$.
(D) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$.

2. If in triangle ABC, $r_1 = 2r_2 = 3r_3$, D is the middle point of BC. Then cos $\angle$ADC is equal to

(A) $\frac{7}{25}$ (B) $-\frac{7}{25}$ (C) $\frac{24}{25}$ (D) $-\frac{24}{25}$

3. Two men P and Q start with velocities v at the same time from the junction of two roads inclined at 45º to each other. If they travel by different roads, find the rate at which they are being separated.

4. ABC is a triangle and D is the middle point of BC. If AD is perpendicular to AC, prove that

$$\cos A \,.\, \cos C = \frac{2(c^2 - a^2)}{3ac}$$

5. With usual notation In a ΔABC, a, c, A are given and $b_2 = 2b_1$, where b_1, b_2 are two values of the thrid side, then prove that $3a = c\sqrt{(1+8\sin^2 A)}$

6. If $2f(x) = f(xy) + f\left(\frac{x}{y}\right)$ for all x, y, $\in R^+$, f(1) = 0 and f ' (1) = 1, then find f(e) and f '(2).

7. Through the origin O, a straight line is drawn to cut the lines $y = m_1x + c_1$ and $y = m_2x + c_2$ at Q and R respectively. Find the locus of the point P on this variable line, such that OP is the geometric mean between OQ and OR.

8. The circle $x^2 + y^2 = 1$ cuts the x-axis at P & Q. Another circle with centre at Q and variable radius intersects the first circle at R above x-axis and the line segment PQ at S. Find the maximum area of the ΔQSR.

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 44

Total Marks : 27

Max. Time : 30 min.

Topic : Sequence & Series

Type of Questions		**M.M., Min.**
Comprehension (no negative marking) Q.1 to Q.3	**(3 marks, 3 min.)**	**[9, 9]**
Single choice Objective (no negative marking) Q.4,5	**(3 marks, 3 min.)**	**[6, 6]**
Subjective Questions (no negative marking) Q.6,7,8	**(4 marks, 5 min.)**	**[12, 15]**

COMPREHENSION : (Q. NO. 1 TO 3)

Let $x \in R^+$ such that {x}, [x], x are in G.P., where [.] & {.} are greatest integer & fractional part functions respectively.

1 Common ratio of this G.P. is

(A) $\frac{-1-\sqrt{5}}{2}$ (B) $\frac{-1+\sqrt{5}}{2}$ (C) $\frac{1-\sqrt{5}}{4}$ (D) $\frac{1+\sqrt{5}}{2}$

2 The value of x is

(A) $\frac{-1-\sqrt{5}}{2}$ (B) $\sqrt{5}$ (C) $\frac{1+\sqrt{5}}{2}$ (D) none of these

3 Sum to n terms of this G.P.

(A) $2^n \cos^n \frac{\pi}{5} - 1$ (B) $2^n \sin^n \frac{\pi}{5} - 1$ (C) $2^n \cos^n \frac{\pi}{5}$ (D) $2^n \sin^n \frac{\pi}{5}$

4. First, second and seventh terms of an A.P. (all the terms are distinct), whose sum is 93, are in G.P. Fourth term of this G.P. is

(A) 21 (B) 31 (C) 75 (D) 375

5. If $\sum_{r=1}^{n} t_r = \frac{1}{12} n(n+1)(n+2)$, then the value of $\sum_{r=1}^{n} \frac{1}{t_r}$ is

(A) $\frac{2n}{n+1}$ (B) $\frac{n}{(n+1)}$ (C) $\frac{4n}{n+1}$ (D) $\frac{3n}{n+1}$

6. Find the number of terms of a G.P. in which the ratio of the sum of the first eleven terms to the sum of the last eleven terms is 1/8, and the ratio of the sum of all the terms without the first nine to the sum of all the terms without the last nine is 2.

7. If $0 < r < 1$ and $m \in N$, then prove that $(2m + 1) r^m (1 - r) < 1 - r^{2m+1}$

8. The value of x + y + z is 15 if a, x, y, z, b are in AP while the value of (1/x) + (1/y) + (1/z) is 5/3 if a, x, y, z, b are in HP. Find a and b.

MATHEMATICS DPP
DAILY PRACTICE PRACTICE PROBLEMS

DPP No. 45

Total Marks : 30

Max. Time : 30 min.

Topics : Solution of Triangle, Application of Derivatives, Straight Line

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3,4	(3 marks, 3 min.)	[12, 12]
Multiple choice objective (no negative marking) Q.5,6	(5 marks, 4 min.)	[10, 8]
Subjective Questions (no negative marking) Q.7,8	(4 marks, 5 min.)	[8, 10]

1. In a ΔABC, a = 5, b = 4 and $\tan\frac{C}{2} = \sqrt{\frac{7}{9}}$, then the side c is equal to

(A) 2 (B) 3 (C) 6 (D) None of these

2. In a triangle ABC, if $a^3\cos(B - C) + b^3\cos(C - A) + c^3\cos(A - B) = \lambda\, abc$, then '$\lambda$' is equal to
(A) 1 (B) 2 (C) 3 (D) None of these

3. With usual notations, in a ΔABC $\frac{r_1}{(s-b)(s-c)} + \frac{r_2}{(s-c)(s-a)} + \frac{r_3}{(s-a)(s-b)}$ is equal to

(A) $\frac{1}{r}$ (B) $\frac{2}{r}$ (C) $\frac{3}{r}$ (D) $\frac{4}{r}$

4. Let $f(x) = \begin{cases} \sin\frac{\pi x}{2} & , 0 \le x < 1 \\ 3 - 2x & , \quad x \ge 1 \end{cases}$ then :

(A) f(x) has local maxima at x = 1
(B) f(x) has local minima at x = 1
(C) f(x) does not have any local extrema at x = 1
(D) f(x) has a global minima at x = 1

5. In a ΔABC, if a + b = 3c, then cosA + cosB is equal to

(A) 3cos C (B) $6\sin^2\frac{C}{2}$ (C) 3 cos (A + B) (D) 3 + 3 cos (A + B)

6. If H ≡ (3, 4) and C ≡ (1, 2) are orthocentre and circumcentre of ΔPQR and equation of side PQ is x – y + 7 = 0, then
(A) equation of circum circle $(x - 1)^2 + (y - 2)^2 = 80$
(B) equation of circum circle $(x - 1)^2 + (y - 2)^2 = 70$

(C) centroid is $\left(\frac{5}{3}, \frac{8}{3}\right)$

(D) circumradius = $\sqrt{70}$

7. The function $f(x) = \sqrt{ax^3 + bx^2 + cx + d}$ has its non zero local minimum and maximum values at x = –2 and x = 2 respectively. If a is a root of the equation $x^2 - x - 6 = 0$. Find all possible values of a, b, c, and d.

8. Let $f(x) = \begin{cases} |x - 2| + a^2 - 9a - 9 & \text{if} \quad x < 2 \\ 2x - 3 & \text{if} \quad x \ge 2 \end{cases}$

Then find the value of 'a' for which f(x) has local minimum at x = 2

MATHEMATICS DPP

DAILY PRACTICE PROBLEMS

DPP No. 46

Total Marks : 37

Max. Time : 38 min.

Topics : Trigonometric Ratio, Quadratic Equation

Type of Questions		M.M.,	Min.
Single choice Objective(no negative marking) Q.1,2,3,6,9	(3 marks, 3 min.)	[15,	15]
Multiple choice objective (no negative marking) Q.5,7	(5 marks, 4 min.)	[10,	8]
Subjective Questions (no negative marking) Q.4,8,10	(4 marks, 5 min.)	[12,	15]

1. The graphs of y = sin x, y = cos x, y = tan x & y = cosec x are drawn on the same axes from 0 to $\pi/2$. A vertical line is drawn through the point where the graphs of y = cos x & y = tan x cross, intersecting the other two graphs at the points A & B. The length of the line segment AB is

(A) 1 (B) $\frac{\sqrt{5}-1}{2}$ (C) $\sqrt{2}$ (D) $\frac{\sqrt{5}+1}{2}$

2. Let m be a positive integer, $m \geq 2$. If $\alpha_1, \alpha_2, \ldots\ldots, \alpha_m$ are the roots of the equation $x^m - 1 = 0$, then the equation whose roots are

$\beta_1 = \alpha_2 + \alpha_3 + \ldots.. + \alpha_m - (m-1)\alpha_1$

$\beta_2 = \alpha_1 + \alpha_3 + \ldots\ldots + \alpha_m - (m-1)\,\alpha_2$

$\vdots$

$\beta_i = \alpha_1 + \ldots. + \alpha_{i-1} + \alpha_{i+1} + \ldots + \alpha_m - (m-1)\,\alpha_i$

$\vdots$

$\beta_m = \alpha_1 + \ldots. + \alpha_{m-1} - (m-1)\,\alpha_m$, is

(A) $x^m + m^m = 0$ (B) $x^m - (-m)^m = 0$ (C) $x^m + (m-1)^m = 0$ (D) $x^m - (m-1)^m = 0$

3. The value of $\sum_{r=1}^{7} \tan^2\left(\frac{r\pi}{16}\right)$ is

(A) 29 (B) 33 (C) 34 (D) 35

4. Find the product of the real roots of the equation : $x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$.

5. The quadratic equation whose roots are $\sec^2\alpha$ and $\text{cosec}^2\alpha$ can be :

(A) $2x^2 - x - 1 = 0$ (B) $x^2 + 3x - 3 = 0$ (C) $x^2 - 9x + 9 = 0$ (D) $x^2 - 12x + 12 = 0$

6. The integral values of x for which $x^2 + 7x + 13$ is perfect square are

(A) – 4, 5, 2 (B) – 3, – 2 (C) – 4, – 3, – 2 (D) – 4, – 3

7. If $b^2 > 4ac$ then roots of equation $ax^4 + bx^2 + c = 0$ are all real & distinct if :

(A) $b < 0, a < 0, c > 0$ (B) $b < 0, a > 0, c > 0$ (C) $b > 0, a > 0, c > 0$ (D) $b > 0, a < 0, c < 0$

8. If α, β are the roots of the equation $x^2 - 2x + 3 = 0$ obtain the equation whose roots are $\alpha^3 - 3\alpha^2 + 5\alpha - 2$ and $\beta^3 - \beta^2 + \beta + 5$

9. If $f(x) = \frac{1 - \sin 2x + \cos 2x}{2\cos 2x}$, then the value of $f(16^\circ) \cdot f(29^\circ)$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 1 (D) $\frac{3}{4}$

10. Solve the equation : $\left(4\sqrt{\cos\frac{x}{2}} - 5 - \frac{\sqrt{2}}{2}\right)^2 + \sqrt{2}\left(4\sqrt{\cos\frac{x}{2}} - 5 - \frac{\sqrt{2}}{2}\right) - \frac{\cos x}{2} = 0$

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 47

Total Marks : 25

Max. Time : 27 min.

