DPP No.- 45 to 48

Time: 24 Min. **M.M.: 18** **DPP. NO.-45**

[SINGLE CORRECT CHOICE TYPE] [3 × 2 = 6]

Q.1 If $f(x) = A\sin\left(\frac{\pi x}{2}\right) + B$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)\,dx = \frac{2A}{\pi}$, Then the constants A and B are respectively

(A) $\frac{\pi}{2} \& \frac{\pi}{2}$ (B) $\frac{2}{\pi} \& \frac{3}{\pi}$ (C) $0 \& -\frac{4}{\pi}$ (D) $\frac{4}{\pi} \& 0$ [JEE'95, 2]

Q.2 The value of $\int_{\pi}^{2\pi} [\,2\sin x\,]\,dx$ where [ ] represents the greatest integer function is

(A) $-\frac{5\pi}{3}$ (B) $-\pi$ (C) $\frac{5\pi}{3}$ (D) -2π [JEE'95, 2]

Q.3 If $g(x) = \int_0^x \cos^4 t\,dt$, then $g(x+\pi)$ equals [JEE'97, 2]

(A) $g(x) + g(\pi)$ (B) $g(x) - g(\pi)$ (C) $g(x)\ g(\pi)$ (D) $[\,g(x)/g(\pi)\,]$

[SINGLE CORRECT CHOICE TYPE] [4 × 3 = 12]

Q.4 If $\int u\frac{d^2v}{dx^2}dx = u\frac{dv}{dx} - v\frac{du}{dx} + w$ then w is equal to

(A) $\int v\frac{d^2u}{dx^2}dx$ (B) $\int u\frac{d^2v}{dx^2}dx$ (C) $\int v\left(\frac{du}{dx}\right)^2 dx$ (D) $\int u\left(\frac{dv}{dx}\right)^2 dx$

Q.5 $\underset{x\to 0}{\text{Limit}} \dfrac{(\sin x - \tan x)^2 - (1-\cos 2x)^4 + x^5}{7.(\tan^{-1}x)^7 + (\sin^{-1}x)^6 + 3\sin^5 x}$ is equal to

(A) 0 (B) $\frac{1}{7}$ (C) $\frac{1}{3}$ (D) 1

Q.6 The value of $\cos\left(2\sin^{-1}\left(\frac{-1}{6}\right)\right) + \tan\left(\sec^{-1}\left(\frac{-13}{12}\right)\right)$ is

(A) $\frac{29}{36}$ (B) $\frac{19}{36}$ (C) $\frac{7}{36}$ (D) $\frac{49}{36}$

Q.7 If $\int_0^{2\pi} \frac{1}{1+\tan^4 x}dx = \frac{\pi}{k}$ $(k \in N)$, then k equals

(A) 1 (B) 2 (C) 3 (D) 4

Time: 30 Min. **M.M.: 23** **DPP. NO.-46**

[MATRIX TYPE] [3+3+3=9]

Q.1

Column-I		Column-II	
(A)	$\lim\limits_{x\to\infty} \dfrac{x^3.\sin\frac{1}{x}+x+1}{x^2+x+1}$	(P)	$\dfrac{1}{2}$
(B)	$\lim\limits_{x\to 0} \dfrac{\left(\sqrt{(1-\cos x)+\sqrt{(1-\cos x)+\sqrt{(1-\cos x)+.........\infty}}}\right)-1}{x^2}$	(Q)	1
(C)	If $\text{Limit}_{x\to 0} \dfrac{\sin nx\,[(a-n).nx-\tan x]}{x^2}=0 \quad (n>0)$ then the minimum value of 'a' is	(R)	2
		(S)	DNE

[INTEGER TYPE / SUBJECTIVE] [2 × 2 = 4]

Q.2 If for non – zero x, $a\,f(x)+b\,f\left(\dfrac{1}{x}\right)=\left(\dfrac{1}{x}\right)-5$; where $a\neq b$ then $\displaystyle\int_1^2 f(x)\,dx=$ _____.

