GGSRDN
Educational Services Private Limited
9th, 10th, NEET, JEE(Main/Advanced)
अभ्यास ही सबसे बड़ा गुरु है।

CLASS : XII (MATHS)

DPP

DAILY PRACTICE PROBLEM

DPP-11 to 20

DPP 11 : **Inverse Trigonometric Function, Set & Relation, Fundamentals of Mathematics, Matrices & Determinants, Quadratic Equation**

DPP 12 : **Inverse Trigonometric Function, Set & Relation, Fundamentals of Mathematics, Matrices & Determinants, Quadratic Equation, Trigonometric Ratio**

DPP 13 : **Fundamentals of Mathematics, Sequence & Series, Trigonometric Ratio, Matrices & Determinants, Binomial Theorem, Straight Line, Permutation & Combination, Complex Number, Circle, Ellipse, Set & Relation**

DPP 14 : **Inverse Trigonometric Function, Matrices & Determinants, Fundamentals of Mathematics, Trigonometric Ratio, Function, Quadratic Equation**

DPP 15 : **Function**

DPP 16 : **Function, Inverse Trigonometric Function**

DPP 17 : **Function, Quadratic Equation**

DPP 18 : **Fundamental of Mathematics, Function, Limits**

DPP 19 : **Fundamentals of Mathematics, Function, Limits**

DPP 20 : **Fundamentals of Mathematics, Function, Limits**

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 11

Total Marks : 25

Max. Time : 26 min.

Topics : **Inverse Trigonometric Function, Set & Relation, Fundamentals of Mathematics, Matrices & Determinants, Quadratic Equation**

Type of Questions **M.M., Min.**

Single choice Objective (no negative marking) Q.1 to Q. 7 (3 marks, 3 min.) [21, 21]
Subjective Questions (no negative marking) Q. 8 (4 marks, 5 min.) [4, 5]

1. Number of solutions of the equation

$$\tan^{-1}\left(\frac{1}{a-1}\right) = \tan^{-1}\left(\frac{1}{x}\right) + \tan^{-1}\left(\frac{1}{a^2 - x + 1}\right)$$

(A) one (B) Two (C) Three (D) Zero

2. Let the matrix A and B be defined as $A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 7 & 3 \end{bmatrix}$, then the value of Det.$(2A^9B^{-1})$ is

(A) 2 (B) 1 (C) – 1 (D) – 2

3. If the quadratic equations, $ax^2 + 2cx + b = 0$ and $ax^2 + 2bx + c = 0$ $(b \neq c)$ have a common root, then $a + 4b + 4c$ is equal to :

(A) – 2 (B) – 2 (C) 0 (D) 1

4. Number of triplets (x, y, z) satisfying $\sin^{-1}x + \cos^{-1}y + \sin^{-1}z = 2\pi$, is

(A) 0 (B) 2 (C) 1 (D) infinite

5. The matrix X for which $\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix} X = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

(A) $\begin{bmatrix} -2 & 4 \\ -3 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} -\frac{1}{5} & \frac{2}{5} \\ \frac{-3}{10} & \frac{1}{5} \end{bmatrix}$ (C) $\begin{bmatrix} 6 & 2 \\ \frac{11}{2} & 2 \end{bmatrix}$ (D) $\begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

6. Let R = {(3, 3), (6, 6), (9, 9), (12, 12) (6, 12), (3, 9), (3, 12), (3, 6)} be relation on the set A = {3, 6, 9, 12}. The relation is-

(A) reflexive and transitive only (B) reflexive only
(C) an equilvalence relation (D) reflexive and symmetric only

7. Let A = {1, 2}, B = {0} then which of the following is correct
(A) number of possible relations from A to B is $2^{o} = 1$
(B) number of void relations from A to B is not possible
(C) number of possible relations from A to B are 4
(D) number of possible relations are equal to $2^{n(A) + n(B)}$

8. Find out the values of 'a' for which any solution of the inequality, $\frac{\log_3(x^2 - 3x + 7)}{\log_3(3x + 2)} < 1$ is also a solution of the inequality, $x^2 + (5 - 2a)\,x \leq 10a$.

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 12

Total Marks : 26

Max. Time : 25 min.

Topics : **Inverse Trigonometric Function, Set & Relation, Fundamentals of Mathematics, Matrices & Determinants, Quadratic Equation, Trigonometric Ratio**

Type of Questions		M.M.,	Min.
Comprehension (no negative marking) Q.1 to Q.3	(3 marks, 3 min.)	[9,	9]
Single choice Objective (no negative marking) Q. 4, 5, 6, 7	(3 marks, 3 min.)	[12,	12]
Multiple choice objective (no negative marking) Q.8	(5 marks, 4 min.)	[5,	4]

COMPREHENSION (FOR Q.NO. 1 TO 3) :

A polynomial P(x) of third degree vanish when $x = 1$ & $x = -2$. This polynomial have the values 4 & 28 when $x = -1$ and $x = 2$ respectively.

1. One of the factor of P(x) is
(A) $x + 1$ (B) $x - 2$ (C) $3x + 1$ (D) none of these

2. If the polynomial P(x) is divided by $(x + 3)$, then remainder is
(A) -32 (B) 100 (C) 32 (D) 0

3. If $i = \sqrt{-1}$, then P(i) is
(A) rational (B) purely imaginary (C) imaginary (D) irrational

4. If $n(A) = 110$, $n(B) = 300$, $n(A - B) = 50$, then $n(A \cup B)$ equals
(1) 350 (B) 410 (C) 160 (D) 460

5. The sum of

$$\cot^{-1}\left(\frac{7}{4}\right) + \cot^{-1}\left(\frac{19}{4}\right) + \cot^{-1}\left(\frac{39}{4}\right) + \cot^{-1}\left(\frac{67}{4}\right) ....\infty \text{ is equal to}$$

(A) $\tan^{-1} 4$ (B) $\tan^{-1} 3$ (C) $\tan^{-1} 2$ (D) None of these

6. If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, then $A^T A^{-1} =$

(A) $\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$ (B) $\begin{bmatrix} -\cos 2x & \sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$

(C) $\begin{bmatrix} \sin 2x & \cos 2x \\ \cos 2x & \sin 2x \end{bmatrix}$ (D) None of these

7. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$

(A) $\frac{56}{33}$ (B) $\frac{19}{12}$ (C) $\frac{20}{7}$ (D) $\frac{25}{16}$

8. If the cubic polynomials $x^3 + ax^2 + 11x + 6$ and $x^3 + bx^2 + 14x + 8$ may have a common factor of the form $x^2 + px + q$, then
(A) $a + p = b + q$ (B) $ap < bq$ (C) pq divides ab (D) p + q divides a + b.

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 13

Total Marks : 45

Max. Time : 45 min.

Topics : **Fundamentals of Mathematics, Sequence & Series, Trigonometric Ratio, Matrices & Determinants, Binomial Theorem, Straight Line, Permutation & Combination, Complex Number, Circle, Ellipse, Set & Relation**

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.1 to Q.13	**(3 marks, 3 min.)**	**[39, 39]**
Assertion and Reason (no negative marking) Q.14, 15	**(3 marks, 3 min.)**	**[6, 6]**

1. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has :

(A) infinite number of real roots
(B) no real roots
(C) exactly one real root
(D) exactly four real roots

2. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to:

(A) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$
(B) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$
(C) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$
(D) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

3. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$ is :

(A) an irrational number
(B) an odd positive integer
(C) an even positive integer
(D) a rational number other than positive integers

4. If 100 times the 100th term of an AP with non zero common difference equals the 50 times its 50th term, then the 150th term of this AP is :

(A) – 150
(B) 150 times its 50th term
(C) 150
(D) zero

5. In a ΔPQR, if $3 \sin P + 4 \cos Q = 6$ and $4 \sin Q + 3 \cos P = 1$, then the angle R is equal to :

(A) $\frac{5\pi}{6}$
(B) $\frac{\pi}{6}$
(C) $\frac{\pi}{4}$
(D) $\frac{3\pi}{4}$

6. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals :

(A) $\frac{29}{5}$
(B) 5
(C) 6
(D) $\frac{11}{5}$

7. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is :

(A) 880
(B) 629
(C) 630
(D) 879

8. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies :

(A) either on the real axis or on a circle passing through the origin.
(B) on a circle with centre at the origin.
(C) either on the real axis or on a circle not passing through the origin.
(D) on the imaginary axis.

9. Let P and Q be 3×3 matrices $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to :

(A) – 2
(B) 1
(C) 0
(D) – 1

10. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is :

(A) $\frac{10}{3}$
(B) $\frac{3}{5}$
(C) $\frac{6}{5}$
(D) $\frac{5}{3}$

11. Let X = {1, 2, 3, 4, 5}. The number of different ordered pairs (Y, Z) that can formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is :

(A) 5^2
(B) 3^5
(C) 2^5
(D) 5^3

12. An ellipse is drawn by taking a diameter of the circle $(x-1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y-2)^2 = 4$ is semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is :

(A) $4x^2 + y^2 = 4$
(B) $x^2 + 4y^2 = 8$
(C) $4x^2 + y^2 = 8$
(D) $x^2 + 4y^2 = 16$

13. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. if the area of the triangle OPQ is least, then the slope of the line PQ is :

(A) $-\frac{1}{4}$
(B) – 4
(C) – 2
(D) $-\frac{1}{2}$

14. **Statement-1 :** The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400) is 8000.

Statement-2 : $\sum_{k=1}^{n} (k^3 - (k-1)^3) = n^3$, for any natural number n.

(A) Statement-1 is false, Statement-2 is true.
(B) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(C) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(D) Statement-1 is true, statement-2 is false.

15. **Statement-1 :** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}\,x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$.

Statement-2 : If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}\,x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.

(A) Statement-1 is false, Statement-2 is true.
(B) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(C) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(D) Statement-1 is true, statement-2 is false.

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 14

Total Marks : 26

Max. Time : 28 min.