Topics : Solution of Triangle, Vector, Application of Derivatives

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.3,4,5,7,8	**(3 marks, 3 min.)**	**[15, 15]**
True or False (no negative marking) Q.1	**(2 marks, 2 min.)**	**[2, 2]**
Subjective Questions (no negative marking) Q.2,6	**(4 marks, 5 min.)**	**[8, 10]**

1. **True/False type questions :**

(i) Length of median AD in ΔABC = $\sqrt{2b^2 + 2c^2 - a^2}$

(ii) Length of angle bisector of angle A in Δ ABC = $\frac{2bc}{b+c}$ cos A

(iii) Every hyperbola has 2 asymptotes.

(iv) Orthocentre of the triangle inscribed in a hyperbola lies on its directrix.

(v) In ΔABC (with usual notation) sin $\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{2bc}}$

(vi) Incentre of pedal triangle of ΔABC is orthocentre of ΔABC

2. Show that for interval $e^{-\pi/4} < x < e^{3\pi/4}$ in which f(x)= sin (ℓn x) – cos(ℓn x) is monotonically increasing

3. Point P is on circumfrence of circle. Chord QR is drawn parallel to tangent at P. Then maximum possible area of Δ PQR is :

(A) $\frac{\sqrt{3}}{4}r^2$ (B) $\frac{3\sqrt{3}}{4}r^2$ (C) $\sqrt{3}\,r^2$ (D) $\frac{\sqrt{3}\,r^2}{4}$

4. If $\vec{a} = 2\hat{i} - 7\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$ and $\vec{a}.m\vec{b}$ = 120 where m is scalar then value of m is equal to

(A) 5 (B) –24 (C) –5 (D) 24

5. A normal is drawn at the point P(a, a^n) on the curve y = x^n in the first quadrant. The normal intersects the y-axis at the point (0, b). If $\lim_{a\to 0}$ b = $\frac{1}{2}$, then 'n' equals

(A) 1/2 (B) 3/2 (C) 2 (D) 4

6. Let $\vec{P}$ = sin x $\hat{i}$ + cos x $\hat{j}$ and $\vec{Q}$ = – $\hat{i}$ – cos x $\hat{j}$, x $\in$ (0, 2nπ), n $\in$ N.

If $\vec{P}$ and $\vec{Q}$ are equal vectors, then find the number of values of x.

7. A, B, C, D, E, are five coplanar points then $\overrightarrow{DA} + \overrightarrow{DB} + \overrightarrow{DC} + \overrightarrow{AE} + \overrightarrow{BE} + \overrightarrow{CE}$ is equal to

(A) $\overrightarrow{DE}$ (B) 3 $\overrightarrow{DE}$ (C) 2 $\overrightarrow{DE}$ (D) 4 $\overrightarrow{DE}$

8. If $\vec{a}$ and $\vec{b}$ are non collinear vector such that vectors (x – 2) $\vec{a} + \vec{b}$ and (2x + 1) $\vec{a} - \vec{b}$ are parallel, then

(A) x = 1/3 (B) no real value of x

(C) infinite values of x (D) x = – 1/3

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 48

Total Marks : 32

Max. Time : 40 min.

Topics : Vector, Application of Derivatives

Type of Questions		M.M., Min.
Subjective Questions (no negative marking) Q.1 to Q.8	(4 marks, 5 min.)	[32, 40]

1. A segment of a line PQ with its extremities on AB and AC bisects a triangle ABC with sides a, b, c into two equal areas. Find the shortest length of the segment PQ.

2. Let f, g : [a, b] $\to$ R be two continuous function, differentiable on (a, b). Assume in addition that g and g' are no where zero on (a, b) and $\frac{f(a)}{g(a)} = \frac{f(b)}{g(b)}$. Prove that there exists c $\in$ (a, b) such that $\frac{f(c)}{g(c)} = \frac{f'(c)}{g'(c)}$

3. Let a, b, c, d, e, f $\in$ R such that ad + be + cf = $\sqrt{(a^2+b^2+c^2)(d^2+e^2+f^2)}$

Use vector to prove that $\frac{a+b+c}{\sqrt{a^2+b^2+c^2}} = \frac{d+e+f}{\sqrt{d^2+e^2+f^2}}$

4. Show that f(x) = $\left(1+\frac{1}{x}\right)^x$ is always an increasing function for all x in its domain.

5. With usual notation in ΔABC if 2b = 3a and $\tan^2 A = \frac{3}{5}$, prove that there are two values of third side, one of which is double the other.

6. Prove that the locus of the centre of a circle , which intercepts a chord of given length '2 a' on the axis of x and passes through a given point on the axis of y, distance b from the origin , is curve, $x^2 \pm 2yb + b^2 = a^2$.

7. Find the sum $\tan\theta + \frac{1}{2}\tan\frac{\theta}{2} + \frac{1}{2^2}\tan\frac{\theta}{2^2} + \frac{1}{2^3}\tan\frac{\theta}{2^3} + ...\infty$ and hence the sum of the series

$$\frac{1}{2^2}\tan\frac{\pi}{2^2} + \frac{1}{2^3}\tan\frac{\pi}{2^3} + \frac{1}{2^4}\tan\frac{\pi}{2^4} + ...\infty$$

8. The two adjacent sides of a paralelogram are represented by the lines x – y + 1 = 0 and 4x–3y–2 = 0. If one of the diagonals of the parallelogram is along the line y = $\frac{3x}{2}$, then find the equation of the other diagonal without finding the vertices of the parallelogram.

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 49

Total Marks : 33

Max. Time : 33 min.

Topics : Sequence & Series, Fundamentals of Mathematics, Quadratic Equation, Straight Line

Type of Questions		M.M.,	Min.
Comprehension (no negative marking) Q.1 to Q.4	(3 marks, 3 min.)	[12,	12]
Single choice Objective (no negative marking) Q.5	(3 marks, 3 min.)	[3,	3]
Multiple choice objective (no negative marking) Q. 6	(5 marks, 4 min.)	[10,	8]
Subjective Questions (no negative marking) Q. 7,8	(4 marks, 5 min.)	[8,	10]

Comprehension (Q. NO. 1 TO 4)

Consider the different positive infinite geometric progression with their sums S_1 and S_2 as

$S_1 = a + ar + ar^2 + ar^3 + ... \infty$

$S_2 = b + bR + bR^2 + bR^3 + ... \infty$

If $S_1 = S_2 = 1$, $ar = bR$ and $ar^2 = \frac{1}{8}$ then answer the following :

1. The sum of their common ratio is

(A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) 1 (D) $\frac{3}{2}$

2. The sum of their first terms is

(A) 1 (B) 2 (C) 3 (D) none of these

3. Common ratio of first G.P. is

(A) $\frac{1}{2}$ (B) $\frac{1-\sqrt{5}}{4}$ (C) $\frac{\sqrt{5}-1}{4}$ (D) $\frac{\sqrt{5}+1}{4}$

4. Common ratio of the second G.P. is

(A) $\frac{3+\sqrt{5}}{4}$ (B) $\frac{3-\sqrt{5}}{4}$ (C) $\frac{1}{2}$ (D) none of these

5. If ω be a imaginary n^{th} root of unity , then $\sum_{r=1}^{n} (ar + b)\, \omega^{r-1}$ is equal to :

(A) $\frac{n(n+1)}{2}a$ (B) $\frac{nb}{1-n}$ (C) $\frac{na}{\omega - 1}$ (D) none of these

6. The complete solution set of the inequation $x - \frac{2(K-1)}{K} \le \frac{2}{3K}(x+1)$ is given by

(A) $(-\infty, 2]$ if $K > \frac{2}{3}$ (B) $[2, \infty)$ if $0 < K < \frac{2}{3}$

(C) $(-\infty, 2]$ if $K < 0$ (D) R if $K = \frac{2}{3}$

7. If α, β are the roots of $x^2 + px + q = 0$ and also of $x^{2n} + p^n x^n + q^n = 0$ and if $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are the roots of $x^n + 1 + (x+1)^n = 0$, then prove that n must be an even integer.

8. The sides of a rhombus are parallel to $y = 2x + 3$ and $2y = x + 5$. The diagonals of the rhombus intersect at (1, 2). If one vertex of the rhombus lies on the y-axis and possible values of the ordinates of this vertex are a & b, then find the value of (a + b).

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 50

Total Marks : 33

Max. Time : 36 min.

Topics : **Vector, Solution of Triangle, Function**

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.1	**(3 marks, 3 min.)**	**[3, 3]**
Multiple choice objective (no negative marking) Q.2, 3	**(5 marks, 4 min.)**	**[10, 8]**
Subjective Questions (no negative marking) Q. 4, 5, 6, 7, 8	**(4 marks, 5 min.)**	**[20, 25]**

1. P, Q have position vectors $\vec{a}$ & $\vec{b}$ relative to the origin 'O' & X, Y divide $\overrightarrow{PQ}$ internally and externally respectively in the ratio 2 : 1 . Vector $\overrightarrow{XY}$ =

(A) $\frac{3}{2}(\vec{b}-\vec{a})$ (B) $\frac{4}{3}(\vec{a}-\vec{b})$ (C) $\frac{5}{6}(\vec{b}-\vec{a})$ (D) $\frac{4}{3}(\vec{b}-\vec{a})$

2. If in a triangle ABC, $b\cos^2\frac{A}{2} + a\cos^2\frac{B}{2} = \frac{3c}{2}$, then

(A) $c^2 \geq ab$ (B) $\frac{a}{c}+\frac{c}{b}+\frac{b}{a} \geq 3$ (C) $\frac{a+c}{2c-a}+\frac{b+c}{2c-b} \geq 4$ (D) a, b, c are in A.P.