[JEE'96, 2]

Q.3 The value of $\displaystyle\int_1^{e^{37}} \frac{\pi\sin(\pi\ln x)}{x}\,dx$ is _______ . [JEE'97, 2]

[INTEGER TYPE] [2 × 5 = 10]

Q.4 If the value $\displaystyle\int \frac{1-(\cot x)^{2008}}{\tan x+(\cot x)^{2009}}\,dx=\frac{1}{k}\ln|\sin^k x+\cos^k x|+C$, then find k.

Q.5 A triangle has side lengths 18, 24 and 30. Find the area of the triangle (in sq. units) whose vertices are the incentre, circumcentre and centroid of the triangle.

Time: 30 Min. **M.M.: 26** **DPP. NO.-47**

[SINGLE CORRECT CHOICE TYPE] [6 × 3 = 18]

Q.1 $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\frac{r}{\sqrt{n^2+r^2}}$ equals [JEE'97, 2]

(A) $1+\sqrt{5}$ (B) $-1+\sqrt{5}$ (C) $-1+\sqrt{2}$ (D) $1+\sqrt{2}$

Q.2 If $\int_0^x f(t)dt = x+\int_x^1 tf(t)dt$, then the value of $f(1)$ is [JEE'98, 2]

(A) $1/2$ (B) 0 (C) 1 (D) $-1/2$

Q.3 The value of the integral $\int_{e^{-1}}^{e^2}\left|\frac{\log_e x}{x}\right|dx$ is [JEE 2000, 1]

(A) 3/2 (B) 5/2 (C) 3 (D) 5

Q.4 Let $g(x)=\int_0^x f(t)\,dt$, where f is such that $\frac{1}{2}\le f(t)\le 1$ for $t\in(0,1]$ and $0\le f(t)\le\frac{1}{2}$ for $t\in(1,2]$. Then $g(2)$ satisfies the inequality : [JEE 2000, 1]

(A) $\frac{-3}{2}\le g(2)<\frac{1}{2}$ (B) $0\le g(2)<2$ (C) $\frac{3}{2}<g(2)\le\frac{5}{2}$ (D) $2<g(2)<4$

Q.5 Let $g(x)$ be an antiderivative for $f(x)$. Then $ln\left(1+(g(x))^2\right)$ is an antiderivative for

(A) $\frac{2f(x)g(x)}{1+(f(x))^2}$ (B) $\frac{2f(x)g(x)}{1+(g(x))^2}$ (C) $\frac{2f(x)}{1+(f(x))^2}$ (D) none

Q.6 If the vertices P and Q of a triangle PQR are given by (2, 5) and (4, –11) respectively, and the point R moves along the line N: $9x+7y+4=0$, then the locus of the centroid of the triangle PQR is a straight line parallel to

(A) PQ (B) QR (C) RP (D) N

[MATRIX TYPE] [2+2+2+2=8]

Q.7

	Column-I		Column-II
(A)	$f(x)=\begin{cases} x+1 & \text{if } x<0 \\ \cos x & \text{if } x\ge 0\end{cases}$ at $x=0$ is	(P)	continuous
(B)	For every $x\in R$ the function $g(x)=\frac{\sin(\pi[x-\pi])}{1+[x]^2}$ where [x] denotes the greatest integer function is	(Q)	differentiability
		(R)	discontinuous
		(S)	non derivable
(C)	$h(x)=\sqrt{\{x\}^2}$ where {x} denotes fractional part function for all $x\in I$, is		
(D)	$k(x)=\begin{cases} x^{\frac{1}{ln x}} & \text{if } x\ne 1 \\ e & \text{if } x=1\end{cases}$ at $x=1$ is		

Time: 30 Min. **M.M.: 26** **DPP. NO.-48**

[SINGLE CORRECT CHOICE TYPE] [2 × 3 = 6]

Q.1 Let f : R → R be a function defined by, f (x) = max [x, x^3]. The set of all points where f (x) is NOT differentiable is

(A) {– 1 , 1} (B) {– 1 , 0} (C) {0 , 1} (D) {– 1 , 0 , 1}

[JEE 2001 (Screening)]

Q.2 For the curve represented implicitly as $3^x - 2^y = 1$, the value of $\lim\limits_{x \to \infty} \left(\frac{dy}{dx}\right)$ is

(A) equal to 1 (B) equal to 0 (C) equal to $\log_2 3$ (D) non existent

[MATRIX TYPE] [2+2+2+2=8]