Topics : **Inverse Trigonometric Function, Matrices & Determinants, Fundamentals of Mathematics, Trigonometric Ratio, Function, Quadratic Equation**

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.1,2,3,4,5,6	**(3 marks, 3 min.)**	**[18, 18]**
Subjective Questions (no negative marking) Q.7,8	**(4 marks, 5 min.)**	**[8, 10]**

1. The set of values of a for which $x^2 + ax + \sin^{-1}(x^2 - 4x + 5) + \cos^{-1}(x^2 - 4x + 5) = 0$ has at least one solution is

(A) $(-\infty, -\sqrt{2\pi}\,] \cup [\sqrt{2\pi}, \infty]$ (B) $(-\infty, -\sqrt{2\pi}\,) \cup (\sqrt{2\pi}, \infty)$

(C) R (D) none of these

2. If A is a square matrix of order 3 such that $|A| = 2$, then $|\text{adj } A^{-1}|$ is :

(A) 2 (B) 4 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

3. If $A = \begin{bmatrix} \alpha^2 & \alpha \\ \beta^2 & \beta \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$, $C = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ are such that AB = C, then absolute value of |A| is

(A) $\frac{1}{6}$ (B) -30 (C) $\frac{2}{27}$ (D) $\frac{1}{36}$

4. If $\sin^2 x + \sin x = 1$, then $\cos^{12}x + 3\cos^{10}x + 3\cos^8 x + \cos^6 x - 1$ is equal to

(A) 1 (B) 0 (C) -1 (D) none of these

5. Domain of the function $f(x) = \log\left(\sin^{-1}\sqrt{x^2 + 3x + 2}\right)$ is

(A) $(-\infty, -2) \cup (-1, \infty)$ (B) $\left(-\frac{3-\sqrt{5}}{2}, \frac{-3+\sqrt{5}}{2}\right)$

(C) $\left[\frac{-3-\sqrt{5}}{2}, -2\right) \cup \left(-1, \frac{-3+\sqrt{5}}{2}\right]$ (D) none of these

6. If $\tan\alpha$, $\tan\beta$, $\tan\gamma$ are the roots of the equation $x^3 - px^2 - r = 0$, then the value of $(1 + \tan^2\alpha)(1 + \tan^2\beta)(1 + \tan^2\gamma)$ is equal to

(A) $(p - r)^2$ (B) $1 + (p - r)^2$ (C) $1 - (p - r)^2$ (D) none of these

7. Find the domain of the following

(i) $f(x) = \sqrt{x + \sqrt{x - 1}}$ (ii) $f(x) = \dfrac{\sqrt{\sin x}}{1 + \sec^2 x}$

(iii) $f(x) = \log_2 \log_{|x+1|}(\sqrt{x - 3})$ (iv) $f(x) = \sin^{-1}(x^2 - x - 1) + \tan^{-1}(x^2 - 5x + 6) + \log_{x-2}|x^2 - 9|$

8. (i) Find the largest integral value of x which satisfies the inequality $\dfrac{4x+19}{x+5} < \dfrac{4x-17}{x-3}$.

(ii) Solve for x : $\sqrt{\dfrac{x-2}{1-2x}} > -1$

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 15

Total Marks : 26

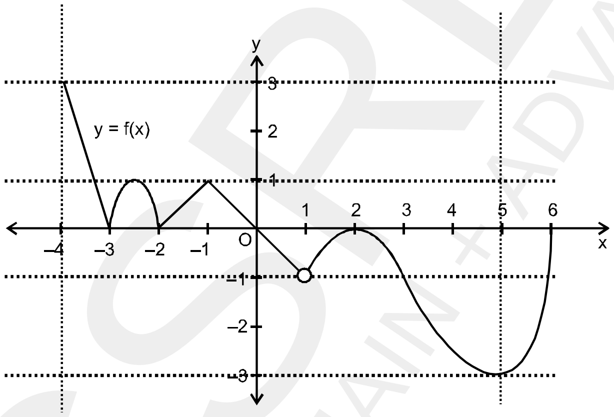
Max. Time : 28 min.

Topic : **Function**

Type of Questions		M.M., Min.
Comprehension (no negative marking) Q.1 to Q.3	(3 marks, 3 min.)	[9, 9]
Single choice Objective (no negative marking) Q.4,5,6	(3 marks, 3 min.)	[9, 9]
Subjective Questions (no negative marking) Q.7,8	(4 marks, 5 min.)	[8, 10]

COMPREHENSION (FOR Q.NO. 1 TO 3)

If graph of a given function y = f(x) is as follows,



1. The range of given function is

(A) [–3, 3] (B) [–4, 6] (C) [–1, 1] (D) [0, 3]

2. The length of longest interval for which the given function in one one

(A) 1 unit (B) 2 unit (C) 3 unit (D) 4 unit

3. Which of the following change in given curve does not represent a function

(A) y = f(|x|) (B) y = |f(x)| (C) |y| = f(x) (D) y = |f(|x|)|

4. Domain of the function $f(x) = \sqrt{\cos(\sin x)} + \sin^{-1}(x^2 - 1)$ is

(A) [– 1, 1] (B) [– 2, 2]

(C) $[-\pi, -\sqrt{2}] \cup [\sqrt{2}, \pi]$ (D) $[-\sqrt{2}, \sqrt{2}]$

5. Let $f(x) = [9^x - 3^x + 1]$ for all $x \in (-\infty, 1]$, then the range of f(x) is ; ([.] denotes the greatest integer function)

(A) {0, 1, 2, 3, 4, 5, 6, 7} (B) {0, 1, 2, 3, 4, 5, 6}

(C) {1, 2, 3, 4, 5, 6, 7} (D) {1, 2, 3, 4, 5, 6}

6. The range of the function $f(x) = \sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left[x^2 - \frac{1}{2}\right]$, where [.] is greatest integer function.

(A) $\left\{\frac{\pi}{2}, \pi\right\}$ (B) $\left\{0, -\frac{1}{2}\right\}$ (C) $\{\pi\}$ (D) $\left(0, \frac{\pi}{2}\right)$

7. Find the range of the following functions .

(i) $f(x) = 4 \tan x . \cos x$

(ii) $g(x) = 9 \cos 3x - 12 \cos^3 3x$

(iii) $h(x) = \cos (2 \sin x)$

(iv) $y = \sqrt{x - x^2}$

8. If f be a function defined on the set of non-negative integers and taking values in the same set. Given that, where [.] denotes greatest integer function

(i) $x - f(x) = 17\left[\frac{x}{17}\right] - 70\left[\frac{f(x)}{70}\right]$ for all non-negative integers

(ii) $1700 < f(1770) < 1800$

Find the possible values f (1770) can take.

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 16

Total Marks : 30

Max. Time : 33 min.

Topics : Function, Inverse Trigonometric Function

Type of Questions		M.M.,	Min.
Single choice Objective (no negative marking) Q.1,2,3	(3 marks, 3 min.)	[9,	9]
Multiple choice objective (no negative marking) Q.4	(5 marks, 4 min.)	[5,	4]
Subjective Questions (no negative marking) Q.5,6,7,8	(4 marks, 5 min.)	[16,	20]

1. Range of the function $f(x) = \tan^{-1}\sqrt{[x]+[-x]} + \sqrt{2-|x|} + \dfrac{1}{x^2}$ is :

(where [.] is the greatest integer function)

(A) $\left[\dfrac{1}{4}, \infty\right)$ (B) $\left\{\dfrac{1}{4}\right\} \cup [2, \infty)$ (C) $\left\{\dfrac{1}{4}, 2\right\}$ (D) $\left[\dfrac{1}{4}, 2\right]$

2. Which of the following functions is periodic

(A) $\cos^2 x + \sin x^3 + \tan (x^4)$ (B) $\cos^2 x + \sin x^3 + \tan^4 x$

(C) $\cos^2 x^2 + \sin x^3 + \tan^4 x$ (D) $\cos 2x + \sin 3x + \tan 4x$

3. Let $f : R \rightarrow \left[0, \dfrac{\pi}{2}\right)$ defined by $f(x) = \tan^{-1}(x^2 + x + a)$, then the set of values of 'a' for which f is onto is

(A) $[0, \infty)$ (B) $\left[\dfrac{1}{4}, \infty\right)$ (C) $\dfrac{1}{4}$ (D) $(0, \infty)$

4. If $x \in [0, 2\pi]$, then $y = \dfrac{\sin x}{|\sin x|}$, $y = \dfrac{|\cos x|}{\cos x}$ are identical functions for $x \in$

(A) $\left(0, \dfrac{\pi}{2}\right)$ (B) $\left(\dfrac{\pi}{2}, \pi\right)$ (C) $\left(\pi, \dfrac{3\pi}{2}\right)$ (D) $\left(\dfrac{3\pi}{2}, 2\pi\right)$

5. If A is domain of $f(x) = \ell n \tan^{-1}((x^3 - 6x^2 + 11x - 6)(x)(e^x - 8))$ and B is the range of $g(x) = \sin^2 \dfrac{x}{4} + \cos \dfrac{x}{4}$. Then find $A \cap B$.

6. Classify one-one, many-one, into, onto function of the following functions

(i) $f(x) = x|x|$, $f : [-1, 1] \rightarrow [-1, 1]$ (ii) $f(x) = \dfrac{x^2}{x^2+1}$, $f : R \rightarrow R$

(iii) $f(x) = \dfrac{x-2}{x-3}$, $f : A \rightarrow B$, where $A = R - \{3\}$, $B = R - \{1\}$

7. Prove that the equality $\left(1-\dfrac{4}{1}\right)\left(1-\dfrac{4}{9}\right)\left(1-\dfrac{4}{25}\right)\ldots\ldots\ldots\ldots\left(1-\dfrac{4}{(2n-1)^2}\right) = \dfrac{1+2n}{1-2n}$ holds true for any natural n.

8. Solve for x, if $\cot^{-1}(x) + \cot^{-1}(17 - x) = \cot^{-1}(3)$.

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 17

Total Marks : 36

Max. Time : 34 min.

Topics : Function, Quadratic Equation

Type of Questions		M.M., Min.	
Single choice Objective (no negative marking) Q.1,2,3	(3 marks, 3 min.)	[9,	9]
Multiple choice objective (no negative marking) Q.4,5,6	(5 marks, 4 min.)	[15,	12]
Subjective Questions (no negative marking) Q.7	(4 marks, 5 min.)	[4,	5]
Match the Following (no negative marking) Q.8	(8 marks, 8 min.)	[8,	8]

1. Suppose f is a real function satisfying f(x + f(x)) = 4f(x) and f(1) = 4. Then the value of f(21) is
(A) 16 (B) 21 (C) 64 (D) 105

2. Let f be a real valued function defined by $f(x) = \dfrac{e^x - e^{-|x|}}{e^x + e^{|x|}}$, then the range of f(x) is :
(A) R (B) [0, 1] (C) [0, 1) (D) $\left[0, \frac{1}{2}\right)$

3. If $f(x) = -\dfrac{x|x|}{1+x^2}$, then $f^{-1}(x)$ equals
(A) $\sqrt{\dfrac{|x|}{1-|x|}}$ (B) $(\text{sgn}(-x))\sqrt{\dfrac{|x|}{1-|x|}}$ (C) $-\sqrt{\dfrac{x}{1-x}}$ (D) $(\text{sgn}(x))\sqrt{\dfrac{|x|}{1+|x|}}$

4. If $f\left(2x+\dfrac{y}{8}, 2x-\dfrac{y}{8}\right)$ = xy, then f(m, n) + f(n, m) is
(A) depends over m and n both (B) periodic and odd function
(C) constant number (D) even function

5. The period of function $\dfrac{|\sin x|+|\cos x|}{|\sin x-\cos x|+|\sin x+\cos x|}$ is
(A) π (B) $\dfrac{\pi}{2}$ (C) 2π (D) $\dfrac{2\pi}{3}$

6. If $\sum_{r=0}^{21} f\left(\dfrac{r}{11}+2x\right)$ = constant $\forall$ x $\in$ R and f(x) is periodic, then period of f(x) is
(A) 1 (B) $\dfrac{1}{11}$ (C) 2 (D) 4

7. For what values of 'a' the equation $x^2 - x(1-a) - (a+2) = 0$ has integral roots.

8.

	Column - I		**Column - II**
(A)	$f : R \rightarrow \left[\dfrac{\pi}{4}, \pi\right)$ and $f(x) = \cot^{-1}(2x - x^2 - 2)$, then f(x) is	(p)	one-one
(B)	f: R $\rightarrow$ R and $f(x) = e^{ax}\sin bx$ where a,b, $\in R^+$, then f(x) is	(q)	into
(C)	$f : R^+ \rightarrow [2, \infty)$ and $f(x) = 2 + 3x^2$, then f(x) is	(r)	many-one
(D)	f: X $\rightarrow$ X and f(f(x)) = x $\forall$ x $\in$ X, then f(x) is	(s)	onto
		(t)	invertible

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 18

Total Marks : 34

Max. Time : 36 min.