3. If 'O' is the circumcentre of the Δ ABC and R_1, R_2 and R_3 are the radii of the circumcircles of triangles OBC, OCA & OAB respectively, then $\frac{a}{R_1}+\frac{b}{R_2}+\frac{c}{R_3}$ has the value equal to

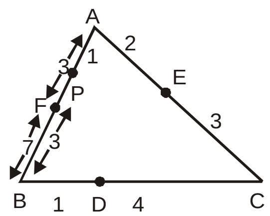
(A) $\frac{abc}{2R^3}$ (B) $\frac{R^3}{abc}$ (C) $\frac{4\Delta}{R^2}$ (D) $\frac{abc}{R^3}$

4. In a ΔABC, prove that $\frac{(a+b+c)^2}{a^2+b^2+c^2} = \frac{\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}}{\cot A+\cot B+\cot C}$

5. If the solution of the equation $\sqrt{2+\sqrt{2+\sqrt{2+x}}} + \sqrt{3}\sqrt{2-\sqrt{2+\sqrt{2+x}}} = 2x$, where x > 0 is given by x = a cos(bπ/c) where argument of cosine function lies in [0, π/2), a, b, c $\in$ N and b, c are relatively prime, find the value of (a + b + c).

6. ABCD is a quadrilateral and E the point of intersection of the lines joining the middle points of opposite sides. Show that the resultant of $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ is equal to 4 $\overrightarrow{OE}$, where O is any point.

7. The point D, E, F divide the respective sides of ΔABC in the ratio as shown in figure. P is a point in AB which divides AB in the ratio 1 : 3 internally prove that $5\left(\overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF}\right) = 2\overrightarrow{CP}$



8. Find all the values of a for which the function $f(x) = (a^2 - 3a + 2)\cos\left(\frac{x}{2}\right) + (a-1)x$ possesses critical points.

DPP 41 TO 50 (ANSWER KEY)

DPP NO. - 41

1. (C) **2.** (B) **3.** (A) **4.** (A)

5. (B) **6.** $(-\infty, -4/3) \cup (2, \infty)$

7. 9/4 **8.** a = b = 3

DPP NO. - 42

1. (B) **2.** (A) **3.** (B)(C)

7. $(\pi/6) + (1/2)\,\ell n\, 3,\ (\pi/3) - (1/2)\,\ell n\, 3$

8. (A) $\rightarrow$ p, s ; (B) $\rightarrow$ q ; (C) $\rightarrow$ p, s ; (D) $\rightarrow$ r, s

DPP NO. - 43

1. (B) **2.** (B) **3.** $v\sqrt{(2-\sqrt{2})}$

6. $f(e) = 1,\ f'(2) = \frac{1}{2}$ **7.** $(y - m_1x)\,(y - m_2x) = c_1c_2$

8. $\frac{4\sqrt{3}}{9}$

DPP NO. - 44

1 (D) **2** (C) **3** (A) **4.** (D)

5. (C) **6.** 38 **8.** a = 1, b = 9 OR b = 1, a = 9

DPP NO. - 45

1. (C) **2.** (C) **3.** (C) **4.** (A)

5. (B)(D) **6.** (A)(C) **7.** a = –2, b = 0, c = 24, d > 32

8. $(-\infty, -1] \cup [10, \infty)$

DPP NO. - 46

1. (A) **2.** (B) **3.** (D) **4.** 20.

5. (C)(D) **6.** (D) **7.** (B)(D) **8.** $x^2 - 3x + 2 = 0$

9. (A) **10.** $x = 4n\pi,\ n \in I$

DPP NO. - 47

1. (i) False (ii) False (iii) True (iv) False

(v) False (vi) True

3. (B) **4.** (C) **5.** (C) **6.** n **7.** (B)

8. (A)

DPP NO. - 48

1. $\sqrt{\frac{(c+a-b)(a+b-c)}{2}}$ **7.** $\frac{1}{\theta} - 2\cot 2\theta,\ \frac{1}{\pi}$

8. $5x - 4y - 1 = 0$

DPP NO. - 49

1. (C) **2.** (A) **3.** (D) **4.** (B)

5. (C) **6.** (A)(B)(C)(D) **8.** 4

DPP NO. - 50

1. (D) **2.** (A)(B)(C) **3.** (C)(D) **5.** 37

8. $(-\infty, 0] \cup [4, \infty) \cup \{1\}$

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CLASS : XII (MATHS)

DPP

DAILY PRACTICE PROBLEM

Solutions

DPP- 41 to 50

DPP NO. - 41

1 to 3. $T_r = \frac{8r}{4r^4+1} = \frac{8r}{(4r^4+4r^2+1)-4r^2}$

$= \frac{1}{(2r^2+1)^2-(2r)^2} \Rightarrow \frac{1}{(2r^2-2r+1)(2r^2+2r+1)}$

$T_n = 2\left[\frac{1}{(2r^2-2r+1)} - \frac{1}{2r^2+2r+1}\right]$

$S_n = 2\left[\frac{1}{1} - \frac{1}{5}\right]$

$= 2\left[\frac{1}{5} - \frac{1}{13}\right]$

$\vdots \quad \vdots$

$= 2\left[\frac{1}{2n^2-2n+1} - \frac{1}{2n^2+2n+1}\right]$

$S_n = \left[1 - \frac{1}{2r^2-2r+1}\right]$

$S_\infty = 2$

$S_{16} = 2\left[1 - \frac{1}{512+32+1}\right] = \frac{1088}{545}$

$S_n = 2\left[\frac{2n^2+2n}{2n^2+2n+1}\right] = \frac{4r^2+4r}{2n^2+2n+1}$

a = 4, b = 4, c = 0 , d = 2, e = 2

4. y = 2 sin x + sin 2x

$\frac{dy}{dx}$ = 2 cos x + 2 cos 2x

at $x = \frac{\pi}{3} \Rightarrow \frac{dy}{dx} = 2.\frac{1}{2} + 2(-1/2) = 0$

point $\left(\frac{\pi}{3}, \frac{2\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) = \left(\frac{\pi}{3}, \frac{3\sqrt{3}}{2}\right)$

length = $\frac{\pi}{3}$, breath = $\frac{3\sqrt{3}}{2}$

Area of rectangle = $\frac{\sqrt{3}\pi}{2}$

5. $\frac{dy}{dx} = \left[\begin{array}{ll} 12x^2 - 2x + 2 & \text{for } 2 < x \le 3 \\ 12x^2 + 2x - 2 & \text{for } 0 \le x < 2 \end{array}\right.$ not derivable

at x = 2 & $\frac{dy}{dx}$ = 0

at x = 1/3, Decreasing in (0, 1/3) & Increasing for (1/3, 2) ∪ (2, 3)

⇒ Minima occurs at x = 1/3. f(1/3) = – 11/27

6. $f'(x) = 4a - 3 + (a-7)\cos x,\ x \ne 2n\pi$

If a = 7, f′(x) = 25 > 0

If $a \ne 7$, $f'(x) = (a-7)\left(\cos x - \frac{3-4a}{a-7}\right)$

$f'(x) > 0 \Rightarrow a - 7 > 0$ and $\cos x > \frac{3-4a}{a-7}$

or $a - 7 < 0$ and $\cos x < \frac{3-4a}{a-7}$

$\Rightarrow a > 7$ and $\frac{3-4a}{a-7} < -1$

or $a < 7$ and $\frac{3-4a}{a-7} > 1$

$\Rightarrow a > 7$ or $2 < a < 7$

$f'(x) < 0 \quad \Rightarrow a - 7 < 0$ and $\cos x < \frac{3-4a}{a-7}$

or $a - 7 > 0$ and $\cos x < \frac{3-4a}{a-7}$

$\Rightarrow a < -\frac{4}{3}$

Hence $a \in \left(-\infty, \frac{-4}{3}\right) \cup (2, 7) \cup \{7\} \cup (7, \infty)$

7. $f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$

f′(x) = –sin x – sin 2x + sin 3x

= 2 cos 2x sin x – 2 sin x cos x

= 2 sin x (cos 2x – cos x)

$f'(x) = 2\sin x\left(-2\sin\left(\frac{3x}{2}\right)\sin\frac{x}{2}\right)$

f′(x) = 0 (for maxima + minima)

$\sin x = 0,\ 0\sin\frac{3x}{2} = 0,\ \sin\left(\frac{x}{2}\right) = 0$

$\Rightarrow x = n\pi,\ \frac{3x}{2} = n\pi,\ \frac{x}{2} = n\pi$

$\Rightarrow x = n\pi,\ x = \frac{2n\pi}{3},\ x = 2n\pi$

$\Rightarrow 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi, ...$

$f(0) = 1 + \frac{1}{2} - \frac{1}{3} = \frac{7}{6}$

$f\left(\frac{2\pi}{3}\right) = \frac{-1}{2} - \frac{1}{4} - \frac{1}{3} = \frac{-13}{12}$ (minimum)

$f(\pi) = -1 + \frac{1}{2} + \frac{1}{3} = \frac{-1}{6}$ (maximum)

$f\left(\frac{4\pi}{3}\right) = \frac{-1}{2} - \frac{1}{4} - \frac{1}{3} = \frac{-13}{12}$ (minimum)

$f(2\pi) = f(0) = \frac{7}{6}$

Difference $= \frac{7}{6} - \left(\frac{-13}{12}\right) = \frac{27}{12} = \frac{9}{4}$

8. $f(x) = \frac{a}{x} + bx$

(1, 6) satisfy $6 = a + b$ ...(1)

and $f'(x) = \frac{-a}{x^2} + b = 0$ at (1, 6)

$\Rightarrow -a + b = 0 \Rightarrow a = 6 \qquad \Rightarrow \; a = b = 3$

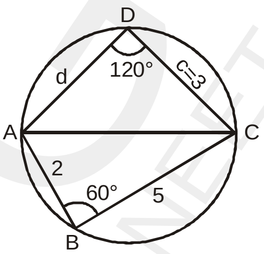
DPP NO. - 42

1. $\frac{s-a}{11} = \frac{s-b}{12} = \frac{s-c}{13} = \frac{3s-(a+b+c)}{11+12+13} = \frac{s}{36}$

Now $\tan^2\left(\frac{A}{2}\right) = \frac{(s-b)(s-c)}{s(s-a)} = \frac{(12)(13)}{(11)(36)} = \frac{13}{33}$

2. Let AB = 2 and BC = 5

$\angle ABC = 60°$ (given).