Q.3

Column-I	Column-I
(A) $\lim\limits_{x \to 0} \dfrac{\sqrt{1 + x\sin x} - \sqrt{\cos 2x}}{\tan^2 \frac{x}{2}}$.	(P) – 3
(B) $\lim\limits_{x \to 1} \dfrac{\sin^2(x^3 + x^2 + x - 3)}{1 - \cos(x^2 - 4x + 3)}$	(Q) 6
(C) $\lim\limits_{x \to 0} \dfrac{6x^2 (\cot x)(\csc 2x)}{\sec\left(\cos x + \pi \tan\left(\frac{\pi}{4\sec x}\right) - 1\right)}$ has the value equal to	(R) 8
(D) If $\lim\limits_{x \to -2} \dfrac{\left(2(a-3)(x+2) - 6\sin^{-1}(x+2)\right)\tan^{-1}(5x+10)}{(x+2)^2} = 0$	(S) 12
then the value of a is equal to	(T) 18

[INTEGER TYPE / SUBJECTIVE] [3 × 4 = 12]

Q.4 Let $\frac{d}{dx} F(x) = \frac{e^{\sin x}}{x}$, x > 0 . If $\int_1^4 \frac{2e^{\sin x^2}}{x} dx = F(k) - F(1)$ then one of the possible values of k is _______.

[JEE'97, 2]

Q.5 $S_n = \frac{1}{1+\sqrt{n}} + \frac{1}{2+\sqrt{2n}} + \ldots\ldots + \frac{1}{n+\sqrt{n^2}}$. Find $\lim\limits_{n \to \infty} S_n$. [REE 2000, Mains, 3 out of 100]

Q.6 Given $\int_0^1 \frac{\sin t}{1+t} dt = \alpha$, find the value of $\int_{4\pi-2}^{4\pi} \frac{\sin \frac{t}{2}}{4\pi + 2 - t} dt$ in terms of α .

DPP No.- 45 to 48

Solutions

Time: 24 Min. **M.M.: 18** **DPP. NO.-45**

[SINGLE CORRECT CHOICE TYPE] [3 × 2 = 6]

Q.1 If $f(x) = A\sin\left(\frac{\pi x}{2}\right) + B$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)\,dx = \frac{2A}{\pi}$, Then the constants A and B are respectively

(A) $\frac{\pi}{2} \& \frac{\pi}{2}$ (B) $\frac{2}{\pi} \& \frac{3}{\pi}$ (C) $0 \& -\frac{4}{\pi}$ (D*) $\frac{4}{\pi} \& 0$

[JEE'95, 2]

[Sol.$_{332/def/SC}$]

Q.2 The value of $\int_{\pi}^{2\pi} [\,2\sin x\,]\,dx$ where [ ] represents the greatest integer function is

(A*) $-\frac{5\pi}{3}$ (B) $-\pi$ (C) $\frac{5\pi}{3}$ (D) -2π [JEE'95, 2]

[Sol.$_{333/def/SC}$]

Q.3 If $g(x) = \int_0^x \cos^4 t\,dt$, then $g(x+\pi)$ equals

(A*) $g(x)+g(\pi)$ (B) $g(x)-g(\pi)$ (C) $g(x)\ \ g(\pi)$ (D) $[\,g(x)/g(\pi)\,]$

[JEE'97, 2]

[Sol.$_{334/def/SC}$]

[SINGLE CORRECT CHOICE TYPE] [4 × 3 = 12]

Q.4 If $\int u\frac{d^2v}{dx^2}dx = u\frac{dv}{dx} - v\frac{du}{dx} + w$ then w is equal to

(A*) $\int v\frac{d^2u}{dx^2}dx$ (B) $\int u\frac{d^2v}{dx^2}dx$ (C) $\int v\left(\frac{du}{dx}\right)^2 dx$ (D) $\int u\left(\frac{dv}{dx}\right)^2 dx$

[Sol.$_{80/inde/SC}$ Differentiate the given integral with respect to x

$$u\frac{d^2v}{dx^2} = u\frac{d^2v}{dx^2} + \frac{du}{dx}\cdot\frac{dv}{dx} - v\frac{d^2u}{dx^2} - \frac{dv}{dx}\cdot\frac{du}{dx} + \frac{dw}{dx} \Rightarrow \frac{dw}{dx} = v\frac{d^2u}{dx^2}$$

