Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-09 to 12

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-09

[SINGLE CORRECT CHOICE TYPE] **[5 × 3 = 15]**

Q.1 If the coefficient of x^{78} in the expansion of $(1+x+2x^2+4x^4)^{20}$ is $\lambda\cdot 2^{40}$ then λ is equal to

(A) 11 (B) 10 (C) 8 (D) 4

Q.2 Let the function $f: D \to R$, $f(x) = \log_5\left(\log_{1/3}\log_8(2x+1)\right)$ where D is the maximum domain of $f(x)$. If S represents the sum of the absolute values of all integers from D. Then the value of S, is

(A) 15 (B) 10 (C) 6 (D) 3

Q.3 The domain of the function, $f(x) = (x+0.5)^{\log_{(0.5+x)}\left(\frac{x^2+2x-3}{4x^2-4x-3}\right)}$ is

(A) $\left(\frac{-1}{2}, \infty\right)$ (B) $[1, 3]$

(C) $\left(\frac{1}{2}, 1\right) \cup \left(\frac{3}{2}, \infty\right)$ (D) $\left(\frac{-1}{2}, \frac{1}{2}\right) \cup \left(\frac{1}{2}, 1\right) \cup \left(\frac{3}{2}, \infty\right)$

Q.4 In a ΔABC, if $r = 1$, $R = 3$ and $\angle C = 90^\circ$, then area of ΔABC is equal to

(A) 7 (B) 8 (C) 9 (D) 10

Q.5 The value of 'α' for which $\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$, $\sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$ and $\sin^{-1}(\alpha)$ are the angles of a triangle, is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{2}{3}$

[INTEGER TYPE] **[2 × 5 = 10]**

Q.6 Let equations $x^2-3x+4=0$ and $4x^2-2(b-5a)x+b=0$ $(a, b \in R)$ have a common root. If thirteen arithmetic means are inserted between a and b, then find arithmetic mean of these means.

Q.7 The three different polynomials x^2+ax+b, x^2+x+ab and ax^2+x+b have exactly one common zero. Where a, b are non-zero real numbers. Find the value of $a+2b$.

Dpp. No.-10

[SINGLE CORRECT CHOICE TYPE] [5 × 3 = 15]

Q.1 The domain of $f(x) = \log_{\left[x+\frac{1}{2}\right]}(x^2+x-2)$ is

[**Note:** [k] denotes greatest integer function less than or equal to k.]

(A) $\left[\frac{3}{2}, \infty\right)$ (B) $\left[\frac{3}{2}, \infty\right) - \{2\}$ (C) $(2, \infty)$ (D) $\left[\frac{1}{2}, \infty\right) - \{2\}$

Q.2 In a triangle ABC, $a \ge b \ge c$. If $\frac{a^3+b^3+c^3}{\sin^3 A+\sin^3 B+\sin^3 C} = 27$, then find the maximum value of a.

[**Note :** All symbols used have usual meaning in triangle ABC.]

(A) 3/2 (B) 2 (C) 3 (D) cannot be determined

Q.3 The domain of definition of the function $f(x) = \log_2\left(1-3^{\frac{1}{x}-1}\right)$, is

(A) $(-\infty, 0) \cup (1, \infty)$ (B) $(-1, 1)$
(C) $(-\infty, \infty)$ (D) $(-\infty, 2) \cup (3, \infty)$

Q.4 The expression $\frac{x^2-y^2}{(x-y)^2} \cdot \frac{x^2-xy+y^2}{x^2-2xy+y^2} \div \frac{x^3+y^3}{(x-y)^4}$ simplifies to

(A) $\frac{1}{xy}$ (B) $(x-y)$ (C) $\frac{x^2-xy+y^2}{x^2+2xy+y^2}$ (D) $(x+y)$

Q.5 Given that $0 \le x \le \frac{1}{2}$, the value of $\tan\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}+\sqrt{\frac{1-x^2}{2}}\right)-\sin^{-1}x\right)$ is

(A) -1 (B) 1 (C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{3}$

[INTEGER TYPE] [1 × 5 = 5]

Q.6 Let N denotes the number of odd integers between 550 and 800 using the digits 4, 5, 6,7, 8 and 9. Find the sum of the digits in N.