Since the quadrilateral is cyclic,

$\angle CDA = 180° - 60° = 120°$,

Let CD = c and DA = d

Also $AB^2 + BC^2 - 2AB \cdot BC \cos 60° = AC^2$

$= CD^2 + DA^2 - 2CD \cdot DA \cos 120°$

by cosine rule.

or $4 + 25 - 2.2.5.\frac{1}{2} = c^2 + d^2 + cd$

$19 = c^2 + d^2 + cd = 9 + d^2 + 3d$

$\therefore \; d^2 + 3d - 10 = 0$

or $(d + 5)(d - 2) = 0 \qquad \therefore d = 2$

3. $y = \cos^{-1}\sqrt{\frac{\sqrt{1+x^2}+1}{2\sqrt{1+x^2}}}$

$\frac{dy}{dx} = \frac{-1}{\sqrt{1-\left(\frac{1}{2}+\frac{1}{2\sqrt{1+x^2}}\right)}} \cdot \frac{1}{2\sqrt{\frac{1}{2}+\frac{1}{2\sqrt{1+x^2}}}} \cdot \frac{1}{2}$

$\frac{2x}{(-2)(1+x^2)^{3/2}}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{\frac{1}{4}-\frac{1}{4(1+x^2)}}} \cdot \frac{x}{4(1+x^2)^{3/2}}$

$\Rightarrow \frac{dy}{dx} = \frac{2\sqrt{1+x^2}}{\sqrt{1+x^2-1}} \cdot \frac{x}{4\sqrt{1+x^2}(1+x^2)}$

$\Rightarrow \frac{dy}{dx} = \frac{x}{2|x|(1+x^2)}$

when $x < 0$

$\frac{dy}{dx} = \frac{-1}{2(1+x^2)}$

when $x > 0$

$\frac{dy}{dx} = \frac{1}{2(1+x^2)}$

Alternate :

put $x = \tan\theta$

$\tan^{-1}x = \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$y = \cos^{-1}\sqrt{\frac{1+|\sec\theta|}{2|\sec\theta|}} = \cos^{-1}\sqrt{\frac{\cos\theta+1}{2}}$

$y = \cos^{-1}(\cos\theta/2)$

$y = \begin{cases} -\theta/2 \; , & -\pi/2 < \theta \le 0 \\ \theta/2 \; , & 0 < \theta < \pi/2 \end{cases}$

$\frac{dy}{dx} = \begin{cases} -\frac{1}{2(1+x^2)} \; , & x \le 0 \\ \frac{1}{2(1+x^2)} \; , & x > 0 \end{cases}$

4. $\cos A + 2\cos B + \cos C = 2$

$2\cos\left(\frac{A+C}{2}\right).\cos\left(\frac{A-C}{2}\right) = 4\sin^2 B/2$

$\cos\left(\frac{A+C}{2}\right) = 2\sin\frac{B}{2}$

$\Rightarrow \cos\left(\frac{A-C}{2}\right) = 2\cos\left(\frac{A+C}{2}\right)$

$\cot\frac{A}{2}.\cot\frac{C}{2} = 3$

$\sqrt{\frac{s(s-a)}{(s-b)(s-c)}} \cdot \sqrt{\frac{s(s-c)}{(s-a)(s-b)}} = 3$

on solving

$\Rightarrow$ a + c = 2b

5. Put $x = \sin^2\theta$, $y = \cos^2\theta$

$\left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right) = (1+\text{cosec}^2\,\theta)(1+\sec^2\theta)$

$= (2+\cot^2\theta)(2+\tan^2\theta) = 4 + 2(\cot^2\theta + \tan^2\theta) + 1$

$= 5 + 2(\tan^2\theta + \cot^2\theta) \geq 9.$

(Since A.M. $\geq$ G.M. $\tan^2\theta + \cot^2\theta \geq 2$)

6. **(i)** Let (i) $f(x) = \frac{x}{1+x} - \ell n(1+x)$

$f'(x) = \frac{(1-x)-x}{(1+x)^2} - \frac{1}{(1+x)} = \frac{1}{(1+x)^2} - \frac{1}{(1+x)}$

$= \frac{1-1-x}{(1+x)^2} = \frac{-x}{(x+1)^2}$

for $x > 0$

$f'(x)$ is negative $\Rightarrow$ f(x) is decreasing

$f(x) < f(0) \quad \Rightarrow \quad \frac{x}{1+x} - \ell n(1+x) < 0$

$\Rightarrow \frac{x}{1+x} < \ell n(1+x)$ ......(i)

also let $g(x) = \ell n(1+x) - x$

$g'(x) = \frac{1}{1+x} - 1 = \frac{-x}{1+x}$

for $x > 0$, $g'(x) < 0 \quad \Rightarrow \quad g(x) < g(0)$

$\ell n(1+x) - x < 0 \quad \Rightarrow \quad \ell n(1+x) < x$ ......(ii)

from (1) and (2)

$\frac{x}{1+x} < \ell n(1+x) < x$

(ii) Let $f(x) = 2x - 3\sin x + x\cos x$

$\Rightarrow f'(x) = 2 - 3\cos x + \cos x - x\sin x$

$= 2 - 2\cos x - x\sin x$

get $g(x) = 2 - 2\cos x - x\sin x$

$g'(x) = 0 + 2\sin x - \sin x - x\cos x = \sin x - x\cos x$

also $h(x) = \sin x - x\cos x$

$h'(x) = \cos x - \cos x + x\sin x = x\sin x$

for $0 < x < \frac{\pi}{2} \quad h'(x) > 0$

$h(x) > h(0)$

$\Rightarrow \sin x - x\cos x > 0 \quad \Rightarrow$ g□′(x) >0

$\Rightarrow$ g(x) is increasing fuction

$\Rightarrow g(x) > g(0) \quad \Rightarrow \quad 2 - 2\cos x - x\sin x > 0$

7. $f'(x) = \frac{1}{\sqrt{1-\frac{x^2}{(x^2+1)}}}\left(\frac{(\sqrt{x^2+1}).1 - \frac{x}{2\sqrt{x^2+1}}.2x}{x^2+1}\right) - \frac{1}{x}$

$= \frac{1}{1+x^2} - \frac{1}{x}$

$= \frac{x-1-x^2}{x(1+x^2)}$

Let $g(x) = x - 1 - x^2$

$g'(x) = 1 - 2x$

for $x \in \left[\frac{1}{\sqrt{3}}, \sqrt{3}\right]$

$g'(x) < 0 \Rightarrow f(x)$ decreasing function

maximum f(x) is $f\left(\frac{1}{\sqrt{3}}\right) = \sin\left(\frac{1}{2}\right) - \ell n\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{\pi}{6} + \frac{1}{2}\ell n3$

minimum f(x) is $= \sin\left(\frac{\sqrt{3}}{2}\right) - \ell n(\sqrt{3})$

$= \frac{\pi}{3} - \frac{1}{2}\ell n3$

8. **(A)** x = 1, is common roots and other roots are a and b.

so cubic equation has three real roots 1, a, b.

(B) $f'(x) = 3x^2 + 2px + q$

$= 3\left(x^2 + \frac{2p}{3}x\right) + q$

$= 3\left(\left(x+\frac{p}{3}\right)^2 - \frac{p^2}{9}\right) + q$

$= 3\left(x+\frac{p}{3}\right)^2 + q - \frac{p^2}{3}$

If $q - \frac{p^2}{3} > 0 \Rightarrow 3q > p^2 \Rightarrow f'(x) > 0$ Monotonic

(C) $f'(x) = 3x^2 + 2px + p - 1$ and $p = q + 1$

Now $D = 4p^2 - 4(3)(p-1)$

$= 4(p^2 - 3p + 3) = 4\left(\left(p-\frac{3}{2}\right)^2 + 3 - \frac{5}{4}\right)$

$= 4\left(p - \frac{3}{2}\right)^2 + 3 \Rightarrow$ positive

$\Rightarrow$ f′(x) has two real roots.

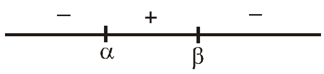
(D) $d(x) = (x^3 + px + qx + 6) - (2x^3 + 2qx)$

$= -x^3 - px^2 - qx + 6$

$Q'(x) = -(3x^2 + 2px + q)$

Here $D = (2p)^2 - 4(3)q > 0$

$\Rightarrow p^2 > 3p \Rightarrow p^2 > 3q$

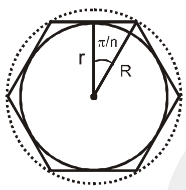


Q′(x) has two real roots

One maxima & minima.

DPP NO. - 43

1. $\frac{r}{R} = \cos\left(\frac{\pi}{n}\right)$



Let $\cos\frac{\pi}{n} = \frac{2}{3}$ for some $n \geq 3, n \in N$

As $\frac{1}{2} < \frac{2}{3} < \frac{1}{\sqrt{2}} \Rightarrow \cos\frac{\pi}{3} < \cos\frac{\pi}{n} < \cos\frac{\pi}{4}$

$\Rightarrow \frac{\pi}{3} > \frac{\pi}{n} > \frac{\pi}{4}$

$\Rightarrow 3 < n < 4$, which is not possible

so option (2) is the false statement

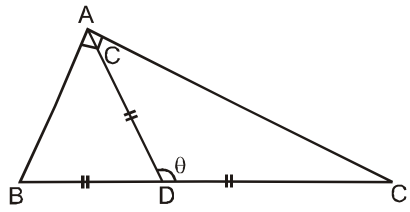
so it will be the right choice

Hence correct option is (2)

2. $\frac{r_1}{6} = \frac{r_2}{3} = \frac{r_3}{2} = k$

$\frac{a}{5} = \frac{b}{4} = \frac{c}{3} = k$

Right angle Δ $\cos C = \frac{4}{5}$



Now $\cos \Delta ADC = \cos(2\pi - 2C)$

$= -\cos 2C$

$= 1 - 2\cos^2 C$

$= 1 - 2\cos^2 C =$

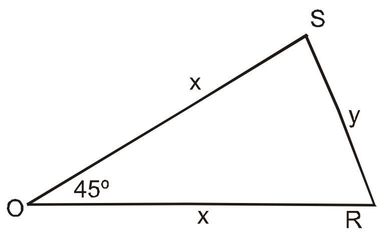
$= 1 - 2\left(\frac{16}{25}\right) = \frac{-7}{25}$

3. OR = OS = x (say)

$\frac{dx}{dt} = v$

SR = y

$y^2 = x^2 + x^2 - 2x.x \cos 45^o$



$= 2x^2 - x^2\sqrt{2}$

$\therefore\ y = x\sqrt{(2-\sqrt{2})}$

$\therefore\ \frac{dy}{dt} = \sqrt{(2-\sqrt{2})}\,\frac{dx}{dt} = v\sqrt{(2-\sqrt{2})}$