$\therefore$ $dw = v\frac{d^2u}{dx^2}dx$

$\therefore$ $w = \int v\frac{d^2u}{dx^2}dx$. **Ans.**]

Q.5 $\underset{x\to 0}{\text{Limit}} \dfrac{(\sin x - \tan x)^2 - (1-\cos 2x)^4 + x^5}{7.(\tan^{-1}x)^7 + (\sin^{-1}x)^6 + 3\sin^5 x}$ is equal to

(A) 0 (B) $\frac{1}{7}$ (C*) $\frac{1}{3}$ (D) 1

[Hint:$_{258/lcd/SC}$ Divide Nr. and Dr. by x^5 and evaluate limit]

Q.6 The value of $\cos\left(2\sin^{-1}\left(\frac{-1}{6}\right)\right) + \tan\left(\sec^{-1}\left(\frac{-13}{12}\right)\right)$ is

(A) $\frac{29}{36}$ (B*) $\frac{19}{36}$ (C) $\frac{7}{36}$ (D) $\frac{49}{36}$

[Sol.$_{194/itf/SC}$ $\sin^{-1}\left(\frac{-1}{6}\right) = \theta \Rightarrow \sin\theta = \frac{-1}{6}$

So, $\cos\left(2\sin^{-1}\left(\frac{-1}{6}\right)\right) = \cos 2\theta = 1 - 2\sin^2\theta = 1 - \frac{2}{36} = \frac{17}{18}$

Also, let $\sec^{-1}\left(\frac{-13}{12}\right) = \theta \Rightarrow \sec\theta = \frac{-13}{12}$ (2^{nd} quadrant) $\Rightarrow \tan\theta = \frac{-5}{12}$

$\therefore$ We have, $\frac{17}{18} - \frac{5}{12} = \frac{34-15}{36} = \frac{19}{36}$. **Ans.**]

Q.7 If $\int_0^{2\pi} \frac{1}{1+\tan^4 x}\,dx = \frac{\pi}{k}$ $(k \in N)$, then k equals

(A*) 1 (B) 2 (C) 3 (D) 4

[Sol.$_{514/def/SC}$ $I = 2\int_0^{\pi} \frac{dx}{1+\tan^4 x} = 4\int_0^{\pi/2} \frac{dx}{1+\tan^4 x}$ ....(1)

Use King

$I = 4\int_0^{\pi/2} \frac{dx}{1+\cot^4 x}$ ....(2)

$\therefore\ 2I = 4\int_0^{\pi/2} dx = 4 \cdot \frac{\pi}{2} \Rightarrow I = \pi.$]

Time: 30 Min. **M.M.: 23** **DPP. NO.-46**

[MATRIX TYPE]

[3+3+3=9]

Q.1 **Column-I** **Column-II**

(A) $\lim\limits_{x\to\infty} \dfrac{x^3.\sin\frac{1}{x}+x+1}{x^2+x+1}$ (P) $\dfrac{1}{2}$

(B) $\lim\limits_{x\to 0} \dfrac{\left(\sqrt{(1-\cos x)+\sqrt{(1-\cos x)+\sqrt{(1-\cos x)+.........\infty}}}\right)-1}{x^2}$ (Q) 1

(C) If $\underset{x\to 0}{\text{Limit}} \dfrac{\sin nx\,[(a-n).nx-\tan x]}{x^2}=0 \quad (n>0)$ (R) 2

then the minimum value of 'a' is (S) DNE

[Ans. (A) Q; (B) P; (C) R]

[Sol.$_{92019/lcd/MTC}$

(B) Let $y=\sqrt{\alpha+\sqrt{\alpha+\sqrt{\alpha+.........\infty}}}$ where $\alpha = 1-\cos x$;as $x\to 0,\ \alpha\to 0$