Dpp. No.-11

[SINGLE CORRECT CHOICE TYPE] **[4 × 3 = 12]**

Q.1 The value of $\sum\limits_{0\le i<j\le 5}\sum \left({}^{5}C_{j}\right)\left({}^{j}C_{i}\right)$ is equal to

(A) $3^5 - 2^5$ (B) $3^5 - 1$ (C) $2^5 - 1$ (D) $3^5 \cdot 2^5$

Q.2 The first two terms of a geometric progression add up to 12. The sum of the third and the fourth tems is 48. If the terms of the geometric progression are alternatively positive and negative, then the first term is

(A) – 4 (B) – 12 (C) 12 (D) 4

Q.3 A function g has domain [0, 2] and range [– 1, 3]. The domain and range of the function f defined by $f(x) = 3 - 2g(9x^2 - 1)$ are equal to

(A) [0, 2] and [– 1, 3] (B) $\left[\frac{-1}{3}, \frac{1}{3}\right]$ and [–3, 5]

(C) $\left[\frac{-1}{\sqrt{3}}, \frac{-1}{3}\right] \cup \left[\frac{1}{3}, \frac{1}{\sqrt{3}}\right]$ and [–3, 5] (D) $\left[\frac{-1}{\sqrt{3}}, \frac{-1}{3}\right]$ and [3, 5]

Q.4 If $\cos^{-1}(2x - 1 - x^2) + \sin^{-1} y = 0$ then the value of $x^2 + y^2 + xy$ is equal to

(A) 0 (B) 1 (C) –1 (D) 3

[MULTIPLE CORRECT CHOICE TYPE] **[3 × 4 = 12]**

Q.5 If $f^2(x) \cdot f\left(\frac{1-x}{1+x}\right) = x^3 \ \forall x \in R - \{-1, 1\}$ and $f(x) \neq 0$, then which of the following is correct.

(A) $f(2) \cdot f(-2) = 16$ (B) $f(2) f(-2) = 8$ (C) $f(2) = -12$ (D) $f(3) = -18$

Q.6 Let $a, b \in R$ satisfying $4a^2 - 5b^2 + 6a + 1 = 0$ and the line $ax + by + 1 = 0$ touches fixed circle, then

(A) centre of circle is (3, 0) (B) radius of circle is $\sqrt{5}$

(C) radius of circle is $\sqrt{3}$ (D) circle passes through M(1, 1)

Q.7 In acute angle triangle ABC with usual notations, $r = r_2 + r_3 - r_1$ and angle $B > \frac{\pi}{3}$ then the number of integer(s) in the range of $\left(\frac{a-c}{b}\right)$ is less than

(A) 0 (B) 1 (C) 2 (D) 3

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-12

[SINGLE CORRECT CHOICE TYPE] [4 × 3 = 12]

Q.1 Let F (x) = f (x) + g (x) + h (x) where $f(x) = \sqrt{\dfrac{\log_{0.3}(x-1)}{x^2-2x-8}}$, $g(x) = \sqrt{{}^{x^2+4x}C_{2x^2+3}}$ and $h(x) = \sqrt{1-\log_3(x^2-6x+11)}$. The number of positive integers in the domain of F (x), is

(A) 2 (B) 3 (C) 9 (D) 11

Q.2 If $a^2 + b^2 + c^2 + ab + bc + ca \le 0$, where $a, b, c \in R$, then domain of the function $f(x) = \sqrt{\text{sgn}(x)\cdot(ax^2+bx+c)}$ will be

(A) $(0, \infty)$ (B) $[0, \infty)$ (C) $(-\infty, 0]$ (D) $(-\infty, \infty)$

Q.3 The number of solutions of the equation $x\sin^{-1}(\sin x) + x = \pi$ in $[0, 2\pi]$ is

(A) 1 (B) 2 (C) 3 (D) 4

Q.4 If $\theta_1 = \sin^{-1}\dfrac{4}{5} + \sin^{-1}\dfrac{1}{3}$ and $\theta_2 = \cos^{-1}\dfrac{4}{5} + \cos^{-1}\dfrac{1}{3}$, then

(A) $\theta_1 > \theta_2$ (B) $\theta_1 = \theta_2$ (C) $\theta_1 < \theta_2$ (D) none of these

[COMPREHENSION TYPE] [2 × 3 = 6]