4. D is the mid-point of BC

BD = DC = a/2

$\angle DAC = \angle BEC = 90^o$

$\angle BEC = \angle DAC = 90^o$

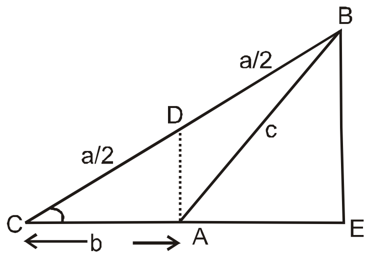
Hence $\angle CDA = \angle CBE = 90 - C$

similar Δ ADC and Δ EBC

$\frac{DC}{BC} = \frac{DA}{BE} = \frac{CA}{CE} \Rightarrow EA = b$

and if DA = y, then BE = 2y

$\cos C = \frac{b}{a/2} = \frac{2b}{a}$ and $\cos(\pi - A) = \frac{b}{c}$



$\cos A = -\frac{b}{c}$

$\cos A.\cos C = -\frac{2b^2}{ac}$ ...(i)

In ΔADC, $y^2 + b^2 = \frac{a^2}{4} \Rightarrow 4y^2 + 4b^2 = a^2$

If ΔBAE, $4y^2 + b^2 = c^2 \Rightarrow c^2 + 3b^2 = a^2$

$\Rightarrow b^2 = \frac{a^2 - c^2}{3}$ ...(ii)

from (i) and (ii) $\cos A . \cos C = \frac{2(c^2 - a^2)}{3ac}$

5. Here the quadratic for third side b is given by

$b^2 - 2bc\cos A + (c^2 - a^2) = 0$

$\therefore b_1 + b_2 = 2c\cos A$

and $b_1 b_2 = c^2 - a^2$

Also it is given that

$b_2 = 2b_1$

Hence from (1) and (3),

$3b_1 = 2c\cos A$

and from (2) and (3),

$2b_1^2 = c^2 - a^2$

Finally from (4) and (5), we have

$$2 . \frac{4c^2\cos^2 A}{9} = c^2 - a^2$$

or $8c^2(1 - \sin^2 A) = 9c^2 - 9a^2$

or $9a^2 = c^2(1 + 8\sin^2 A)$

Hence $3a = c\sqrt{1 + 8\sin^2 A}$

6. Given $2f(x) = f(xy) + f\left(\frac{x}{y}\right)$ ...(1)

Replacing x by y and y by x in (1),

then $2f(y) = f(xy) + f\left(\frac{x}{y}\right)$ ...(2)

Subtract (2) from (1),

we get $2\{f(x) - f(y)\} = f\left(\frac{x}{y}\right) - f\left(\frac{y}{x}\right)$ ...(3)

Putting x = 1 in (1) then $2f(1) = f(y) + f\left(\frac{1}{y}\right) = 0$

$(\because f(1) = 0)$

$\therefore f(y) = -f\left(\frac{1}{y}\right)$

$\therefore f\left(\frac{y}{x}\right) = -f\left(\frac{x}{y}\right)$ ...(4)

Now from (3) and (4), we get

$$2\{f(x) - f(y)\} = 2f\left(\frac{x}{y}\right)$$

or $f(x) - f(y) = f\left(\frac{x}{y}\right)$ ...(5)

Now, $f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0}\frac{f\left(1+\frac{h}{x}\right)}{h} \quad \{\text{from (5)}\}$$

$$= \lim_{h\to 0}\frac{f\left(1+\frac{h}{x}\right)}{\frac{h}{x}\bullet x} = \frac{1}{x}f'(1) = \frac{1}{x} \quad [\because f'(1) = 1]$$

$\therefore f'(x) = \frac{1}{x} \Rightarrow f'(2) = \frac{1}{2}$

and $f(x) = \ell n\, x + \ell n\, c$ for x = 1, and $f(1) = \ell n\, 1 + \ell n\, c$

$\Rightarrow 0 = 0 + \ell n\, c \qquad \therefore \ell n\, c = 0$

then $f(x) = \ell n\, x \qquad \therefore f(e) = \ell n\, e = 1$

7. $(OP)^2 = (OQ)(OR)$

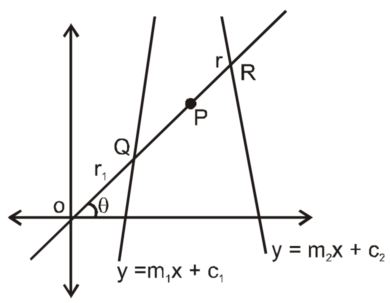
Let $OQ = r_1$

$OP = r$, $OR = r_2$

$r_1\cos\theta = m_1 r_1\sin\theta + c_1$

$\Rightarrow r_1\cos\theta = m_1 r_1\sin\theta + c_1$

$\Rightarrow r_1 = \frac{c_1}{(\cos\theta - m_1\sin\theta)}$ and $r_2 = \frac{c_2}{(\cos\theta - m_2\sin\theta)}$



$$r^2 = \frac{c_1 c_2}{(\cos\theta - m_1\sin\theta)(\cos\theta - m_2\sin\theta)}$$

$\Rightarrow (r\cos\theta - m_1 r\sin\theta)(r\cos\theta - m_2 r\sin\theta) = c_1c_2$

Put $y = r\cos\theta$, $x = r\sin\theta$

Locus $(y - m_1x)(y - m_2x) = c_1c_2$.

8. $r^2 - x^2 = 1^2 - (1 - x)^2$

$r^2 = 2x$

$\Delta = \frac{1}{2} r\sqrt{r^2 - x^2}$

$\Delta' = 4\Delta^2 = r^2 (r^2 - x^2)$

$= (2x)(2x - x^2)$

$= 4x^2 - 2x^3$

$\frac{d(\Delta')}{dx} = 8x - 6x^2 = 0$

$\Rightarrow x = \frac{4}{3}$ $\quad \therefore r^2 = \frac{8}{3}$

$\Delta = \frac{1}{2}\frac{\sqrt{8}}{3}\sqrt{\frac{8}{3} - \frac{16}{9}}$

$= \frac{4\sqrt{3}}{9}$

DPP NO. - 44

1 to 3. a b c

f I I + f

f fr fr²

$I^2 = f(I + f) \Rightarrow f^2 + If - I^2 = 0$

$f = \left(\frac{\sqrt{5}-1}{2}\right) I \Rightarrow R = \frac{I}{f} = \frac{\sqrt{5}+1}{2}$

$0 \le f < 1 \Rightarrow 0 \le I\, \frac{\sqrt{5}-1}{2} < 1$

$I = 1$

$x = I + f$

$= 1 + \left(\frac{\sqrt{5}-1}{2}\right) = \frac{\sqrt{5}+1}{2}$ Ans.

4. a, a+d , a + 6d

$(a + d)^2 = a(a + 6d)$

$d^2 + 2ad = 6ad$

$d^2 = 4ad$

$d = 4a$

$3a + 7d = 93$

$a = 3$

$d = 12$

3 , 15 , 75

$ar^3 \Rightarrow 3(5)^3 = 375$

5. $\sum_{r=1}^{n} t_r = \frac{1}{12}(n)(n + 1)(n + 2) = S_n$

$t_n = S_n - S_{n-1}$

$= \frac{1}{12}[n(n + 1)(n + 2) - (n - 1) n (n + 1)]$

$= \frac{1}{12} n(n + 1)[n + 2 - n + 1]$

$\Rightarrow t_n = \frac{n(n+1)}{4}$

So $\frac{1}{t_n} = \frac{4}{n(n+1)} \Rightarrow \frac{1}{t_n} = 4\left[\frac{1}{n} - \frac{1}{n+1}\right]$

So $\sum_{r=1}^{n} \frac{1}{t_r} = \sum_{r=1}^{n} 4\left[\frac{1}{r} - \frac{1}{r+1}\right]$

$= 4\left[1 - \frac{1}{n+1}\right]$

$= \frac{4n}{n+1}$ Ans.

6. $\frac{a\frac{(r^{11}-1)}{r-1}}{ar^{n-11}\frac{(r^{11}-1)}{r-1}} = \frac{1}{8} \Rightarrow r^{n-11} = 8$ .... (i)

and $\frac{\frac{ar^9(r^{n-9}-1)}{r-1)}}{\frac{a(r^{n-9}-1)}{r-1}} = 2 \Rightarrow r^9 = 2$

by equation (i)

$(r^9)^{n-11} = 8^9 = 2^{27}$

$2^{n-11} = 2^{27} \Rightarrow n = 38$ Ans.

7. $\frac{1 + r + r^2 + \ldots\ldots r^{2m}}{2m+1} > (1.r.r^2\ldots\ldots r^{2m})^{\frac{1}{2m+1}}$

$\frac{1 - r^{2m+1}}{1 - r} > (2m + 1) . \left(r^{\frac{2m(2m+1)}{2}}\right)^{\frac{1}{2m+1}}$

$(2m + 1) r^m (1 - r) < 1 - r^{2m+1}$ H.P.

8. $x + y + z = 15$ ....(i)

a, x, y, z, b are in AP

Suppose d is common difference $d = \frac{b-a}{4}$

$\therefore x = a + \frac{b-a}{4} = \frac{b+3a}{4}$, $y = \frac{2b+2a}{4}$

and $z = \frac{3b+a}{4}$

on substituting the values of X, Y and Z in (i), we get

$\Rightarrow \frac{6a+6b}{4} = 15$

$\Rightarrow a + b = 10$ ..... (ii)

$\because \quad \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{5}{3}$ ....(iii)

and a, x, y, z, b are in H.P.

$\therefore \quad \frac{1}{a}, \frac{1}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{b}$ are in A.P.