$y=\sqrt{\alpha+y}$; $y^2 = a + y \Rightarrow y^2 - y - \alpha = 0$

$y=\dfrac{1\pm\sqrt{1+4\alpha}}{2}$ (neglecting – ve sign as y can not be – ve) $\Rightarrow$ $y=\dfrac{1+\sqrt{1+4\alpha}}{2}$

now $l=\lim\limits_{\substack{x\to 0\\ \alpha\to 0}} \dfrac{\left[\dfrac{1+\sqrt{1+4\alpha}}{2}-1\right]}{\dfrac{x^2}{1-\cos x}\cdot(1-\cos x)} = \lim\limits_{\alpha\to 0}\dfrac{\sqrt{1+4\alpha}-1}{2\cdot 2\cdot\alpha}$ (as $\lim\limits_{x\to 0}\dfrac{x^2}{1-\cos x}=2$)

$=\lim\limits_{\alpha\to 0}\dfrac{(1+4\alpha)-1}{4\alpha(\sqrt{1+4\alpha}+1)}=2=\dfrac{4}{2}$ (rationalising the D^r) $=\lim\limits_{\alpha\to 0}\dfrac{1}{\sqrt{1+4\alpha}+1}=\dfrac{1}{2}$]

(C) $\underset{x\to 0}{\text{Limit}}\dfrac{\sin nx[(a-n)nx-\tan x]}{x^2}=0 \Rightarrow \underset{x\to 0}{\text{Limit}}\dfrac{n.\sin nx}{nx}\left[(a-n)n-\dfrac{\tan x}{x}\right]=0$

$= n\,[(a-n)\,n-1\,]=0 \quad \Rightarrow (a-n)n=1$

$= an - n^2 = 1 \quad \Rightarrow a=\dfrac{n^2+1}{n}=n+\dfrac{1}{n} \Rightarrow a_{min.}=2$]

[INTEGER TYPE / SUBJECTIVE]

[2 × 2 = 4]

Q.2 If for non – zero x, $a\,f(x)+b\,f\left(\dfrac{1}{x}\right)=\left(\dfrac{1}{x}\right)-5$; where $a\neq b$ then $\displaystyle\int_1^2 f(x)\,dx =$ _____.

[JEE'96, 2]

[Ans. $\dfrac{\left(a\,ln\,2-5a+\dfrac{7b}{2}\right)}{(a^2-b^2)}$]

[Sol.$_{81515/def/SUB}$]

Q.3 The value of $\int_1^{e^{37}} \frac{\pi \sin(\pi \ln x)}{x} dx$ is _______ . [JEE'97, 2]

[Ans. 2]

[Sol.$_{81518/def/SUB}$ Let $\pi \ln x = t \Rightarrow dt = \frac{\pi}{x} dx$

$$\int_0^{37\pi} \sin t\, dt = 37 \int_0^{\pi} \sin t\, dt = 37 \cdot 2 = 74 \text{ **Ans.**]}$$

[INTEGER TYPE] **[2 × 5 = 10]**

Q.4 If the value $\int \frac{1-(\cot x)^{2008}}{\tan x + (\cot x)^{2009}} dx = \frac{1}{k} \ln|\sin^k x + \cos^k x| + C$, then find k. [Ans. 2010]

[Sol.$_{50705/inde/OMR}$ L.H.S. $\int \frac{(\sin x)^{2008} - (\cos x)^{2008}}{(\sin x)^{2008}\left(\frac{\sin x}{\cos x} + \left(\frac{\cos x}{\sin x}\right)^{2009}\right)} dx$

$$= \int \frac{\sin x \cos x \left((\sin x)^{2008} - (\cos x)^{2008}\right)}{(\sin x)^{2010} + (\cos x)^{2010}} dx$$

$$= \int \frac{\left((\sin x)^{2009} \cos x - (\cos x)^{2009} \sin x\right)}{(\sin x)^{2010} + (\cos x)^{2010}} dx$$

put $(\sin x)^{2010} + (\cos x)^{2010} = t$

$$= \frac{1}{2010} \int \frac{dt}{t}$$

$$= \frac{1}{2010} \ln\left|(\sin x)^{2010} + (\cos x)^{2010}\right| + c$$

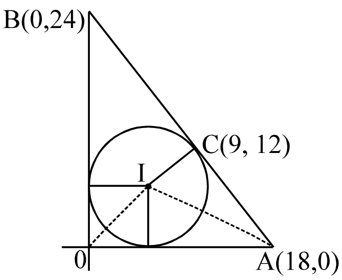
$\Rightarrow$ k = 2010]