Topics : Fundamental of Mathematics, Function, Limits

Type of Questions		**M.M.,**	**Min.**
Single choice Objective (no negative marking) Q.1,2,3	**(3 marks, 3 min.)**	**[9,**	**9]**
Multiple choice objective (no negative marking) Q.4	**(5 marks, 4 min.)**	**[5,**	**4]**
Subjective Questions (no negative marking) Q.5,6,7	**(4 marks, 5 min.)**	**[12,**	**15]**
Match the Following (no negative marking) Q.8	**(8 marks, 8 min.)**	**[8,**	**8]**

1. Total number of positive integers x for which $f(x) = x^3 - 8x^2 + 20x - 13$ is a prime number, is

(A) 1 (B) 2 (C) 3 (D) 4

2. Let f be a real valued function such that for any real x

$f(15 + x) = f(15 - x)$ and $f(30 + x) = -f(30 - x)$

Then which of the following statements is true ?

(A) f is odd and periodic (B) f is odd but not periodic

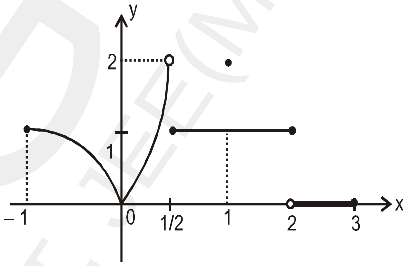
(C) f is even and periodic (D) f is even but not periodic

3. Which of the following functions is **not** periodic, where [.] denotes greatest integer function

(A) $f(x) = 1^{[x]} + (-1)^{[x]}$ (B) $g(x) = 1^{[5x]} + (-1)^{[5x]}$

(C) $h(x) = 2^{[x]} - (-2)^{[x]}$ (D) $\phi(x) = 1^{[x]} - (-1)^{[x]}$

4. Which of the following statements are true for the function f defined for $-1 \le x \le 3$ in the figure shown.



(A) $\underset{x \to -1^+}{\text{Limit}} f(x) = 1$

(B) $\underset{x \to 2}{\text{Limit}} f(x)$ does not exist

(C) $\underset{x \to 1^-}{\text{Limit}} f(x) = 1$

(D) $\underset{x \to 0^+}{\text{Limit}} f(x) = \underset{x \to 0^-}{\text{Limit}} f(x)$

(E) $\underset{x \to c}{\text{Limit}} f(x)$ exists at every c between – 1 & 1

(F) $\underset{x \to c}{\text{Limit}} f(x)$ exists at every c between – 1 & 0.

5. Find the fundamental period of the functions

(i) $f(x) = \sin\left(2\pi x + \frac{\pi}{3}\right) + 2\sin\left(3\pi x + \frac{\pi}{4}\right) + 3\sin 5\pi x$

(ii) $f(x) = \sin\left(\frac{\pi}{3}x\right) + \cos\left(\frac{\pi}{4}x\right)$

6. If $f(x) = 4x^3 - x^2 - 2x + 1$ and $g(x) = \begin{cases} \text{Min}\ \{f(t) : 0 \le t \le x\} & ;\ 0 \le x \le 1 \\ 3 - x & ;1 < x \le 2 \end{cases}$ then find the value of

$g\left(\frac{1}{4}\right) + g\left(\frac{3}{4}\right) + g\left(\frac{5}{4}\right)$.

7. Identify the indeterminate forms (if any) in the following limits :

(i) $\lim\limits_{x\to 0} \frac{\sin x^3}{x^2}$

(ii) $\lim\limits_{x\to 0} \frac{\sin[x^2]}{[x^2]}$; [.] represents the greatest integer function

(iii) $\lim\limits_{x\to 0} |x|^{[\sin^2 x]}$; [.] represents the greatest integer function

(iv) $\lim\limits_{x\to 0^+} \frac{\text{cosec}^{-1} x}{\cot^{-1} x}$

(v) $\lim\limits_{x\to 0^-} \frac{\text{cosec}^{-1} x}{\cot^{-1} x}$

8. Let $f(x) = x + \frac{1}{x}$ and $g(x) = \frac{x+1}{x+2}$.

Match the composite function given in Column–I with respective domains given in Column–II.

Column I		**Column II**	
(A)	fog(x)	(p)	R – {–2, –5/3}
(B)	gof(x)	(q)	R – {–1,0}
(C)	fof(x)	(r)	R – {0}
(D)	gog(x)	(s)	R – {–2, –1}
		(t)	R – {– 1}

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 19

Total Marks : 28

Max. Time : 30 min.

Topics : Fundamentals of Mathematics, Function, Limits

Type of Questions		**M.M., Min.**	
Single choice Objective (no negative marking) Q.1,2	**(3 marks, 3 min.)**	**[6,**	**6]**
Multiple choice objective (no negative marking) Q.3	**(5 marks, 4 min.)**	**[5,**	**4]**
Assertion and Reason (no negative marking) Q.4	**(3 marks, 3 min.)**	**[3,**	**3]**
Subjective Questions (no negative marking) Q.5,6,8	**(4 marks, 5 min.)**	**[12,**	**15]**
True or False (no negative marking) Q.7	**(2 marks, 2 min.)**	**[2,**	**2]**

1. The solution set of the inequality max $\{1 - x^2, |x - 1|\} < 1$ is

(A) $(-\infty, 0) \cup (1, \infty)$ (B) $(-\infty, 0) \cup (2, \infty)$ (C) $(0, 2)$ (D) $(-1, 1)$

2. If $h(x) = \log_{10} x$, then the value of $\sum_{n=1}^{89} h(\tan n^{o}) =$

(A) 1 (B) 0 (C) – 1 (D) none of these

3. If f = sin |cos x|, g = cos |sin y|, then

(A) least value of f + g is cos 1 (B) greatest value of f + g is sin 1

(C) period of g is $\frac{\pi}{2}$ (D) greatest value of f + g is 1 + sin 1

4. In both the statements [.] represents greatest integer function.

STATEMENT-1 : The greatest value of $\sin\left(\frac{3}{2}x - \frac{3}{2}[x]\right)$ is $\sin\frac{3}{2}$.

STATEMENT-2 : The greatest value of [sin x] is 1, where $x \in R$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

5. Solve for x : $(|x| - 5)(|x - 1| - 1) < 0$

6. Evaluate the following limits

(i) $\lim\limits_{x\to 0} \dfrac{\sin x^4 - x^4 . \cos x^4}{x^4(e^{2x^4} - 1 - 2x^4)}$ (ii) $\lim\limits_{x\to 0^+} (1 + 2\cos x)^2$

7. The function, $\sqrt{x-1} + \sqrt[3]{x-3} + \sqrt[4]{5-x}$ and $\sin^{-1}\left(\frac{x-3}{2}\right)$ have identical domains . **[True or False]**

8. If f(x) is non - zero polynomial function such that $f(2x) = f'(x)\, f''(x)$, then f(x) = _______ .

MATHEMATICS
DPP
DAILY PRACTICE PROBLEMS

DPP No. 20

Total Marks : 33

Max. Time : 37 min.

Topics : Fundamentals of Mathematics, Function, Limits

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,4	(3 marks, 3 min.)	[9, 9]
Subjective Questions (no negative marking) Q.3,5,6,7	(4 marks, 5 min.)	[16, 20]
Match the Following (no negative marking) Q.8	(8 marks, 8 min.)	[8, 8]

1. If f(x) = $\frac{x-1}{x+1}$, then f(f(ax)) in terms of f(x) is equal to

(A) $\frac{f(x)-1}{a(f(x)-1)}$ (B) $\frac{f(x)+1}{a(f(x)-1)}$ (C) $\frac{f(x)-1}{a(f(x)+1)}$ (D) $\frac{f(x)+1}{a(f(x)+1)}$

2. If f(x) = $((\text{sgn } x)^{\text{sgn } x})^n$; n is an odd integer. Then

(A) f(x) is an odd function (B) f(x) is an even function
(C) f(x) = 0 (D) none of these

3. Let f(x) = $\frac{2}{4^x+2}$ for real numbers x. Evaluate : $f\left(\frac{1}{2011}\right)+f\left(\frac{2}{2011}\right)+......+f\left(\frac{2010}{2011}\right)$.

4. In which of the following functions, range is singleton set.

(A) f(x) = [x] + [–x] (B) f(x) = {x} + {– x} (C) f(x) = $|\text{sgn}(x)|$ (D) f(x) = $\left[\sqrt{x-[x]}\right]$

where [x], {x} and sgn(x) are greatest integer function, fractional part function and signum function respectively.

5. Evaluate % (i) $\lim\limits_{x\to\pi}\frac{1-\sin\frac{x}{2}}{\cos\frac{x}{2}\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)}$ (ii) $\lim\limits_{x\to 0}\left(\frac{\sqrt{1+x\sin x}-\sqrt{\cos 2x}}{\tan^2(x/2)}\right)$

6. Solve the inequality : $\left(2\log_3^2 x-3\log_3 x-8\right)\left(2\log_3^2 x-3\log_3 x-6\right)\ge 3$.

7. Let f(x) = $\frac{|x^3-6x^2+11x-6|}{x^3-6x^2+11x-6}$. Find the set of points 'a' where $\lim\limits_{x\to a}$ f(x) does not exist.

8.

Column - I		Column - II	
(A)	$\lim\limits_{x\to 0}[\sin|x|-|x|]=$	(p)	0
(B)	$\lim\limits_{x\to 0}\left[\frac{x}{[x]}\right]=$	(q)	1
(C)	$\lim\limits_{x\to \frac{1}{2}}\left[x\left[\frac{1}{x}\right]\right]=$	(r)	Does not exist
(D)	$\lim\limits_{x\to -1}\left[\frac{[x]}{x}\right]=$	(s)	– 1

($\because$ where [.] denotes greatest integer function)

DPP 11 TO 20 (ANSWER KEY)

DPP NO. - 11

1. (B) **2.** (D) **3.** (C) **4.** (C)
5. (C) **6.** (1) **7.** (C)
8. $a \geq \frac{5}{2}$

DPP NO. - 12

1. (C) **2.** (A) **3.** (C) **4.** (1)
5. (C) **6.** (A) **7.** (A) **8.** (A)

DPP NO. - 13

1. (B) **2.** (D) **3.** (A) **4.** (D)
5. (B) **6.** (C) **7.** (D) **8.** (A)
9. (C) **10.** (A) **11.** (B) **12.** (D)
13. (C) **14.** (B) **15.** (B)

DPP NO. - 14

1. (D) **2.** (D) **3.** (D) **4.** (B)
5. (C) **6.** (B)
7. (i) $[1, \infty)$
(ii) $x \in [2n\pi, 2n\pi + \pi] - (2n + 1)\frac{\pi}{2}, n \in I$
(iii) $(4, \infty)$ (iv) ϕ
8. (i) $x = 2$ (ii) $(1/2, 2]$

DPP NO. - 15

1. (A) **2.** (C) **3.** (C) **4.** (D)
5. (A) **6.** (C)
7. (i) $(-4, 4)$ (ii) $[-3, 3]$ (iii) $[\cos 2, 1]$(iv) $[0, 1/2]$
8. 1752