$\therefore \quad \frac{1}{x}=\frac{1}{a}+\frac{\left(\frac{1}{b}-\frac{1}{a}\right)}{4}, \quad \frac{1}{y}=\frac{1}{a}+\frac{2}{4}\left(\frac{1}{b}-\frac{1}{a}\right)$ and

$\frac{1}{z}=\frac{1}{a}+\frac{3}{4}\left(\frac{1}{b}-\frac{1}{a}\right)$

on substituting the value of $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ in (iii), we get

$\frac{3}{a}+\frac{6}{4}\left(\frac{1}{b}-\frac{1}{a}\right)=\frac{5}{3}$

$\Rightarrow \quad \frac{3}{2a}+\frac{3}{2b}=\frac{5}{3}$

$\frac{1}{a}+\frac{1}{b}=\frac{10}{9}$ ..... (iv)

By equations (ii) & (iv), we get

a = 9, b = 1 or a = 1, b = 9

DPP NO. - 45

1. $\tan\frac{C}{2}=\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

$\frac{7}{9}=\frac{(s-a)(s-b)}{s(s-c)}$

$s=\frac{a+b+c}{2}$

$s=\frac{a+c}{2}$

$\frac{7}{9}=\frac{c^2-1}{81-c^2}$

$\Rightarrow$ $16c^2 = 24a^2$

$c=\frac{24}{4}=6$

3. $r_1=\frac{\Delta}{3-a}$

$\frac{\Delta}{(s-a)(s-b)(s-c)}+\frac{\Delta}{(s-a)(s-b)(s-c)}$

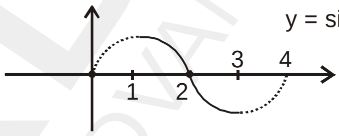
$+\frac{\Delta}{(s-a)(s-b)(s-c)}$

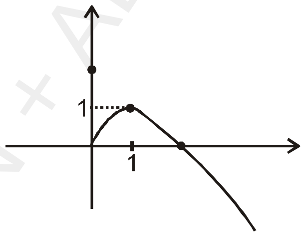
Multiply by s divide by s

$\frac{s}{\Delta}+\frac{s}{\Delta}+\frac{s}{\Delta}=\frac{3}{r}$

4. $f(x)=\begin{cases}\sin\frac{\pi x}{2} & , 0\le x<1 \\ 3-2x & , \quad x\ge 1\end{cases}$

$T=\frac{2\pi}{\pi}x^2=4$





5. a + b = 3c cos A + cosB

sin A + sin B = 3 sin C

$2\sin\frac{A+B}{2}\cos\frac{A-B}{2}==6\sin^2\frac{C}{2}$

$2\sin\frac{C}{2}\cos\frac{A-B}{2}==6\sin^2\frac{C}{2}$

$\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)=6\sin^2\frac{C}{2}$

$=\cos A+\cos B=6\sin^2\frac{C}{2}$

7. Since f(x) is minimum at x = – 2 and maximum at x = 2,

let $g(x) = ax^3 + bx^2 + cx + d$

$\therefore$ g(x) is also minimum at x = – 2 and maximum at x = 2

$\therefore$ a < 0

a is root of $x^2 - x - 6 = 0$ $\Rightarrow$ a = 3, – 2

$\therefore$ a = – 2

then $g(x) = -2x^3 + bx^2 + cx + d$

$g'(x) = -6x^2 + 2bx + c = -6(x + 2)(x - 2)$

on comparing we get b = 0, c = 24

Since minimum and maximum values are positive

$\therefore$ g(–2) < 0 $\Rightarrow$ 16 – 48 + d > 0 $\Rightarrow$ d > 32

and g(2) > 0 $\Rightarrow$ – 16 + 48 + d > 0

$\Rightarrow$ d> – 32

$\Rightarrow$ d > 32

Hence a = –2, b = 0, c = 24, d > 32

8. $\lim_{x\to 2^-} - f(x) \geq f(2)$

$\lim_{h\to 0} f(2-h) \geq f(2)$

$\Rightarrow \lim_{h\to 0} (\,|2-h-2| + a^2 - 9a - 9) \geq 1$

$a^2 - 9a - 10 \geq 0$

$(a + 1)(a - 10) \geq 0$

$\Rightarrow a \leq -1$ or $a \geq 10$

$(-\infty, -1] \cup [10, \infty)$

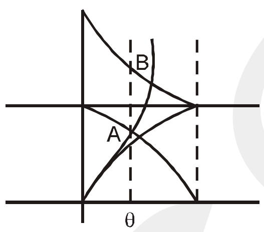
DPP NO. - 46

1. tan x = C9x

tan nq = cos θ $\Rightarrow \frac{\sin\theta}{\cos\theta} = \cos\theta$

AB = cosec θ – cos θ

$= \frac{1}{\sin\theta}$



$\sin\theta = 1 - \sin^2 b$

$\sin^2\theta + \sin\theta - 1 = 0$

$\sin\theta = \frac{-1 \pm \sqrt{1+4}}{2}$

$\sin\theta = \frac{\sqrt{5}-1}{2}$, $\cos\theta = \frac{\sqrt{5}+1}{2}$

$AB = \frac{2}{\sqrt{5}+1} - \frac{2}{\sqrt{5}+1} = \frac{4}{4} = 1$

3. $\tan^2$

$\frac{\pi}{16} + \tan^2\frac{2\pi}{16} + \tan^2\frac{3\pi}{16} + \tan^2\frac{4\pi}{16}$

$+ \tan^2\frac{5\pi}{16} + \tan^2\frac{6\pi}{16} + \tan^2\frac{7\pi}{16}$

$= \left(\tan^2\frac{\pi}{16} + \cot^2\frac{\pi}{16}\right)$

$+ \left(\tan^2\frac{2\pi}{16} + \cot^2\frac{2\pi}{16}\right) + \left(\tan^2\frac{3\pi}{16} + \cot^2\frac{3\pi}{16}\right) + \tan^2\frac{\pi}{4}$

Let $\frac{\pi}{16} = \theta$

then solve $\tan^2\theta + \cot^2\theta = \frac{\sin^4 + \cos^4\theta}{\sin^2\theta\cos^2\theta}$

$= \frac{2(3+\cos 4\theta)}{1-\cos 4\theta}$

now put $\theta = \frac{\pi}{16}$

and solve

$2\left[\left(\frac{3+\cos\frac{\pi}{4}}{1-\cos\frac{\pi}{4}}\right) + \frac{3+\cos\frac{\pi}{2}}{1-\cos\frac{\pi}{2}} + \frac{3+\cos\frac{3\pi}{4}}{1-\cos\frac{3\pi}{4}}\right] + 1$

= 35 Ans.

4. Let $x^2 + 18x + 30 = t$

$t = 2\sqrt{t+15}$

square $t^2 - 4t - 60 = 0 \Rightarrow t = 10, -6\ (t \geq 0)$

$t = 10 = x^2 + 18x + 30$

$x^2 + 18x + 20 = 0$

product of roots = 20 Ans.

5. $\sec^2\alpha + \cos\sec^2\alpha$

$\frac{1}{\cos^2\alpha} + \frac{1}{\sin^2\alpha} = \frac{1}{\cos\alpha.\sin\alpha} = \frac{4}{(\sin 2\theta)^2}$

sum of root [4, α)

propduct of root

$= \frac{4}{(\sin 2\alpha)^2} \geq 4$

both sum and product of rotate equal

7. $x^2 = t$

$at^2 + bt + c = 0$

$t = \frac{-b \pm (+ve)}{a}$

$t = \frac{-b + (+ve)}{a} > 0$

$$\frac{-b-(+ve)}{a} > 0$$

9. $f(x) = \dfrac{1-\sin 2x+\cos 2x}{2\cos 2x} = \dfrac{1}{2}\left[\dfrac{(\cos x-\sin x)^2}{\cos^2 x-\sin^2 x}+1\right]$

$= \dfrac{1}{2}\,[\tan(45° - x) + 1]$

$f(16°) = \dfrac{1}{2}\,[\tan 29° + 1]$

$f(29°) = \dfrac{1}{2}\,[\tan 16° + 1]$

$\Rightarrow f(16°)\,f(29°) = \dfrac{1}{4}\,[1 + \tan 16°]\,[1 + \tan 29°]$

$= \dfrac{1}{4}\,(2) = \dfrac{1}{2}$

10. $\left(4\sqrt{\cos\dfrac{x}{2}}-5-\dfrac{\sqrt{2}}{2}\right)^2 + \sqrt{2}\left(4\sqrt{\cos\dfrac{x}{2}}-5-\dfrac{\sqrt{2}}{2}\right) - \dfrac{\cos x}{2} = 0$

let $t = 4\sqrt{\cos\dfrac{x}{2}} - 5 - \dfrac{1}{\sqrt{2}}$, $t < 0$.

$t^2 + \sqrt{2}\,t = \dfrac{\cos x}{2}$

$\left(t+\dfrac{1}{\sqrt{2}}\right)^2 = \dfrac{1+\cos x}{2} = \dfrac{2\cos^2 x/2}{2}$

$t + \dfrac{1}{\sqrt{2}} = -\cos\dfrac{x}{2}$

$4\sqrt{\cos\dfrac{x}{2}} - 5 + \dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}} = -\cos\dfrac{x}{2}$

let $\sqrt{\cos\dfrac{x}{2}} = y,\ y \ge 0$

$y^2 + 4y = 5$

$\Rightarrow (y+2)^2 = 3^2$ $\qquad \Rightarrow y + 2 = 3$

$\Rightarrow y = 1$ $\qquad \Rightarrow \sqrt{\cos\dfrac{x}{2}} = 1$

$\Rightarrow \cos\dfrac{x}{2} = 1$ $\qquad \Rightarrow \dfrac{x}{2} = 2n\pi$

$\Rightarrow x = 4n\pi,\ n \in I$

DPP NO. - 47

2. Here domain x > 0 as log x exists when x > 0

Then $f'(x) = \dfrac{\cos(\log_e x)+\sin(\log_e x)}{x}$

or $f'(x) = \dfrac{\sqrt{2}\left\{\dfrac{1}{\sqrt{2}}\cos(\log x)+\dfrac{1}{\sqrt{2}}\sin(\log x)\right\}}{x}$

or $f'(x) = \dfrac{\sqrt{2}\left\{\sin\dfrac{\pi}{4}\cos(\log x)+\cos\dfrac{\pi}{4}\sin(\log x)\right\}}{x}$

or $f'(x) = \dfrac{\sqrt{2}\left\{\sin\left(\dfrac{\pi}{4}+\log x\right)\right\}}{x}$

as domain x > 0

$\therefore$ f(x) strictily increases when $f'(x) \ge 0$ i.e.,

$$\sin\left(\frac{\pi}{4}+\log x\right) \ge 0$$

i.e., $(2n)\pi \le \dfrac{\pi}{4} + \log x \le (2n+1)\pi$

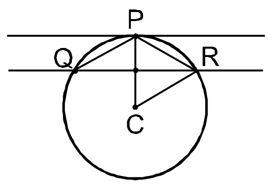
$\Rightarrow 2n\pi - \dfrac{\pi}{4} \le \log x \le 2n\pi + \pi - \dfrac{\pi}{4}$

$\Rightarrow e^{2n\pi-\frac{\pi}{4}} \le x \le e^{2n\pi+\frac{3\pi}{4}}$

Therefore, f(x) is strictly increasing when,

$$x \in \left[e^{2n\pi-\frac{\pi}{4}},\ e^{2n\pi+\frac{3\pi}{4}}\right]$$

3.