Paragraph for question nos. 5 & 6

Consider $f(x) = (a-1)x^2 - 8x + 1$, $a \in R$, $a \ne 1$, $x \in R$ and $g(x) = x^2 - 8x + 13$, $x \in R$

Q.5 The largest integral value of 'a' for which maximum finite value of f(x) > minimum value of g(x) is

(A) 0 (B) 1 (C) 5 (D) 6

Q.6 The least integral value of 'a' for which minimum finite value of f(x) > minimum value of g(x) is

(A) 0 (B) 1 (C) 5 (D) 6

[MATRIX TYPE] [2+2+2+2=8]

Q.7 The circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 2x - 2y + 1 = 0$ intersect at A and B, then

	Column-I		Column-II
(A)	if $ax + 2by = 5$ is common chord of given circles, then $\left(\dfrac{b}{a}\right)$ is equal to	(P)	$\sqrt{\dfrac{7}{2}}$
		(Q)	$\dfrac{3}{4}$
(B)	if θ is acute angle between given circles then the value of $\cos\theta$ is equal to	(R)	$\sqrt{\dfrac{5}{3}}$
(C)	length of common tangent of given circles is equal to	(S)	$\dfrac{1}{2}$
(D)	diameter of smallest circle which is passing through A and B is equal to	(T)	1

Dpp. No.-09 TO 12
(SOLUTIONS)

Dpp. No.-09

[SINGLE CORRECT CHOICE TYPE] [5 × 3 = 15]

Q.1 If the coefficient of x^{78} in the expansion of $(1 + x + 2x^2 + 4x^4)^{20}$ is $\lambda \cdot 2^{40}$ then λ is equal to

(A) 11 (B*) 10 (C) 8 (D) 4

[Sol.$_{181/bin/SC}$ $(1 + x + 2x^2 + 4x^4)^{20}$

$T_{r+1} = {}^{20}C_r \cdot (1 + x + 2x^2)^{20-r} \cdot (4x^4)^r$

For $r = 19$

$T_{19+1} = {}^{20}C_{19} \cdot (1 + x + 2x^2)^1 \cdot (4x^4)^{19}$

$\therefore$ Coefficient of $x^{78} = 2 \cdot {}^{20}C_{19} \cdot 4^{19} = 2 \cdot 20 \cdot 2^{38} = 10 \cdot 2^{40}$ **Ans.**]

Q.2 Let the function $f : D \to R$, $f(x) = \log_5\left(\log_{1/3} \log_8 (2x+1)\right)$ where D is the maximum domain of $f(x)$. If S represents the sum of the absolute values of all integers from D. Then the value of S, is

(A) 15 (B) 10 (C*) 6 (D) 3

[Sol.$_{71/func/SC}$ Domain is $0 < x < 3.5$

$S = 1 + 2 + 3 = 6$ Ans.]

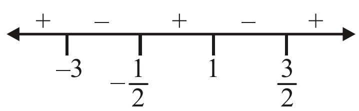
Q.3 The domain of the function, $f(x) = (x + 0.5)^{\log_{(0.5+x)}\left(\frac{x^2+2x-3}{4x^2-4x-3}\right)}$ is

(A) $\left(\frac{-1}{2}, \infty\right)$ (B) $[1, 3]$

(C) $\left(\frac{1}{2}, 1\right) \cup \left(\frac{3}{2}, \infty\right)$ (D*) $\left(\frac{-1}{2}, \frac{1}{2}\right) \cup \left(\frac{1}{2}, 1\right) \cup \left(\frac{3}{2}, \infty\right)$

[Sol.$_{91/func/SC}$ $x + 0.5 > 0$ $\Rightarrow$ $x + 0.5 \neq 1$

Also, $\frac{x^2 + 2x - 3}{4x^2 - 4x - 3} > 0 \Rightarrow \frac{(x+3)(x-1)}{(2x+1)(2x-3)} > 0$



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Q.4 In a ΔABC, if $r = 1$, $R = 3$ and $\angle C = 90^\circ$, then area of ΔABC is equal to

(A*) 7 (B) 8 (C) 9 (D) 10

[Sol.$_{235/sot/SC}$ $\because$ $c = 2R \sin C = 6$ and $r = (s - c) \tan \frac{C}{2} \Rightarrow s = 7$

$\because r = \frac{\Delta}{s} \Rightarrow \Delta = 7.$]

Q.5 The value of 'α' for which $\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$, $\sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$ and $\sin^{-1}(\alpha)$ are the angles of a triangle, is equal to

(A) $\frac{1}{2}$ (B*) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{2}{3}$

[Sol.$_{422/itf/SC}$ $\tan^{-1}(2), \tan^{-1}(3)$ and $\tan^{-1}\left(\frac{\alpha}{\sqrt{1-\alpha^2}}\right)$ are angles of a triangle.]