DPP NO. - 16

1. (C) **2.** (D) **3.** (C) **4.** (A C)
5. (0, 1)
6. (i) one-one, onto (ii) many-one, into
(iii) one-one, onto **8.** $x = 4, 13$

DPP NO. - 17

1. (C) **2.** (D) **3.** (B) **4.** (B C D)
5. (A B C) **6.** (C D) **7.** – 2, 0
8. (A)→(q,r), (B)→(r,s), (C)→(p,q), (D)→(p,s, t)

DPP NO. - 18

1. (C) **2.** (A) **3.** (C) **4.** (A B D)
5. (i) 2 (ii) 24 **6.** 5/2
7. (i) $\frac{0}{0}$ (ii) not defined (iii) non indeterminate
(iv) not defined (v) not defined
8. (A) → (s) ; (B) → (q) ; (C) → (r) ; (D) → (p)

DPP NO. - 19

1. (C) **2.** (B) **3.** (A)(D) **4.** (D)
5. $x \in (-5, 0) \cup (2, 5)$ **6.** (i) 1/6 (ii) 9
7. True **8.** $\frac{4x^3}{9}$

DPP NO. - 20

1. (C) **2.** (A) **3.** 1005 **4.** (D)
5. (i) $\frac{1}{\sqrt{2}}$ (ii) 6
6. $\left(0, \frac{1}{\sqrt{27}}\right] \cup \left[\frac{1}{3}, \sqrt{243}\right] \cup [27, \infty)\ ]$
7. a = 1, 2, 3.
8. (A) → (s), (B) → (r), (C) → (p), (D) → (q)

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अभ्यास ही सबसे बड़ा गुरू है।

CLASS : XII (MATHS)

DPP

DAILY PRACTICE PROBLEM

Solutions

DPP-11 to 20

DPP 11 : **Inverse Trigonometric Function, Set & Relation, Fundamentals of Mathematics, Matrices & Determinants, Quadratic Equation**

DPP 12 : **Inverse Trigonometric Function, Set & Relation, Fundamentals of Mathematics, Matrices & Determinants, Quadratic Equation, Trigonometric Ratio**

DPP 13 : **Fundamentals of Mathematics, Sequence & Series, Trigonometric Ratio, Matrices & Determinants, Binomial Theorem, Straight Line, Permutation & Combination, Complex Number, Circle, Ellipse, Set & Relation**

DPP 14 : **Inverse Trigonometric Function, Matrices & Determinants, Fundamentals of Mathematics, Trigonometric Ratio, Function, Quadratic Equation**

DPP 15 : **Function**

DPP 16 : **Function, Inverse Trigonometric Function**

DPP 17 : **Function, Quadratic Equation**

DPP 18 : **Fundamental of Mathematics, Function, Limits**

DPP 19 : **Fundamentals of Mathematics, Function, Limits**

DPP 20 : **Fundamentals of Mathematics, Function, Limits**

DPP NO. - 11

1. $\tan^{-1}\left(\frac{1}{a-1}\right) - \tan^{-1}\left(\frac{1}{a^2-x+1}\right) = \tan^{-1}\left(\frac{1}{x}\right)$

$\Rightarrow \tan^{-1}\left\{\frac{a^2-x+1-a+1}{(a-1)(a^2-x+1)+1}\right\} = \tan^{-1}\left(\frac{1}{x}\right)$

$\Rightarrow \frac{a^2+1-x-a+1}{(a-1)(a^2+1-x)+1} = \frac{1}{x}$

Solving we set $x = a, x = a^2 - a + 1$

2. $|2A^9 B^{-1}|$

$2^2|A^9| |B^{-1}|$

$\frac{2^2 |A|^9}{|B|} = \frac{2^2(-1)^9}{2} = -2$

3. $ax^2 + 2cx + b = 0$ and $ax^2 + 2bx + c = 0$ .... (2)

Subtracting $2 (c-b) x = (c-b)$

$x = \frac{1}{2}$ is a common root

Put in equation $\frac{a}{4} + \frac{2c}{2} + b = 0$

$\Rightarrow a + 4c + 4b = 0$

4. $\sin^{-1}x + \cos^{-1}y + \sin^{-1}z = 2\pi$

Possible when

$\sin^{-1}x = \frac{\pi}{2} \Rightarrow x = \sin\frac{\pi}{2} = 1$

$\cos^{-1}y = \pi \Rightarrow y = \cos\pi = -1$

$\sin^{-1}(z) = \frac{\pi}{2} \Rightarrow z = \sin\frac{\pi}{2} = 1$

only one triplet (1, – 1, 1)

5. $X = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}^{-1} \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix} = \frac{1}{10}$

$\begin{bmatrix} -2 & 4 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ \frac{11}{2} & 2 \end{bmatrix}$

7. As $A = \{1, 2\}, B = \{0\}$

$\therefore$ n(A) = 2, n(B) = 1

$\therefore$ number of relations from A to B is $= 2^{n(A)\times n(B)}$

$= 2^{2\times1} = 4$

8. $\log_{(3x+2)} (x^2 - 3x + 7) < 1$

Case - I When $0 < 3x + 2 < 1$

$\Rightarrow x^2 - 3x + 7 > 3x + 2$

$-\frac{2}{3} < x < \frac{-1}{3} \Rightarrow x^2 - 6x + 5 > 0$ ....(i)

$\Rightarrow (x - 1) (x - 5) > 0$

$x \in (-\infty , 1) \cup (5, \infty)$ ....(ii)

and $x^2 - 3x + 7 >$ $\Rightarrow$ always positive $D < 0$

$x \in \left(\frac{-2}{3}, \frac{-1}{3}\right)$

case-II; when $3x + 2 > 1$

$\Rightarrow x > \frac{-1}{3}$ ....(ii)

and $x^2 - 3x + 7 < 3x + 2 \Rightarrow x^2 - 6x + 5 < 0$

$(x -1) (x - 5) < 0$

$x \in (1, 5)$ ....(iv)

$\Rightarrow$ from (iii) and (iv) $x \in (1, 5)$

From (1) $1^2 + 5 - 2a \le 10a \Rightarrow \frac{1}{2} \le a$

and $25 + (5 - 2a)5 \le 10a \Rightarrow \frac{5}{2} \le a$

DPP NO. - 12

1 to 3. $f(x) = (x + 1) (x + 2) (ax + b)$

$f(-1) = 4 \Rightarrow -2(-a + b) = 4$

$\Rightarrow a - b = 2$

$f(2) = 28 \Rightarrow 4 (2a + b) = 28$

$\Rightarrow 2a + b = 7$

On solving a = 3, b = 1 factor is (3x + 1)

and polynomial $f(x) = (x - 1) (x + 2) (3x + 1)$

$f(-3) = (-4) (-1) (-8) = -32$

Also $P(i) = (i - 1) (i + 2) (3i + i)$

$= (i^2 + i - 2) (3i + 1) = (i - 3) (3i + 1)$

$= 3i^2 - 8i - 3 = -6 - 8i$

4. $n(1) = n(A - B) + n(A \cap B)$

$\therefore n(A \cap B) = n(A) - n(A - B)$

$\therefore n(A \cap B) = 110 - 50 = 60$

Now $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 350$

5. $S_n = \cot^{-1}\left(\frac{7}{4}\right) + \cot^{-1}\left(\frac{19}{4}\right) + \cot^{-1}\left(\frac{39}{4}\right) +$....... to

nth terms

$\therefore S_\infty = \lim_{n\to\infty}\left[\cot^{-1}\left(1+\frac{3}{4}\right)+\cot^{-1}\left(4+\frac{3}{4}\right)+\cot^{-1}\left(9+\frac{3}{4}\right)+......\text{to n terms}\right]$

$= \lim_{n\to\infty}\sum_{r=1}^{n}\cot^{-1}\left(r^2+\frac{3}{4}\right) = \lim_{n\to\infty}\sum_{r=1}^{n}\cot^{-1}\left(\frac{4r^2+3}{4}\right)$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{4}{4r^2+3}\right)$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{r^2+\frac{3}{4}}\right)$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r^2-\frac{1}{4}}\right)$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left[\frac{\left(r+\frac{1}{2}\right)-\left(r-\frac{1}{2}\right)}{1+\left(r+\frac{1}{2}\right)\left(r-\frac{1}{2}\right)}\right]$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\left[\tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\left(r-\frac{1}{2}\right)\right]$$

$$= \lim_{n\to\infty}\left[\tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\frac{1}{2}\right]$$

$$= \tan^{-1}\infty - \tan^{-1}\frac{1}{2} = \frac{\pi}{2} - \tan^{-1}\frac{1}{2}$$

$$= \cot^{-1}\frac{1}{2} = \tan^{-1} 2$$

6. $A^TA^{-1} = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix} \bullet \frac{1}{1+\tan^2\theta}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$$

7. $\tan 2\alpha = \tan((\alpha + \beta) + (\alpha - \beta))$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)} = \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}} = \frac{(9+5)4}{48-15}$$

$$= \frac{14\times 4}{33} = \frac{56}{33}$$

8. $x^3 + ax^2 + 11x + 6$ Roots α, β, γ

$\alpha + \beta + \gamma = -a$ ...(i)
$\alpha\beta + \beta\gamma + \gamma\alpha = 11$ ...(ii)
$\alpha\beta\gamma = -6$ ...(iii)
$x^3 + bx^2 + 14x + 8$ Roots are α, β, δ
$\alpha + \beta + \delta = -b$ ...(iv)
$\alpha\beta + \beta\delta + \alpha\delta = 14$ ...(v)
$\alpha\beta\delta = -8$ ...(vi)

Now $x^2 + px + q$ have roots α, β

$\alpha + \beta = -p, \alpha\beta = q \quad \frac{\gamma}{\delta} = \frac{3}{4}$, $-p + \gamma = -a$ and $\gamma = \frac{-6}{q}$ and $\delta = p - b$

also $(a - b)x^2 - 3x - 2 = 0$ and $x^2 + px + q = 0$

comparing $\frac{a-b}{1} = \frac{-3}{p} = \frac{-2}{q} \Rightarrow \frac{2p}{3} = q$

also from (ii) $q + \gamma(-p) = 11 \Rightarrow \quad q + \frac{-6}{q}(-p) = 11 \Rightarrow$

$$\frac{2p}{3} + 6\left(\frac{3}{2}\right) = 11$$

$\Rightarrow \frac{2p}{3} = 2 \Rightarrow p = 3$ and $q = 2$ & γ

$= -3,\ \delta = \frac{4\gamma}{3} = -4$

$a = p - \gamma = 3 - (-3) = 6,\ b = 3 - (-4) = 7$ and check option.