$CM = r\cos\theta$

$D = \dfrac{1}{2}\,(r - r\cos\theta)\,(2r\sin\theta) = r^2\,(1-\cos\theta)\sin\theta$

$\dfrac{d\Delta}{d\theta} = r^2\,(\,(1-\cos\theta)\cos\theta + (\sin\theta)\sin\theta)$

$\dfrac{d\Delta}{d\theta} = r^2\,(\cos\theta - \cos^2\theta + \sin^2\theta) = 0$

$\cos\theta = \cos 2\theta$

$\theta = 2n\pi \pm 2\theta$

$3\theta = 2\pi$

$\theta = \dfrac{2\pi}{3}$

$\Delta = r^2\left(1-\cos\dfrac{2\pi}{3}\right)\sin\left(\dfrac{2\pi}{3}\right) = r^2\left(1+\dfrac{1}{2}\right)\left(\dfrac{2\pi}{3}\right)$

$= \frac{3\sqrt{3}}{4}r^2$

4. $m(\vec{a}.\vec{b}) = 120 \quad \Rightarrow \quad m(2 - 21 - 5) = 120$

$m = \frac{120}{-24} = -5$

5. $y = x^n, (a, a^n)$

$\frac{dy}{dx} = n.x^{n-1}$ slope of tangent $= n\,a^{n-1}$

Equation of normal

$y - a^n = \frac{-1}{na^{n-1}}(x - a)$

at x = 0

$y = b = a^n + \frac{-1}{n.a^{n-2}}$

$\lim_{a\to 0} b = \lim_{a\to 0}\left(a^n + \frac{1}{n.a^{n-2}}\right) = \frac{1}{2}$

If n = 2 we get

$b = 0 + \frac{1}{2}$

6. $\Rightarrow \vec{P} = \vec{Q} \quad \Rightarrow \quad \sin x\,\hat{i} + \cos x\,\hat{j} = -\hat{i} - \cos x\,\hat{j}$

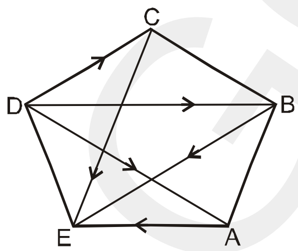
$\sin x = -1$ and $\cos x = -\cos x$ and $\cos x = 0$

$\Rightarrow x = \frac{3\pi}{2}$

only one value in $(0, 2\pi)$

therefore n values in $(0, 2n\pi)$, $x \in N$

7.



$(\overrightarrow{DA} + \overrightarrow{AE}) + (\overrightarrow{DB} + \overrightarrow{BE}) + (\overrightarrow{DE} + \overrightarrow{CE})$

$= \overrightarrow{DE} + \overrightarrow{DE} + \overrightarrow{DE}$

$= 3\overrightarrow{DE}$

8. $(x - 2)\vec{a} + \vec{b} = \lambda((2x + 1)\vec{a} - \vec{b})$

$\Rightarrow x - 2 = \lambda\,(2x+1) \qquad$ or $\qquad 1 = -\lambda$

$\Rightarrow x - 2 = -1\,(2x+1)$

$3x = 1$

$x = \frac{1}{3}$

DPP NO. - 48

1. $\cos A = \frac{x^2 + y^2 - \ell^2}{2xy}$

$\Rightarrow \ell^2 = x^2 + y^2 - 2xy\cos A$

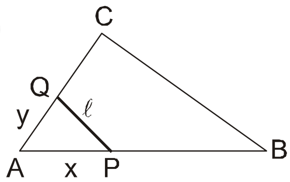
and $\frac{1}{2}xy\sin A = \frac{1}{2}\left(\frac{1}{2}bc\sin A\right)$

$\Rightarrow 2xy = bc$

$\frac{1}{2}(AP)(PQ) = \frac{1}{2} \times \frac{1}{2}bc\sin A$

$\frac{1}{2}\left(x\cos A + \sqrt{y^2 - x^2\sin^2 A}\right).\,x\sin A = \frac{1}{2}bc\sin A$

$\Rightarrow y^2 = \left(\frac{bc}{2x} - x\cos A\right)^2 + x^2\sin^2 A$



$\Rightarrow \ell = y^2 = \frac{b^2c^2}{4x^2} + x^2 - bc\cos A$

for minimum $\frac{d\ell}{dx} = 0 \quad \Rightarrow \quad x^2 = \frac{bc}{2}$

$\Rightarrow y^2 = bc\,(1 - \cos A)$

$= bc\left(1 - \frac{b^2 + c^2 - a^2}{2bc}\right)$

$\Rightarrow y = \sqrt{\frac{(c + a - b)(a + b - c)}{2}}$

2. Let $\phi(x) = \frac{f(x)}{g(x)}$

= both are continuous and differentiable in (a,b)

and $\frac{f(a)}{g(a)} = \frac{f(b)}{g(b)} \Rightarrow \phi(a) = \phi(b)$

then by Rolle's theorem g'(c)=0

$\frac{g(c)f'(c) - g'(c)f(c)}{(g(c))^2} = 0$

$\frac{f(c)}{g(c)} = \frac{f'(c)}{g'(c)}$

3. $\vec{A} = a\hat{i} + b\hat{j} + c\hat{k} \quad \Rightarrow \quad |\vec{A}| = \sqrt{a^2 + b^2 + c^2}$

$\vec{B} = d\hat{i} + e\hat{j} + f\hat{k} \quad \Rightarrow \quad |\vec{B}| = \sqrt{d^2 + e^2 + f^2}$

Here $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|$

$|\vec{A}||\vec{B}| \cos\theta = |\vec{A}||\vec{B}|$

$\Rightarrow \cos\theta = 1$

$\vec{A}$ and $\vec{B}$ are collinear $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$

$= \frac{a+b+c}{d+e+f} = \frac{\sqrt{a^2+b^2+c^2}}{\sqrt{d^2+e^2+f^2}}$

4. $f(x) = \left(1 + \frac{1}{x}\right)^x$

$f'(x) = \left(1+\frac{1}{x}\right)^x \left[1.\ell n\left(1+\frac{1}{x}\right) + x.\frac{1}{\left(1+\frac{1}{x}\right)}.\left(\frac{-1}{x^2}\right)\right]$

$= \left(1+\frac{1}{x}\right)^x \left[\ell n\left(1+\frac{1}{x}\right) - \frac{1}{x+1}\right]$

Let $g(x) = \ell n\left(1+\frac{1}{x}\right) - \frac{1}{x+1}$

$\Rightarrow g'(x) = \frac{1}{1+\frac{1}{x}}\left(\frac{-1}{x^2}\right) - \frac{1}{(x+1)^2}$

$= \frac{-(x+1)-x}{x(x+1)^2} = \frac{-(2x+1)}{x(x+1)^2}$

5. $\sec^2 A = 1 + \tan^2 A = \frac{8}{5}$

$\therefore \cos A = \sqrt{\left(\frac{5}{8}\right)}$

or $b^2 + c^2 - a^2 = 2bc.\frac{\sqrt{5}}{2\sqrt{2}}$. put $a = \frac{2b}{3}$

$\therefore b^2 + c^2 - \frac{4b^2}{9} = bc\frac{\sqrt{5}}{\sqrt{2}}$

$c^2 - bc\sqrt{\left(\frac{5}{2}\right)} + \frac{5b^2}{9} = 0$

$\therefore c_1 + c_2 = b\sqrt{\left(\frac{5}{2}\right)}, \ c_1c_2 = \frac{5b^2}{9}$

$\therefore (c_1 - c_2)^2 = (c_1 + c_2)^2 - 4c_1c_2$

$= \frac{5b^2}{2} - \frac{20}{9}b^2 = \frac{5}{18}b^2$

$\therefore c_1 - c_2 = \frac{b}{3}\sqrt{\left(\frac{5}{2}\right)}$

$\therefore c_1 = \frac{2b}{3}\sqrt{\left(\frac{5}{2}\right)}, \ c_2 = \frac{b}{3}\sqrt{\left(\frac{5}{2}\right)}$

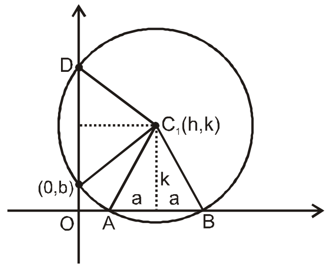
$\therefore c_1 = 2c_2$

6. $(h-0)^2 + (k \pm b)^2 = a^2 + k^2$

$h^2 + k^2 \pm 2kb + b^2 = a^2 + k^2$

$\Rightarrow h^2 \pm 2kb + b^2 - a^2 = 0$

required locus $x^2 \pm 2by + b^2 - a^2 = 0$.



7. $\tan\theta = \cot\theta - 2\cot 2\theta$

i.e. $t_1 = \cot\theta - 2\cot 2\theta$

similarly $t_2 = \frac{1}{2}\cot\frac{\theta}{2} - \cot\theta$

$t_3 = \frac{1}{2^2}\cot\frac{\theta}{2^2} - \frac{1}{2}\cot\frac{\theta}{2}$

....