Q.5 A triangle has side lengths 18, 24 and 30. Find the area of the triangle (in sq. units) whose vertices are the incentre, circumcentre and centroid of the triangle. **[Ans.** 3]

[Sol.$_{50732/st.line/OMR}$ radius of the incircle,

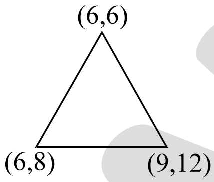
$$r = \frac{\Delta}{s} = \frac{(18 \cdot 24)2}{2(18+24+30)} = \frac{18 \cdot 24}{72} = 6$$

Hence coordinates of incentre (6, 6)



centroid is = $\left(\frac{18}{3}, \frac{24}{3}\right) = (6, 8)$

Hence D formed by I, G and circumcentre is



$\therefore \quad A = \frac{1}{2}\begin{vmatrix} 6 & 6 & 1 \\ 9 & 12 & 1 \\ 6 & 8 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 6 & 6 & 1 \\ 3 & 6 & 0 \\ 0 & 2 & 0 \end{vmatrix} = 3$ sq. units]

Time: 30 Min. **M.M.: 26** **DPP. NO.-47**

[SINGLE CORRECT CHOICE TYPE] [6 × 3 = 18]

Q.1 $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\frac{r}{\sqrt{n^2+r^2}}$ equals

(A) $1+\sqrt{5}$ (B*) $-1+\sqrt{5}$ (C) $-1+\sqrt{2}$ (D) $1+\sqrt{2}$

[JEE'97, 2]

[Sol.$_{335/def/SC}$]

Q.2 If $\int_0^x f(t)dt = x+\int_x^1 tf(t)dt$, then the value of $f(1)$ is

(A*) 1/ 2 (B) 0 (C) 1 (D) – 1/ 2

[JEE'98, 2]

[Sol.$_{336/def/SC}$]

Q.3 The value of the integral $\int_{e^{-1}}^{e^2}\left|\frac{\log_e x}{x}\right|dx$ is

(A) 3/2 (B*) 5/2 (C) 3 (D) 5

[JEE 2000, 1]

[Sol.$_{340/def/SC}$]

Q.4 Let $g(x)=\int_0^x f(t)\,dt$, where f is such that $\frac{1}{2}\le f(t)\le 1$ for $t\in(0, 1]$ and $0\le f(t)\le\frac{1}{2}$ for $t\in(1, 2]$. Then $g(2)$ satisfies the inequality :

(A) $\frac{-3}{2}\le g(2)<\frac{1}{2}$ (B*) $0\le g(2)<2$ (C) $\frac{3}{2}<g(2)\le\frac{5}{2}$ (D) $2<g(2)<4$

[JEE 2000, 1]

[Sol.$_{341/def/SC}$]

Q.5 Let $g(x)$ be an antiderivative for $f(x)$. Then $ln\left(1+(g(x))^2\right)$ is an antiderivative for

(A) $\frac{2f(x)g(x)}{1+(f(x))^2}$ (B*) $\frac{2f(x)g(x)}{1+(g(x))^2}$ (C) $\frac{2f(x)}{1+(f(x))^2}$ (D) none

[Sol.$_{27/inde/SC}$ Given $\int f(x)\,dx = g(x) \Rightarrow g'(x)=f(x)$

now $\frac{d}{dx}\left(ln(1+g^2(x))\right)=\frac{2g(x)g'(x)}{1+g^2(x)}=\frac{2f(x)g(x)}{1+g^2(x)} \Rightarrow$ (B)]

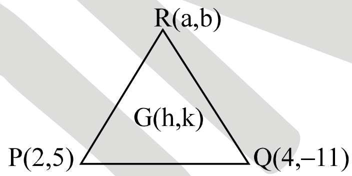
Q.6 If the vertices P and Q of a triangle PQR are given by (2, 5) and (4, –11) respectively, and the point R moves along the line N: $9x + 7y + 4 = 0$, then the locus of the centroid of the triangle PQR is a straight line parallel to