DPP NO. - 13

1. Let $e^{\sin x} = t$

$\Rightarrow t^2 - 4t - 1 = 0$

$\Rightarrow t = \frac{4 \pm \sqrt{16+4}}{2}$

$\Rightarrow t = e^{\sin x} = 2 \pm \sqrt{5}$

$\Rightarrow e^{\sin x} = 2 - \sqrt{5}$, $\quad e^{\sin x} = 2 + \sqrt{5}$

$e^{\sin x} = 2 - \sqrt{5} < 0, \Rightarrow \quad \sin x = ln(2 + \sqrt{5}) > 1$

so rejected   so rejected

hence no solution

2. $A(u_1 + u_2) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \qquad |A| = 1$

$$A^{-1} = \frac{1}{|A|}\text{ adj } A$$

$$u_1 + u_2 = A^{-1}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

3. $(\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n}$

$= 2[{}^{2n}C_1(\sqrt{3})^{2n-1} + {}^{2n}C_3(\sqrt{3})^{2n-3} + {}^{2n}C_5(\sqrt{3})^{2n-5} + \ldots\ldots..]$

= which is an irrational number

4. $100(a + 99d) = 50(a + 49d)$
$2a + 198d = a + 49d$
$a + 149d = 0$
$T_{150} = a + 149d = 0$

5. $3\sin P + 4\cos Q = 6$ ...(i)
$4\sin Q + 3\cos P = 1$ ...(ii)

Squaring and adding (i) & (ii) we get $\sin(P + Q) = \frac{1}{2}$

$\Rightarrow P + Q = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ $\Rightarrow R = \frac{5\pi}{6}$ or $\frac{\pi}{6}$

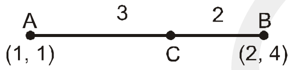
If $R = \frac{5\pi}{6}$ then $0 < P, Q < \frac{\pi}{6}$

$\Rightarrow \cos Q < 1$ and $\sin P < \frac{1}{2}$

$\Rightarrow 3\sin P + 4\cos Q < \frac{11}{2}$

So $R = \frac{\pi}{6}$

6.



$\therefore C\left(\frac{8}{5}, \frac{14}{5}\right)$

Line $2x + y = k$ passes $C\left(\frac{8}{5}, \frac{14}{5}\right)$

$\frac{2 \times 8}{5} + \frac{14}{5} = k$
$k = 6$

7. $(10 + 1)(9 + 1)(7 + 1) - 1$
$= 11.10.8 - 1 = 879$

8. $\frac{z^2}{z-1} = \frac{\bar{z}^2}{\bar{z}-1}$

$\Rightarrow z\bar{z}z - z^2 = z\bar{z}\,\bar{z} - \bar{z}^2$

$\Rightarrow |z|^2(z - \bar{z}) - (z - \bar{z})(z + \bar{z}) = 0$

$\Rightarrow (z - \bar{z})(|z|^2 - (z + \bar{z})) = 0$

Either $z = \bar{z} \Rightarrow$ real axis

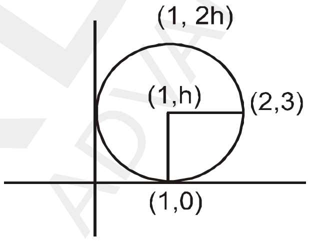
or $|z|^2 = z + \bar{z} \Rightarrow z\bar{z} - z - \bar{z} = 0$

represents a circle passing through origin.

9. Subtracting $P^3 - P^2Q = Q^3 - Q^2P$
$P^2(P - Q) + Q^2(P - Q) = 0$
$(P^2 + Q^2)(P - Q) = 0$
If $|P^2 + Q^2| \neq 0$ then $P^2 + Q^2$ is invertible
$\Rightarrow P - Q = 0$ contradiction
Hence $|P^2 + Q^2| = 0$

10. Now
$h^2 = (1 - 2)^2 + (h - 3)^2$
$0 = 1 - 6h + 9$
$6h = 10$

$h = \frac{5}{3}$



Now diameter is $2h = \frac{10}{3}$

11. Every element has 3 options. Either set Y or set Z or none
so number of ordered pairs = 3^5

12. $\Rightarrow$ Length of semi minor axis is = 2
Length of semi major axis is 4
then equation of ellipse is

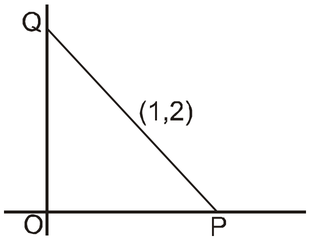
$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$

$x^2 + 4y^2 = 16$

13. $(y - 2) = m(x - 1)$

$OP = 1 - \frac{2}{m}$

$OQ = 2 - m$



Area of ΔPOQ = $\frac{1}{2}$(OP)(OQ) = $\frac{1}{2}\left(1-\frac{2}{m}\right)(2-m)$

$$= \frac{1}{2}\left[2-m-\frac{4}{m}+2\right]$$

$$= \frac{1}{2}\left[4-\left(m+\frac{4}{m}\right)\right]$$

m = –2

14. $T_n = (n-1)^2 + (n-1)n + n^2$

$$= \frac{\left((n-1)^3 - n^3\right)}{(n-1)-n} = n^3 - (n-1)^3$$

$T_1 = 1^3 - 0^3$
$T_2 = 2^3 - 1^3$
⋮
$T_{20} = 20^3 - 19^3$
$S_{20} = 20^3 - 0^3 = 8000$

15. Equation of tangent to the ellipse $\frac{x^2}{2}+\frac{y^2}{4} = 1$ is

$y = mx \pm \sqrt{2m^2+4}$ .....(1)

equation of tangent to the parabola $y^2 = 16\sqrt{3}\,x$

is $y = m\,x + \frac{4\sqrt{3}}{m}$ .....(2)

On comparing (1) and (2)

$$\frac{4\sqrt{3}}{m} = \pm\sqrt{2m^2+4}$$

$\Rightarrow$ $48 = m^2(2m^2+4)$ $\Rightarrow$ $2m^4 + 4m^2 - 48 = 0$
$\Rightarrow$ $m^4 + 2m^2 - 24 = 0$ $\Rightarrow$ $(m^2+6)\ (m^2-4) = 0$
$\Rightarrow$ $m^2 = 4$ $\Rightarrow$ $m = \pm 2$
$\Rightarrow$ equation of common tangents are

$y = \pm 2x \pm 2\sqrt{3}$

statement -1 is true.
statement-2 is obviously true.

DPP NO. - 14

1. $\because$ $x^2 + ax + \sin^{-1}(x^2 - 4x + 5) + \cos^{-1}(x^2 - 4x + 5) = 0$ ....(1)

for equation (1) to be defined.
$-1 \le x^2 - 4x + 5 \le 1$
$\because$ $x^2 - 4x + 5 > 0\ \forall\ x \in R$
$\Rightarrow$ $x^2 - 4x + 4 \le 0$
$(x-2)^2 \le 0$
$\Rightarrow$ x = 2
$\therefore$ equation (1) will be defined if x = 2, we get.

$$4 + 2a + \frac{\pi}{2} = 0.$$

$$2a = -\left(4+\frac{\pi}{2}\right) \Rightarrow a = -\left(2+\frac{\pi}{4}\right) \text{ Ans.}$$

2. $|\text{adj } A^{-1}| = |A^{-1}|^2 = \frac{1}{|A|^2} = \frac{1}{4}$

3. $\begin{bmatrix}\alpha^2 & \alpha\\ \beta^2 & \beta\end{bmatrix}\begin{bmatrix}6\\-5\end{bmatrix} = \begin{bmatrix}-1\\-1\end{bmatrix}$

$\Rightarrow$ $6\alpha^2 - 5\alpha = -1$ and $6\beta^2 - 5\beta = -1$
$\Rightarrow$ α & β are roots of the equation $6x^2 - 5x + 1 = 0$
$\Rightarrow$ $(\alpha,\ \beta) = \left(\frac{1}{2},\frac{1}{3}\right)$ or $\left(\frac{1}{3},\frac{1}{2}\right)$

Now $|A| = \alpha^2\beta - \beta^2\alpha = \alpha\beta\,(\alpha-\beta) = \frac{1}{36}$ or $-\frac{1}{36}$

4. $\sin^2 x + \sin x = 1 \Rightarrow \sin x = \cos^2 x$
$\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 1$
$= (\sin^6 x) + 3\sin^5 x + 3\sin^4 x + (\sin^3 x) - 1$
$= (\sin^2 x)^3 + 3(\sin^2 x)^2 \sin x + 3\sin^2 x \sin^2 x + (\sin x)^3 - 1$
$= (\sin^2 x + \sin x)^3 - 1 = 1 - 1 = 0$

5. defined for $\sin^{-1}\sqrt{x^2+3x+2} > 0$

$\Rightarrow$ $\sqrt{x^2+3x+2} > 0$ true for all x.

also $-1 \le \sqrt{x^2+3x+2} \le 1$
Here $x^2 + 3x + 2 > 0$ or $x^2 + 3x + 1 \le 0$

$(x+1)(x+2) > 0$ or $\frac{-3-\sqrt{5}}{2} \le x \le \frac{-3+\sqrt{5}}{2}$

Ans. $\left[\frac{-3-\sqrt{5}}{2},-2\right) \cup \left(-1,\frac{-3+\sqrt{5}}{2}\right]$

6. Let $1 + \tan^2\alpha = t$ $\Rightarrow$ $\tan\alpha = \sqrt{t-1}$

equation $(\sqrt{t-1})^3 - p\,(t-1) - r = 0$

$\Rightarrow$ $(\sqrt{t-1})^3 = pt + r - p$
$\Rightarrow$ $t^3 - 1 - 3t^2 + 3t = p^2t^2 + (r-p)^2 + 2p\,(r-p)t$
$\Rightarrow$ $t^3 + (-3-p^2)\,t^2 + (3 - 2p(r-p))t + (-1-(r-p)^2) = 0$
then product of roots $= 1 + (r-p)^2$

7. (i) $x + \sqrt{x-1} \ge 0,\ x - 1 \ge 0$
$\Rightarrow$ $x \ge 1$
$[1, \infty)$
(ii) $1 + \sec^2 x > 0$
$\sin x \ge 0$

$x \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

$x \in [2n\pi, 2n\pi + \pi] - (4n + 1)\,\frac{\pi}{2}, n \in I$

(iii) $|x + 1| \neq 1 \;\Rightarrow\; x + 1 = \pm 1,\; x \neq 0, x \neq -2$

$x \neq -1$

$x - 3 > 0 \;\Rightarrow\; x > 3$

for $x > 3$

$\log_{|x+1|} \sqrt{x-3} > 0$

$\sqrt{x-3} > 1$

$x - 3 > 1$

domain $(4, \infty)$

(iv) $-1 \leq x^2 - x - 1 \leq 1$

$x^2 - x \geq 0$ or $x^2 - x - 2 \leq 0$

$x(x - 1) \geq 0$ or $(x - 2)(x + 1) \leq 0$

$[-1, 0] \cup [1, 2]$ and $x^2 - 9 \neq 0$

$x \neq \pm 3$

$x - 2 > 0$

$x > 2$ and $x - 2 \neq 1 \;\Rightarrow\; x \neq 3$

domain ϕ

8. (i) $\frac{4x+19}{x+5} - \frac{4x-17}{x-3} < 0$

$$\Rightarrow \frac{(4x^2+19x-12x-57)-(4x^2-17x+20x-85)}{(x-3)(x+5)} < 0$$

$$\Rightarrow \frac{(x+7)}{(x-3)(x+5)} < 0$$

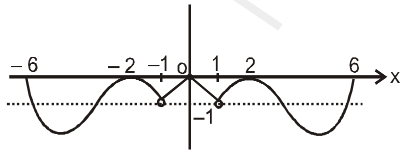
$x \in (-\infty, -7) \cup (-5, 3)$ largest integral $x = 2$.