$t_n = \frac{1}{2^{n-1}}\cot\frac{\theta}{2^{n-1}} - \frac{1}{2^{n-2}}\cot\frac{\theta}{2^{n-1}}$

....

adding we get, $S_n = \frac{1}{2^{n-1}}\cot\frac{\theta}{2^{n-1}} - 2\cot 2\theta$

Required sum $S = \underset{n\to\infty}{\text{Lt}} S_n$

$= \underset{n\to\infty}{\text{Lt}} \ \underset{n\to\infty}{\text{Lt}}\left(\frac{1}{2^{n-1}}\cot\frac{\theta}{2^{n-1}} - 2\cot 2\theta\right)$

$$= \lim_{n\to\infty}\left(\frac{1}{2^{n-1}}\frac{1}{\tan\frac{\theta}{2^{n-1}}} - 2\cot 2\theta\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{\theta}\frac{\theta}{2^{n-1}}\frac{1}{\tan\frac{\theta}{2^{n-1}}} - 2\cot 2\theta\right)$$

$$= \frac{1}{\theta} - 2\cot 2\theta$$

$$\left[\because \text{as } n\to\infty, \frac{\theta}{2^{n-1}} \to 0. \text{ and } \lim_{\theta\to 0}\frac{\tan\theta}{\theta} = 1\right]$$

Second part : The given series

$$= \frac{1}{2^2}\tan\frac{\pi}{2^2} + \frac{1}{2^2}\tan\frac{\pi}{2^3} + \frac{1}{2^4}\tan\frac{\pi}{2^4} + ...\infty$$

$$= \frac{1}{2^2}\left[\tan\frac{\pi}{2^2} + \frac{1}{2}\tan\frac{\pi}{2^3} + \frac{1}{2^4}\tan\frac{\pi}{2^4}\tan\frac{\pi}{2^4} .....\infty\right]$$

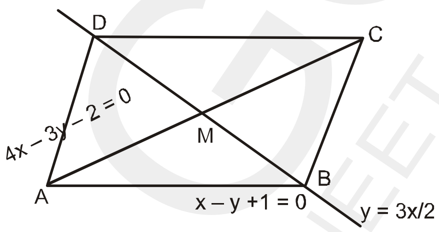
$$= \frac{1}{2^2}\left[\tan\theta\frac{1}{2}\tan\frac{\theta}{2} + \frac{1}{2^2}\tan\frac{\pi}{2^2} + ...\infty\right]$$

$$= \frac{1}{4}\left[\frac{1}{\theta} - 2\cot 2\theta\right]$$

$$= \frac{1}{4}\left[\frac{1}{\left(\frac{\pi}{2^2}\right)} - \cot 2.\left(\frac{\pi}{2^2}\right)\right]$$

$$= \frac{1}{4}\left[\frac{4}{\pi} - 2\cot\frac{\pi}{2}\right] = \frac{1}{\pi}$$

8.



$4x - 3y - 2 = 0$ ... (1)

$x - y + 1 = 0$

$3x - 3y + 3 = 0$ ... (2)

$x - 5 = 0$

$x = 5, \quad y = 5 + 1 = 6$

A (5, 6)

(1) & $y = \frac{3x}{2}$

$\Rightarrow 4x - \frac{9x}{2} - 2 = 0 \quad \Rightarrow \quad \frac{-x}{2} = 2$

$x = -4$

D(– 4, – 6)

(2) and $y = \frac{3x}{2}$

$x - \frac{3x}{2} + 1 = 0$

$\frac{-x}{2} = -1 \quad \Rightarrow \quad x = 2$

D(2, 3)

mid point of BD = m(– 1, – 3/2)

equation of AM is

$$y + \frac{3}{2} = \frac{6+\frac{3}{2}}{5+1}(x+1)$$

$$y + \frac{3}{2} = \frac{15}{12}(x+1)$$

$$\Rightarrow y + \frac{3}{2} = \frac{5}{4}(x+1) \quad \Rightarrow \quad 5x - 4y - 1 = 0$$

DPP NO. - 49

1 to 3. $s_1 = a + ar + ar^2 + ...$

$rs_2 = ar + ar + ar^2 + ...$

$s_1 = \frac{a}{1-r}$

$s_1 = \frac{b}{1-R}$

5. $(a + b) + (2a + b)\omega + (3a + b)\omega^2 + (4a + b)\omega^3 + ...+ (an + b)\omega^{n-1}$

$\Rightarrow a(1 + 2\omega + 3\omega^2 + 4\omega^3 + ...+ n\omega^{n-1}) + b(1 + \omega + \omega^2 + \omega^3 +.....+ \omega^{n-1})$

$\lambda = 1 + 2\omega + 3\omega^2 + 4\omega^3 +.......+ n\omega^{n-1}$

$\lambda\omega = + \omega + 2\omega^2 + 3\omega^3 +.......+ (n-1)\omega^{n-1} + n\omega^n$

$\lambda(1 - \omega) = 1 + \omega + \omega^2 + \omega^3 +.......+ \omega^{n-1} - n\omega^n$

$$\lambda(1-\omega) = \frac{1-\omega^n}{1-\omega} - n\omega^n$$

$$\lambda = 0 - \frac{n\omega^n}{1-\omega} = \frac{n\omega^n}{\omega - 1}$$

$$S = \frac{an(1)}{\omega - 1}, \ (\omega^n = 1)$$

6. Now the inequality becomes

$$\left(1 - \frac{2}{3K}\right)x \le \frac{2}{3K} + 2 - \frac{2}{K}$$

$$\text{i.e. } x\left(1 - \frac{2}{3K}\right) \le 2\left(1 - \frac{2}{3K}\right)$$

$$\text{i.e. } \left(1 - \frac{2}{3K}\right)(x-2) \le 0 \qquad \text{i.e. } \frac{3k-2}{3k}(x-2) \le 0$$

7. We have $\alpha + \beta = -p$ and $\alpha\beta = q$. ...(i)

Also since α, β are the roots of

$x^{2n} + p^n x^n + q^n = 0$

We have

$\alpha^{2n} + p^n \alpha^n + q^n = 0$ and $\beta^{2n} + p^n\beta^n + q^n = 0$

Subtracting the above relations, we get

$(\alpha^{2n} - \beta^{2n}) + p^n(\alpha^n - \beta^n) = 0$

$\therefore\ \alpha^n + \beta^n = -p^n$ ...(ii)

If α/β or β/α is a root of

$x^n + 1 + (x + 1)^n = 0$, then

$(\alpha/\beta)^n + 1 + [(\alpha/\beta) + 1]^n = 0$

or $(\alpha^n + \beta^n) + (\alpha + \beta)^n = 0$

or $-p^n + (-p)^n = 0$, by (1) and (2)

Above is possible only when n is even.

8. $\alpha^2 - 2\alpha + 3 = 0$

$\alpha(\alpha^2 - 3\alpha + 5) - 2$

$\alpha[(\alpha^2 - 2\alpha + 3) + (2 - \alpha)] - 2$

$= \alpha(2 - \alpha) - 2 = 2\alpha - \alpha^2 - 2 = -(\alpha^2 - 2\alpha + 2)$

$= -(-3 + 2) = 1$

$\beta(\beta - \beta + 1) + 5$

$\beta(\beta^2 - 2\beta + 3 + \beta - 2) + 5$

$\beta(\beta - 2) + 5$

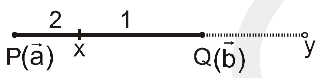
$\beta^2 - 2\beta + 5 - 3 + 5 = 2$

$x^2 - (2 + 1)x + 2 = 0$

$x^2 - 3x + 2 = 0$

DPP NO. - 50

1.



$$\overrightarrow{OX} = \frac{2\vec{b} + \vec{a}}{3}, \quad \overrightarrow{OY} = \frac{2\vec{b} - \vec{a}}{2 - 1}$$

$\Rightarrow$ Now $\overrightarrow{XY} = \overrightarrow{OY} - \overrightarrow{OX}$

$$= (2\vec{b} - \vec{a}) - \left(\frac{2\vec{b} + \vec{a}}{3}\right) = \frac{6\vec{a} - 3\vec{a} - 2\vec{b} - \vec{a}}{3}$$

$$= \frac{4}{3}(\vec{b} - \vec{a})$$

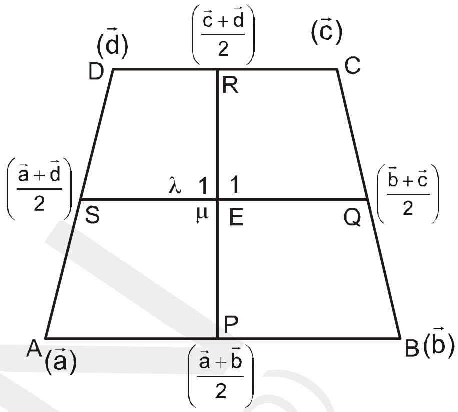
6. $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = \vec{a} + \vec{b} + \vec{c} + \vec{d}$

Now for point E

$$\frac{\lambda\left(\frac{\vec{b} + \vec{c}}{2}\right) + \left(\frac{\vec{a} + \vec{d}}{2}\right)}{\lambda + 1} = \frac{\mu\left(\frac{\vec{c} + \vec{d}}{2}\right) + 1.\left(\frac{\vec{a} + \vec{b}}{2}\right)}{\mu + 1}$$

$\Rightarrow$ comparing coff.

$$\frac{1}{\lambda + 1} = \frac{1}{\mu + 1} \Rightarrow \lambda = \mu, \qquad \frac{\lambda}{2(\lambda + 1)} = \frac{1}{2(\mu + 1)}$$



$\Rightarrow \lambda = 1$

$\mu = 1$

Position vector of E is

$$\overrightarrow{OE} = \frac{\frac{\vec{a} + \vec{b}}{2} + \frac{\vec{c} + \vec{d}}{2}}{2} = \frac{\vec{a} + \vec{b} + \vec{c} + \vec{d}}{4}$$

$$4\,\overrightarrow{OE} = \vec{a} + \vec{b} + \vec{c} + \vec{d}$$

7. $D = \dfrac{\vec{c} + 4\vec{b}}{5}$, $E = \dfrac{2}{5}\vec{c}$, $F = \dfrac{3}{10}\vec{b}$

$P = \dfrac{1}{4}\vec{b}$, $\overrightarrow{AD} = \dfrac{\vec{c} + 4\vec{b}}{5}$, $\overrightarrow{BE} = \dfrac{2}{5}\vec{c} - \vec{b}$

$\overrightarrow{CF} = \dfrac{3}{10}\vec{b} - \vec{c}$, $\overrightarrow{CP} = \dfrac{1}{4}\vec{b} - \vec{c}$

Now L.H.S.

$$= 5\left[\frac{\vec{c} + 4\vec{b}}{5} + \frac{2\vec{c} - 5\vec{b}}{5} + \frac{3\vec{b} - 10\vec{c}}{10}\right]$$

$$= 2\left(\frac{1}{4}\vec{b} - \vec{c}\right) = 2\,\overrightarrow{CP}$$

8. $$f'(x) = -\frac{(a^2 - 3a + 2)}{2}\sin\left(\frac{x}{2}\right) + (a - 1) = 0$$

$$\Rightarrow (a - 1)\left(1 - \left(\frac{a - 2}{2}\right)\sin\frac{x}{2}\right) = 0$$

$$\Rightarrow a = 1, \ \sin\left(\frac{x}{2}\right) = \frac{2}{a - 2}$$

$$-1 \le \sin\frac{x}{2} \le 1$$

$$\Rightarrow -1 \le \frac{2}{a - 2} \le 1$$

$\Rightarrow a \in (-\infty, 0] \cup [4, \infty)$

$(-\infty, 0] \cup [4, \infty) \cup \{1\}$ **Ans.**