(A) PQ (B) QR (C) RP (D*) N

[Sol.$_{165/st.line/SC}$ R (a, b) lies on $9x + 7y + 4 = 0$ $\Rightarrow$ $9a + 7y + y = 0$

$$\Rightarrow R\left(a, \frac{(4+9a)}{7}\right), \text{ centroid of } \Delta \text{ PQR} = (h, k)$$

$$h = \left(\frac{2+4+a}{3}\right) = \frac{6+a}{3} \qquad ....(1)$$



$$k = \frac{5 - 11 - \frac{(4+9a)}{7}}{3} = \frac{-46-9a}{7\times 3} \qquad ....(2)$$

from (1) and (2) we get

equating x $\quad 3h - 6 = \dfrac{-(21k-46)}{9} \Rightarrow 27h + 21k - 54 + 46 = 0$

or locus is $9x + 7y - 8/3 = 0$

this line is || to N.]

[MATRIX TYPE] [2+2+2+2=8]

Q.7

Column-I	Column-II
(A) $f(x) = \begin{cases} x+1 & \text{if } x<0 \\ \cos x & \text{if } x \ge 0 \end{cases}$ at $x = 0$ is	(P) continuous
(B) For every $x \in R$ the function $g(x) = \dfrac{\sin(\pi[x-\pi])}{1+[x]^2}$ where [x] denotes the greatest integer function is	(Q) differentiability
(C) $h(x) = \sqrt{\{x\}^2}$ where {x} denotes fractional part function for all $x \in I$, is	(R) discontinuous
(D) $k(x) = \begin{cases} x^{\frac{1}{\ln x}} & \text{if } x \ne 1 \\ e & \text{if } x = 1 \end{cases}$ at $x = 1$ is	(S) non derivable

[Ans. (A) P, S; (B) P, Q; (C) R, S; (D) P, Q]

[Sol.$_{92022/lcd/MTC}$

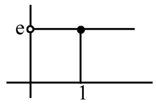
(A) $f'(0) = \lim\limits_{h\to 0} \dfrac{\cos h - 0}{h}$ does not exist. Obviously $f(0) = f(0^-) = f(0^+) = 1$

Hence continuous and not derivable

(B) $g(x) = 0$ for all x, hence continuous and derivable

(C) as $0 \le \{f(x)\} < 1$, hence $h(x) = \sqrt{\{x\}^2} = \{x\}$ which is discontinuous hence non derivable all $x \in I$

(D) $\lim\limits_{x\to1} x^{\frac{1}{\ln x}} = \lim\limits_{x\to1} x^{\log_x e} = e = f(1)$



Hence k (x) is constant for all x > 0 hence continuous and differentiable at x = 1.]

Class:XII **Discussion Date: 13/07/2018** **Time: 30 Min.** **M.M.: 26** **DPP. NO.-48**

[SINGLE CORRECT CHOICE TYPE] [2 × 3 = 6]

Q.1 Let f : R → R be a function defined by, f (x) = max [x, x^3]. The set of all points where f(x) is NOT differentiable is

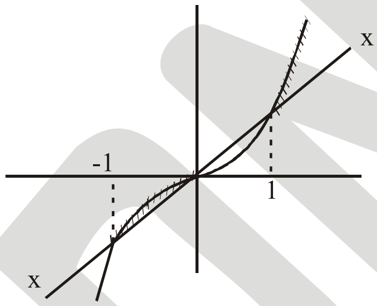
(A) {–1 , 1} (B) {–1 , 0} (C) {0 , 1} (D*) {–1 , 0 , 1}

[JEE 2001 (Screening)]

[Sol.$_{286/lcd/SC}$ **[D]**

f (x) = Max. [x , x^3]

not derivable at –1, 0, 1



$$f(x) = \begin{bmatrix} x^3 & \text{if} & x \ge 1 \\ x & & 0 \le x < 1 \\ x^3 & & -1 \le x < 0 \\ x & & x < -1 \end{bmatrix}]$$

Q.2 For the curve represented implicitly as $3^x - 2^y = 1$, the value of $\lim_{x\to\infty}\left(\frac{dy}{dx}\right)$ is