(ii) $\sqrt{\frac{x-2}{1-2x}} > -1$

True for all $\frac{x-2}{1-2x} \geq 0 \Rightarrow \frac{x-2}{2x-1} \leq 0$

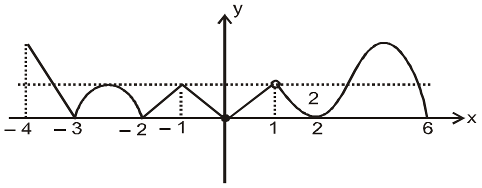
DPP NO. - 15

1 to 3. (A) $y = f(|x|)$



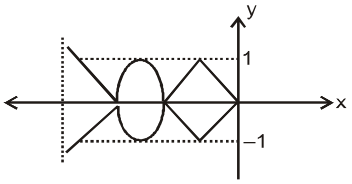
Represent a function since line parallel to y-axis cut once

(B) $y = |f(x)|$



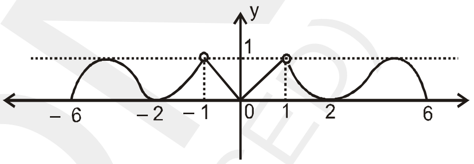
Represent a function since line parallel to y-axis cut once

(C) $|y| = f(x)$



Not represent a function

(D) $y = |f(|x|)|$



Represent a function since line parallel to y-axis cut once

4. defined of $\cos(\sin x) \geq 0$ and $-1 \leq x^2 - 1 \leq 1$

$\because \sin x \in [-1, 1] \;\forall\; x \in R$ and $0 \leq x^2 \leq 2$

$\forall\; t \in [-1, 1] \;\Rightarrow\; \cos t \geq 0 \;\Rightarrow\; x \in R$

and $-\sqrt{2} \leq x \leq \sqrt{2}$ Ans. $[-\sqrt{2}, \sqrt{2}]$

5. Let $y = 9^x - 3^x + 1$

Let $3^x = t$ $y = t^2 - t + 1$

$y = (t - 1/2)^2 + 3/4$ for $(-\infty, 1]$

at $x = 1$ $y = 9 - 3 + 1 = 7$

Range of y is $\left[\frac{3}{4}, 7\right]$

and Range of f(x) is {0, 1, 2, 3, 4, 5, 6, 7}.

6. $\left[x^2+\frac{1}{2}\right] = \left[x^2-\frac{1}{2}+1\right] = 1 + \left[x^2-\frac{1}{2}\right]$

for domain $\left[x^2-\frac{1}{2}\right] = 0, -1$

$\Rightarrow \left[x^2+\frac{1}{2}\right] = 1, 0$

$f(x) = \sin^{-1}(1) + \cos^{-1}(0)$

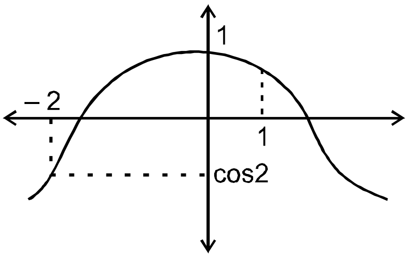
or $f(x) = \sin^{-1}(0) + \cos^{-1}(-1)$

Range of $f(x) = \{\pi\}$

7. (i) $f(x) = 4\tan x \cos x = 4\sin x$ But $x \neq (2n + 1)\frac{\pi}{2}$

Range of f(x) is $(-4, 4)$

(ii) $g(x) = 9\cos 3x - 12\cos^3 3x$



= 9 cos 3x – (3 cos 9x + 9 cos 3x)

= – 3 cos 9x

Range = [–3, 3]

(iii) h(x) = cos(2 sinx)

$-2 \le 2\sin x \le 1$

From figure range [cos 2, 1]

(iv) $y = \sqrt{x - x^2} \Rightarrow$ y is positive

$x^2 - x + y^2 = 0$

$\Rightarrow \forall x \in R \quad D \ge 0$

$\Rightarrow 1 - 4y^2 \ge 0 \Rightarrow \frac{-1}{2} \le y \le \frac{1}{2}$ But y is positive

$\Rightarrow$ Range $\left[0, \frac{1}{2}\right]$

8. Put x = 1770

$$\frac{1700}{70} < \frac{f(1770)}{70} < \frac{1800}{70}$$

$$1770 - f(1770) = 17\left[\frac{1770}{17}\right] - 70\left[\frac{f(1770)}{70}\right]$$

$\Rightarrow$ 1770 – f(1770) = 17 (104) – 70 (24)

$\Rightarrow$ f(1770) = 1770 – 1768 + 1680 = 1682 which is not possible

$\therefore$ 1770 – f (1770) = 17(104) – 70(25)

f(1770) = 2 + 1750 = 1752

DPP NO. - 16

1. Domain

$2 - |x| \ge 0 \Rightarrow |x| \le 2 \Rightarrow x = -2, -1, 0, 1, 2$

at x = ±1 y = 2

$x = \pm 2 \qquad y = \frac{1}{4}$

2. $\because$ $\sin x^3$, $\tan(x^4)$ are not periodic function

3. $f(x) = \tan^{-1}\left(\left(x + \frac{1}{2}\right)^2 + a - \frac{1}{4}\right)$

$\because$ Range is $\left[0, \frac{\pi}{2}\right) \Rightarrow \left(x + \frac{1}{2}\right)^2 + a - \frac{1}{4} \ge 0 \Rightarrow$

for onto function

$a = \frac{1}{4}$ Ans.

4. $x \in [0, 2\pi]$, $y_1 = \frac{\sin x}{|\sin x|}$, $y_2 = \frac{|\cos x|}{\cos x}$

when $x \in \left(0, \frac{\pi}{2}\right)$, $y_1 = \frac{\sin x}{\sin x} = 1$, $y_2 = \frac{\cos x}{\cos x} = 1$

identical

when $x \in \left(\frac{\pi}{2}, \pi\right)$, $y_1 = 1$, $y_2 = -1$ Not identical

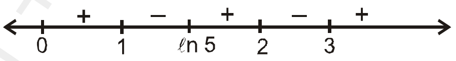
when $x \in \left(\pi, \frac{3\pi}{2}\right)$, $y_1 = -1$, $y_2 = -1$

identical

when $x \in \left(\frac{3\pi}{2}, 2\pi\right)$, $y = -1$, $y_2 = 1$ Not identical

5. f(x) = ℓn tan^{-1} ((x – 1) (x – 2) (x – 3) x (e^x – 5))

for defined (x – 1) (x – 2) (x – 3)x (e^x – 5) > 0



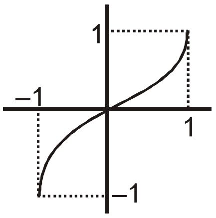
A = Domain (0, 1) ∪ (ℓn 5, 2) ∪ (3, ∞)

and $g(x) = 1 - \cos^2\frac{x}{4} + \cos\frac{x}{4} = \frac{5}{4} - \left(\cos\frac{x}{4} - \frac{1}{2}\right)^2$

B = Range $\left[-1, \frac{5}{4}\right]$ A ∩ B = (0, 1)

6. (i) $f(x) = \begin{cases} -x^2 & , \ -1 \le x < 0 \\ 0 & , \quad x = 0 \\ x^2 & , \ 0 < x \le 1 \end{cases}$

co-domain = [–1, 1]



Range = [–1, 1]

f(x) is one one, onto f(x)

(ii) $y = f(x) = \frac{x^2}{x^2 + 1}$

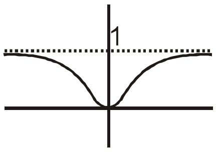
$$= \frac{(x^2 + 1)(2x) - x^2(2x)}{(x^2 + 1)^2} = \frac{2x}{(x^2 + 1)^2}$$

for x > 0, f′(x) > 0 and for x < 0, f′(x) < 0

f(x) is many one

$yx^2 + y = x^2 \quad \Rightarrow \quad x^2 = \frac{y}{1-y}$

$\because \quad x^2 \ge 0 \quad \Rightarrow \quad \frac{y}{1-y} \ge 0$



$\Rightarrow \quad \frac{y}{y-1} \le 0, \; y \in [0, 1]$

Range = [0, 1) ≠ codomain f(x) is into f(x)

(iii) $f(x) = \frac{x-2}{x-3} = y$

$f'(x) = \frac{(x-3)-1(x-2)}{(x-3)^2} = \frac{-1}{(x-3)^2}$

$\because$ f′(x) < 0 decreasing so f(x) is one-one $\quad f'(x) < 0$

also $y = \frac{x-2}{x-3}$

$\Rightarrow \quad xy - 3y = x - 2$

$x(y - 1) = 3y - 2$

$x = \frac{3y-2}{y-1}$

Range = R – {1}

f(x) is onto

7. $\prod\left(1 - \frac{4}{(2r-1)^2}\right) = \prod\left(\frac{(2r-1)^2 - 2^2}{(2r-1)^2}\right)$

$= \prod\left(\frac{(2r-3)(2r+1)}{(2r-1)(2r-1)}\right)$

Put r = 1, 2, 3, 4,..............n

$= \left(\frac{-1}{1}\cdot\frac{3}{1}\right)\left(\frac{1}{3}\cdot\frac{5}{3}\right)\left(\frac{3}{5}\cdot\frac{7}{5}\right)\left(\frac{5}{7}\cdot\ldots\right)$

$= (-1)\left(\frac{2n+1}{2n-1}\right).$

8. $\cot^{-1}(17 - x)$

$= \tan^{-1}\left(\frac{1}{3}\right) - \tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}\left(\frac{\frac{1}{3} - \frac{1}{x}}{1 + \frac{1}{3x}}\right)$

$\Rightarrow \quad \tan^{-1}\left(\frac{1}{17-x}\right) = \tan^{-1}\left(\frac{x-3}{3x+1}\right)$

$\Rightarrow \quad 3x + 1 = (x - 3)(17 - x)$

$\Rightarrow \quad 3x + 1 = 17x - 51 - x^2 + 3x$

$\Rightarrow \quad x^2 - 17x + 52 = 0 \quad \Rightarrow \quad (x - 13)(x - 4) = 0$

$\Rightarrow \quad x = 4, x = 13$

DPP NO. - 17

1. f(x + f(x)) = 4f(x)

Put x = 1, f(1 + f(1)) = 4f(1)

$\Rightarrow$ f(1 + 4) = 4(4) $\quad \Rightarrow \quad$ f(5) = 16

again put x = 5

f(5 + 16) = 4f(5) $\quad \Rightarrow \quad$ f(21) = 4(16) = 64.

2. $f(x) = \frac{e^x - e^{-|x|}}{e^x + e^{|x|}}$

as. $x \ge 0$, $\quad f(x) = \frac{e^x - e^{-x}}{e^x + e^x}$

$= \frac{e^x - e^{-x}}{2e^x} = \frac{1}{2} - \frac{1}{2e^{2x}} \qquad \text{Range} = \left[0, \frac{1}{2}\right)$

as x < 0, $f(x) = \frac{e^x - e^x}{e^x + e^{-x}} = 0$

Range [0, 1/2)

3. $y = f(x) = \begin{cases} \frac{x^2}{1+x^2}, & x < 0 \\ 0, & x = 0 \\ \frac{-x^2}{1+x^2}, & x > 0 \end{cases}$

when $\quad x < 0 \Rightarrow y = \frac{x^2}{1+x^2} \quad \Rightarrow \quad x^2(1 - y) = y$

$\Rightarrow \quad x = -\sqrt{\frac{y}{1-y}} \qquad \because (x < 0)$

when $\quad x > 0 \Rightarrow y = \frac{-x^2}{1+x^2} \quad \Rightarrow \quad x^2 = \frac{-y^2}{1+y}$

$\Rightarrow \quad x = +\sqrt{\frac{-y}{1+y}}$

$\Rightarrow \quad f^{-1}(y) = x = \text{sgn}(-y)\sqrt{\dfrac{|y|}{1-|y|}}$.

4. $f\left(2x+\dfrac{y}{8}, 2x-\dfrac{y}{8}\right) = xy$

Let $m = 2x + \dfrac{y}{8}$, $\quad n = 2x - \dfrac{y}{8}$

$x = \dfrac{m+n}{4}$, $\quad m - n = \dfrac{y}{4}$

$y = 4(m - n)$

$f(m, n) = \left(\dfrac{m+n}{4}\right) . 4(m-n) = m^2 - n^2$

Similarly $\quad f(n, m) = n^2 - m^2$

$f(m, n) + f(n, m) = 0$

5. Fundamental period is $|\sin x + \cos x|$ is $\dfrac{\pi}{2}$

Now $f\left(\dfrac{\pi}{2}+x\right)$

$= \dfrac{|\cos x|+|\sin x|}{|\cos x+\sin x|+|\cos x-\sin x|} = f(x)$

6. $f(2x) + f\left(\dfrac{1}{11}+2x\right) + f\left(\dfrac{2}{11}+2x\right) + f\left(\dfrac{3}{11}+2x\right) + ...+$

$f\left(\dfrac{21}{11}+2x\right) = k$

Now $2x \to 2x + \dfrac{1}{11}$

$f\left(2x+\dfrac{1}{11}\right) + f\left(2x+\dfrac{2}{11}\right) + f\left(2x+\dfrac{3}{11}\right)$

$+ f\left(2x+\dfrac{4}{11}\right) + ... + f\left(2x+\dfrac{23}{11}\right) = k$

on subtracting $f(2x) = f(2x + 2)$

7. $D = (1 - a)^2 + 4(a + 2) = a^2 + 2a + 9$

$= (a + 1)^2 + 8 = \alpha^2$ (let) where a is an integer

$\Rightarrow \alpha^2 - (a + 1)^2 = 8$

$\Rightarrow (\alpha - a - 1)(\alpha + a + 1) = 8$

Case- I $\alpha - a - 1 = 4$ and $\alpha + a + 1 = 2 \Rightarrow \alpha = 3 \Rightarrow a = -2, 0$

Case- II $\alpha - a - 1 = 2$ and $\alpha + a + 1 = 4 \Rightarrow \alpha = 3 \Rightarrow a = -2, 0$

Case- III $\alpha - a - 1 = -4$ and $\alpha + a + 1 = -2 \Rightarrow \alpha = -3 \Rightarrow a = -2, 0$

Case- IV $\alpha - a - 1 = -2$ and $\alpha + a + 1 = -4 \Rightarrow \alpha = -3 \Rightarrow a = -2, 0$

Case- V $\alpha - a - 1 = 8$ and $\alpha + a + 1 = 1 \Rightarrow \alpha = \dfrac{9}{2}$ not possible since α is integer

Case- VI $\alpha - a - 1 = 1$ and $\alpha + a + 1 = 8$

$\Rightarrow \alpha = \dfrac{9}{2}$ not possible since α is integer

Case- VII $\alpha - a - 1 = -8$ and $\alpha + a + 1 = -1$

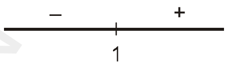
$\Rightarrow \alpha = -\dfrac{9}{2}$ not possible since α is integer

Case- VIII $\alpha - a - 1 = -1$ and $\alpha + a + 1 = -8$

$\Rightarrow \alpha = -\dfrac{9}{2}$ not possible since α is integer

8. (A) $f'(x) = \dfrac{-1}{1+(2x-x^2-2)^2}\,(2-2x)$

$= \dfrac{2(x-1)}{1+(2x-x^2-2)^2}$



For $\quad x < 1 \quad f'(x)$ is negative

for $\quad x > 1 \quad f'(x)$ is positive

$\Rightarrow$ many one

$y = 2x - x^2 - 2 \qquad \Rightarrow \quad x^2 - 2x + 2 + y = 0$

$x \in R, \; D \geq 0 \qquad \Rightarrow \quad 4 - 4(2 + y) \geq 0$

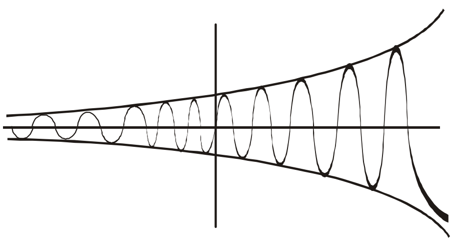
$1 - 2y \geq 0$

$\Rightarrow \quad -1 - y \geq 0$

$\Rightarrow \quad y \leq -1$

Range $\left[\dfrac{3\pi}{4}, \pi\right)$

(B)



(C) $f(x) = 2 + 3x^2$

$f'(x) = 6x > 0, \quad$ for $x > 0$

Range of $f(x) = (2, \infty)$

(D) $f(x) = f^{-1}(x)$ one - one onto

DPP NO. - 18

1. $f(x) = x^3 - 8x^2 + 20x - 13$

$= x^2 (x-1) - 7x (x-1) + 13 (x-1)$

$= (x-1)(x^2 - 7x + 13)$ is prime $\Rightarrow x - 1 = 1$

or $x^2 - 7x + 13 = 1$

$x = 2$ or $(x-4)(x-3) = 0 \Rightarrow x = 4, 3$

2. $f(15 + x) = f(15 - x)$ .... (i)

and $f(30 + x) = -f(30 - x)$

$x \to 15 - x$

$\Rightarrow f(30 - x) = f(x)$

by (2) $f(x) = -f(30 + x)$ .... (iii)

$x \to x + 30$

then $f(x + 30) = -f(x + 60)$ .... (iv)

from (iii) & (iv) $f(x) = f(x+ 60)$

$\Rightarrow$ f(x) is periodic with period 60

again $f(30 - x) = f(x)$

$x \to -x \Rightarrow f(30 + x) = f(-x)$

from (ii) $f(-x) = -f(x)$

f(x) is odd

3. (A) $f(x + T) = 1^{[x+T]} + (-1)^{[x+T]}$

$= 1^{[x]+T} + (-1)^{[x]+T}$

Periodic for T is even

Similarly for B and D

(C) $h(x + T) = 2^{[x]+T} - (-2)^{[x]+T}$

$= 2^T (2^{[x]} - (-2)^{[x]} (-1)^T) \neq h(x)$

5. (i) $\text{period} = \text{LCM}\left\{\frac{2\pi}{2\pi}, \frac{2\pi}{3\pi}, \frac{2\pi}{5\pi}\right\}$

$= \frac{\text{LCM}\{2,2,2\}}{\text{HCF}\{2,3,5\}} = \frac{2}{1} = 2$

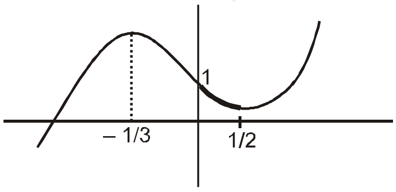
(ii) $\text{period} = \text{LCM}\left\{\frac{2\pi}{\pi/3}, \frac{2\pi}{\pi/4}\right\}$

$= \text{LCM}\{6, 8\} = 24$

6. $f'(x) = 12x^2 - 2x - 2$

$= 2(6x^2 - x - 1) = 2(3x + 1)(2x - 1) = 0$

$x = \frac{-1}{3}$ $x = \frac{1}{2}$ $f''(x) = 24x - 2$



Now $g\left(\frac{1}{4}\right) = \frac{1}{16} - \frac{1}{16} - \frac{1}{2} + 1 = \frac{1}{2}$

$\Rightarrow g\left(\frac{1}{2}\right) = 4\left(\frac{1}{8}\right) - \frac{1}{4} - 2\left(\frac{1}{2}\right) + 1 = \frac{1}{4}$

$g\left(\frac{5}{4}\right) = 3 - \frac{5}{4} = \frac{7}{4}$

$\Rightarrow g\left(\frac{1}{4}\right) + g\left(\frac{1}{2}\right) + g\left(\frac{5}{4}\right) = \frac{1}{2} + \frac{1}{4} + \frac{7}{4} = \frac{10}{4} = \frac{5}{2}$ Ans.

7. (i) $\lim_{x\to 0} \frac{\sin x^3}{x^2} \left(\frac{0}{0} \text{ form}\right)$

(ii) $\lim_{x\to 0} \frac{\sin[x^2]}{[x^2]} = \frac{\text{exactly0}}{\text{exactly0}}$ Not defined

(iii) $\lim_{x\to 0} |x|^{[\sin^2 x]} = (\text{tending to zero})^{\text{exactly zero}}$

(iv) $\text{cosec}^{-1}x$ does not exists
since domain $(-\infty, -1] \cup [1, \infty)$

(v) $\text{cosec}^{-1}x$ does not exists
since domain $(-\infty, -1] \cup [1, \infty)$

8. (A) $f(g(x)) = f\left(\frac{x+1}{x+2}\right) = \frac{x+1}{x+2} + \frac{x+2}{x+1}$

Domain $R - \{-2, -1\}$

(B) $g(f(x)) = g\left(x + \frac{1}{x}\right) = \frac{x + \frac{1}{x} + 1}{x + \frac{1}{x} + 2}$

$= \frac{x^2 + x + 1}{x^2 + 2x + 1}$

$= \frac{x^2 + x + 1}{(x+1)^2}$ also $x \neq 0$,

Domain $= R - \{-1, 0\}$

(C) $fof(x) = f\left(x + \frac{1}{x}\right)$

$= \left(x + \frac{1}{x}\right) + \frac{1}{x + \frac{1}{x}} = \frac{x^2 + 1}{x} + \frac{x}{x^2 + 1}$ Domain $R - \{0\}$

(D) $gog(x) = g\left(\frac{x+1}{x+2}\right)$

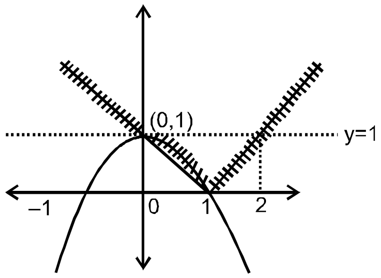
$= \frac{\frac{x+1}{x+2} + 1}{\frac{x+1}{x+2} + 2} = \frac{2x + 3}{3x + 5}$

defined $x + 2 \neq 0$ and $3x + 5 \neq 0$

$x \neq -2$ and $x = \frac{-5}{3}$

DPP NO. - 19

1.