(A) equal to 1 (B) equal to 0 (C*) equal to $\log_2 3$ (D) non existent

[Sol.$_{98/mod/SC}$ $2^y = 3^x - 1$

$y\ ln\ 2 = ln(3^x - 1)$

$$ln\ 2\ \frac{dy}{dx} = \frac{3^x \cdot ln\,3}{3^x - 1} = \frac{ln\,3}{1 - 3^{-x}}$$

$$\lim_{x\to\infty}\frac{dy}{dx} = \frac{ln\,3}{ln\,2}\cdot\lim_{x\to\infty}\frac{1}{1-3^{-x}} = \log_2 3 \text{ Ans.]}$$

[MATRIX TYPE] [2+2+2+2=8]

Q.3 **Column-I** **Column-I**

(A) $\lim_{x\to 0}\dfrac{\sqrt{1 + x\sin x} - \sqrt{\cos 2x}}{\tan^2\frac{x}{2}}$. (P) – 3

(B) $\lim_{x\to 1}\dfrac{\sin^2(x^3 + x^2 + x - 3)}{1 - \cos(x^2 - 4x + 3)}$ (Q) 6

(C) $\lim_{x\to 0}\dfrac{6x^2(\cot x)(\csc 2x)}{\sec\left(\cos x + \pi\tan\left(\frac{\pi}{4\sec x}\right) - 1\right)}$ has the value equal to (R) 8

(D) If $\lim_{x\to -2}\dfrac{\left(2(a-3)(x+2) - 6\sin^{-1}(x+2)\right)\tan^{-1}(5x+10)}{(x+2)^2} = 0$ (S) 12

then the value of a is equal to (T) 18

[Ans. (A) Q; (B) T; (C) P; (D) Q]

[Sol.$_{92015/lcd/MTC}$

(B) $\lim_{x\to 1} \frac{\sin^2(x^3+x^2+x-3)}{(x^3+x^2+x-3)^2} \cdot \frac{(x^3+x^2+x-3)^2}{1-\cos(x^2-4x+3)}$

$= (1) \cdot \lim_{x\to 1} \frac{(x^2-4x+3)^2}{1-\cos(x^2-4x+3)} \cdot \frac{(x^3+x^2+x-3)^2}{(x^2-4x+3)^2}$

$= (1)\ (2) \lim_{x\to 1} \left(\frac{x^3+x^2+x-3}{x^2-4x+3}\right)^2 = 2l^2$ where $l = \lim_{x\to 1} \frac{3x^2+2x+1}{2x-4} = \frac{6}{-2} = -3$

$\therefore \quad L = 2(-3)^2 = 18$ Ans.]

(C) $N^r = 6\ \frac{x}{\tan x} \cdot \frac{x}{\sin 2x} = 3$

$D^r = \sec(1 + \pi - 1) = -1$ $\Big] \Rightarrow l = -3$]

(D) $\lim_{x\to -2} \left(2(a-3) - \frac{6\sin^{-1}(x+2)}{(x+2)}\right) \cdot \frac{\tan^{-1} 5(x+2)}{x+2} = 0$

$\Rightarrow \quad 5\big(2(a-3)-6\big) = 0 \quad \Rightarrow \quad a = 6$]

[INTEGER TYPE / SUBJECTIVE] [3 × 4 = 12]

Q.4 Let $\frac{d}{dx} F(x) = \frac{e^{\sin x}}{x}$, $x > 0$. If $\int_1^4 \frac{2\, e^{\sin x^2}}{x} dx = F(k) - F(1)$ then one of the possible values of k is ______.

[JEE'97, 2]
[Ans. 16]

[Sol.$_{81519/def/SUB}$]

Q.5 $S_n = \frac{1}{1+\sqrt{n}} + \frac{1}{2+\sqrt{2n}} + \ldots\ldots + \frac{1}{n+\sqrt{n^2}}$. Find $\lim_{n\to\infty} S_n$.

[REE 2000, Mains, 3 out of 100]
[Ans. 2 ln 2]

[Sol.$_{81528/def/SUB}$]

Q.6 Given $\int_0^1 \frac{\sin t}{1+t} dt = \alpha$, find the value of $\int_{4\pi-2}^{4\pi} \frac{\sin \frac{t}{2}}{4\pi+2-t} dt$ in terms of α.

[REE 2000, Mains, 3 out of 100]
[Ans. – α]

[Sol.$_{81529/def/SUB}$]