2. $h(x) = \log_{10} x = \sum_{n=1}^{89} h(\tan n^o)$

$= \log_{10} \tan 1° + \log_{10} \tan 2° + \log_{10} \tan 3° + \log_{10} \tan 4° + ...+ \log_{10} \tan 89°$

$= \log_{10}(\tan 1° \tan 2° \tan 3° \tan 4° \tan 5° ..... \tan 89°) = \log_{10}(1) = 0.$

4. Statement-1 $\frac{3x}{2} - \frac{3}{2}$ [x] = $\frac{3}{2}$ {x}

$\therefore \quad 0 \leq \frac{3x}{2} - \frac{3}{2}[x] < \frac{3}{2}$

$\therefore \quad 0 \leq \sin\left(\frac{3}{2}x - \frac{3}{2}[x]\right) < \sin\frac{3}{2}$

$\therefore$ there is no greatest value and so statement-1 is false.

Statement-2 Obviously true

5. **Case-I :** When x < 0

$(-x-5)(-(x-1)-1) < 0$

$(x+5)(x) < 0$

$x \in (-5, 0)$

Case-II : When 0 < x < 1

$(x-5)(x) < 0$

$x \in (0, 1)$

Case-III : When x > 1

$(x-5)(x-2) < 0$

$x \in (2, 5)$

6. (i) $\lim\limits_{x\to0} \frac{\sin x^4 - x^4 \cos x^4}{x^4(e^{2x^4} - 1 - 2x^4)}$

$$= \lim_{x\to0} \frac{\left(\left(x^4 - \frac{x^{12}}{3!} + ...\right) - x^4\left(1 - \frac{(x^4)^2}{2!} + ...\right)\right)}{x^4\left(1 + 2x^4 + \frac{(2x^4)^2}{2!} + 0......... - 1 - 2x^4\right)}$$

$$= \lim_{x\to0} \frac{\frac{1}{3}x^{12} + ...}{2.x^{12}} = \frac{1}{6}$$

(ii) $\lim\limits_{x\to0} (1 + 2\cos x)^2 = (1+2)^2 = 9$

7. Domain D_1 is

$x - 1 \geq 0$ and $5 - x \geq 0$

$\Rightarrow \quad 1 \leq x \leq 5$

also domain D_2 is

$-1 \leq \frac{x-3}{2} \leq 1$

$\Rightarrow \quad 1 \leq x \leq 5$

8. Let $f(x) = ax^3 + bx^2 + cx + d$

$f(2x) = 8ax^3 + 4bx^2 + 2cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

Now $f(2x) = f'(x) f''(x)$

$(8ax^3 + 4bx^2 + 2cx + d) \equiv (3ax^2 + 2bx + c)(6ax + 2b)$

Comparing cofficient

$8a = 18a^2 \quad \Rightarrow \quad a = \frac{4}{9}$ and $4b = 6ab + 12ab$

$\Rightarrow \quad 18ab = 4b \Rightarrow a = \frac{4}{18} = \frac{2}{9}$

But $a = \frac{4}{9} \Rightarrow b = 0$ also $2c = 4b^2 + 6ac$

$\Rightarrow \quad 2c = 0 + 6ac \Rightarrow 2c = 6\left(\frac{4}{9}\right) c \Rightarrow c = 0$

and $\quad d = 2bc = 0 \quad \Rightarrow \quad f(x) = \frac{4}{9}x^3.$

DPP NO. - 20

1. Let $\frac{x-1}{x+1} = f(x) = y$

$\therefore \quad x - 1 = yx + y \Rightarrow x(y-1) = -1 - y$

$\Rightarrow \quad x = \frac{y+1}{1-y}$

Now,

$$f(f(x)) = f\left(\frac{x-1}{x+1}\right) = \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1}$$

$$= \frac{(x-1)-(x+1)}{(x-1)+(x+1)} = \frac{-2}{2x} = -\frac{1}{x}$$

$$f(f(ax)) = -\frac{1}{ax} = -\frac{1}{a\left(\frac{y+1}{1-y}\right)}$$

$$= \frac{y-1}{a(y+1)} = \frac{f(x)-1}{a(f(x)+1)}$$

2. $f(x) = \begin{cases} (1)^n = 1 & , \quad x > 0 \\ ((-1)^{-1})^n = -1 & , \quad x < 0 \end{cases}$ n is an odd integer

3. $f(x) = \dfrac{2}{4^x + 2}$

$f\,(1 - x) = \dfrac{2}{4^{1-x}+2} = \dfrac{2.4^x}{4+2.4^x}$

$\Rightarrow \quad f(x) + f(1 - x) = 1$ ..... (i)

Now put $x = \dfrac{1}{2011}$

$\Rightarrow \quad f\left(\dfrac{1}{2011}\right)+f\left(1-\dfrac{1}{2011}\right) = 1$

$f\left(\dfrac{1}{2011}\right)+f\left(\dfrac{2010}{2011}\right) = 1$

Put $x = \dfrac{2}{2011} \Rightarrow f\left(\dfrac{2}{2011}\right)+f\left(\dfrac{2009}{2011}\right) = 1$

Put $x = \dfrac{1004}{2011} \Rightarrow f\left(\dfrac{1004}{2011}\right)+f\left(\dfrac{1007}{2011}\right) = 1$

put $x = \dfrac{1005}{2011} \Rightarrow f\left(\dfrac{1005}{2011}\right)+f\left(\dfrac{1006}{2011}\right) = 1$

on adding we get sum = 1005

4. (A) when $x = I$; $f(x) = 0$ when $x \neq I \Rightarrow x = I + f$
then $f(x) = I + (- I - 1) = - 1$ Range = $\{-1, 0\}$
(B) when $x = I$, $f(x) = 0$ when $x \neq I$, $f(x) = 1$
Range = $\{0, 1\}$
(C) Range = $\{0, 1\}$
(D) $f(x) = [\sqrt{\{x\}}] = 0$ Range = $\{0\}$

5. (i) $$\lim_{x\to\pi} \frac{\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)^2}{\cos\frac{x}{2}\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)} = \lim_{x\to\pi} \frac{\cos\frac{x}{4}-\sin\frac{x}{4}}{\cos\left(\frac{x}{2}\right)}$$

$$= \lim_{x\to\pi} \frac{-\frac{1}{4}\sin\frac{x}{4}-\frac{1}{4}\cos\frac{x}{4}}{-\frac{1}{2}\sin\frac{x}{2}}$$

$$= \frac{1}{2}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right) = \frac{2}{2.\sqrt{2}} = \frac{1}{\sqrt{2}}$$

(ii) $$\lim_{x\to 0} \frac{\sqrt{1+x\sin x}-\sqrt{\cos 2x}}{\tan^2\left(\frac{x}{2}\right)}$$

$$= \lim_{x\to 0} \frac{(1-\cos 2x)+x\sin x}{\tan^2\left(\frac{x}{2}\right)(\sqrt{1+x\sin x}+\sqrt{\cos 2x})}$$

$$= \lim_{x\to 0} \frac{\sin x(2\sin x + x)}{2\tan^2\left(\frac{x}{2}\right)}$$

$$= \lim_{x\to 0} \frac{\left(\frac{\sin x}{x}\right)\left(2\frac{\sin x}{x}+1\right)}{\frac{2}{4}.\left(\frac{\tan\frac{x}{2}}{\frac{x}{2}}\right)^2} = 2(1)(3) = 6$$

6. Put $2\,\log_3^2 x - 3\,\log_3 x = t$

$(t - 8)\,(t - 6) \geq 3$

$\Rightarrow \quad t^2 - 14t + 48 \geq 3$

$\Rightarrow \quad t^2 - 14t + 45 \geq 0$

$\Rightarrow \quad (t - 9)\,(t - 5) \geq 0$

$\Rightarrow \quad t \geq 9$ or $t \leq 5$

let $\log_3 x = z$

$2z^2 - 3z \geq 9$ OR $2z^2 - 3z - 5 \leq 0$

$\Rightarrow \quad 2z^2 - 3z - 9 \geq 0 \quad \Rightarrow \quad 2z^2 - 5z + 2z - 5 \leq 0$

$\Rightarrow \quad 2z^2 - 6z + 3z - 9 \geq 0$

$\Rightarrow \quad z\,(2z - 5) + 1\,(2z - 5) \leq 0$

$\Rightarrow \quad 2z\,(z - 3) + 3\,(z - 3) \geq 0$

$\Rightarrow \quad (z+1)\,(2z-5) \leq 0$

$\Rightarrow \quad (z - 3)\,(2z + 3) \geq 0$

$z \geq 3$ or $z \leq -3/2$

$-1 \leq \log_3 z \leq 5/2$

$1/3 \leq z \leq \sqrt{243}$

$\log_3 x \geq 3$ OR $\log_3 x \leq -3/2$

$\Rightarrow$ $x \geq 27$ OR $x \leq \frac{1}{\sqrt{27}}$

7. $\lim\limits_{x\to a} \frac{|(x-1)(x-2)(x-3)|}{(x-1)(x-2)(x-3)}$

$\lim\limits_{x\to a} f(x)$ does not exist when a = 1, 2, 3

$\therefore$ $\lim\limits_{x\to a} \frac{|x|}{x}$ = does not exist

8. (A) $\mathop{\text{Lim}}\limits_{x\to 0} [\sin|x| - |x|] =$

LHL = $\mathop{\text{Lim}}\limits_{x\to 0^-} [-\sin x + x] = -1$

($\therefore$ $\sin x < x$ for $x < 0$)

RHL= $\lim\limits_{x\to 0^+} [\sin x - x] = -1$

(B) $\mathop{\text{Lim}}\limits_{x\to 0} \left[\frac{x}{[x]}\right]$ LHL = $\lim\limits_{x\to 0^-} \left[\frac{x}{[x]}\right]$

$= \lim\limits_{x\to 0^-} \left[\frac{x}{-1}\right] = 0$ $\therefore$ x is negative

RHL = $\lim\limits_{x\to 0^+} \left[\frac{x}{[x]}\right] = \left[\frac{x}{0}\right]$ = does not exist

(C) $\mathop{\text{Lim}}\limits_{x\to \frac{1}{2}} \left[x\left[\frac{1}{x}\right]\right]$

LHL = $\mathop{\text{Lim}}\limits_{x\to \frac{1}{2}^-} \left[x\left[\frac{1}{x}\right]\right] = \lim\limits_{h\to 0} \left[\left(\frac{1}{2}-h\right)\left[\frac{1}{\frac{1}{2}-h}\right]\right]$

$= \lim\limits_{h\to 0} \left[\left(\frac{1}{2}-h\right)(2)\right] = \lim\limits_{h\to 0} [1-2h] = 0$

RHL = $\lim\limits_{x\to \frac{1}{2}^+} \left[x\left[\frac{1}{x}\right]\right] = \lim\limits_{h\to 0} \left[\left(\frac{1}{2}+h\right)\left[\frac{1}{\frac{1}{2}+h}\right]\right] =$

$\lim\limits_{h\to 0} \left[\left(\frac{1}{2}+h\right)(1)\right] = 0$

(D) $\mathop{\text{Lim}}\limits_{x\to -1} \left[\frac{[x]}{x}\right]$

LHL = $\lim\limits_{h\to 0} \left[\frac{[-1-h]}{-1-h}\right] = \lim\limits_{h\to 0} \left[\frac{-2}{-1-h}\right]$

$= \lim\limits_{h\to 0} \left[\frac{2}{1+h}\right] = 1$

RHL $= \lim\limits_{h\to 0} \left[\frac{[-1+h]}{-1+h}\right] = \lim\limits_{h\to 0} \left[\frac{-1}{-1+h}\right]$

$= \lim\limits_{h\to 0} \left[\frac{1}{1-h}\right] = 1$