[INTEGER TYPE] **[2 × 5 = 10]**

Q.6 Let equations $x^2 - 3x + 4 = 0$ and $4x^2 - 2(b-5a)x + b = 0$ $(a, b \in R)$ have a common root. If thirteen arithmetic means are inserted between a and b, then find arithmetic mean of these means.

[Ans. 9]

[Sol.$_{50083/seq/OMR}$ $\because$ Roots of $x^2 - 3x + 4 = 0$ are imaginary

$\therefore$ Both the roots should be common

$\therefore \frac{4}{1} = \frac{-2(b-5a)}{-3} = \frac{b}{4} \Rightarrow b = 16$

and $b - 5a = 6 \Rightarrow 5a = 10 \Rightarrow a = 2$

Arithmetic mean of all the mean $= \frac{\text{sum of all the means}}{\text{number of means}}$

$= \frac{13 \cdot (\text{single A.M. between a and b})}{13} = \frac{2+16}{2} = 9$ **Ans.**]

Q.7 The three different polynomials $x^2 + ax + b$, $x^2 + x + ab$ and $ax^2 + x + b$ have exactly one common zero. Where a, b are non-zero real numbers. Find the value of $a + 2b$.

[Ans. 0]

[Sol.$_{50089/qe/OMR}$ Let $x = \alpha$ be common zero then

$\alpha^2 + a\alpha + b = 0$ ……(1)

$\alpha^2 + \alpha + \alpha b = 0$ ……(2)

$a\alpha^2 + \alpha + b = 0$ ……(3)

$(1) - (2) \Rightarrow (a-1)(\alpha - b) = 0$

$a \neq 1 \Rightarrow \alpha = b$

(1) or (3) gives $b^2 + ab + b = 0$ ……(4)

and (3) gives $ab^2 + 2b = 0$

$b \neq 0 \Rightarrow ab = -2$

if $ab = -2$, $b^2 + b - 2 = 0$

$b = 1$ or -2

$a = -2$ or 1

$a = -2$ $b = 1$ and $a = 1, b = -2$ (does not satisfy the mentioned condition polynomial will be same)

$a + 2b = 0.$]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-10

[SINGLE CORRECT CHOICE TYPE] **[5 × 3 = 15]**

Q.1 The domain of $f(x) = \log_{\left[x+\frac{1}{2}\right]}(x^2 + x - 2)$ is

[**Note:** [k] denotes greatest integer function less than or equal to k.]

(A*) $\left[\frac{3}{2}, \infty\right)$ (B) $\left[\frac{3}{2}, \infty\right) - \{2\}$ (C) $(2, \infty)$ (D) $\left[\frac{1}{2}, \infty\right) - \{2\}$

[Sol.$_{327/func/SC}$ $\because$ $x^2 + x - 2 > 0 \Rightarrow (x + 2)(x - 1) > 0 \Rightarrow x < -2$ or $x > 1$

and base $= \left[x + \frac{1}{2}\right] > 0, \neq 1 \Rightarrow \left[x + \frac{1}{2}\right] \geq 2 \Rightarrow x + \frac{1}{2} \geq 2 \Rightarrow x \geq \frac{3}{2}$

$\therefore$ Domain $= \left[\frac{3}{2}, \infty\right)$.]

Q.2 In a triangle ABC, $a \geq b \geq c$. If $\dfrac{a^3 + b^3 + c^3}{\sin^3 A + \sin^3 B + \sin^3 C} = 27$, then find the maximum value of a.

[**Note :** All symbols used have usual meaning in triangle ABC.]

(A) 3/2 (B) 2 (C*) 3 (D) cannot be determined

[Sol.$_{80/sot/SC}$ Using $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$

$8R^3 = 27 \quad \Rightarrow \quad R = 3/2$

Now, $\dfrac{a}{\sin A} = 2R \quad \Rightarrow \quad a = 2R \sin A = 3 \sin A$

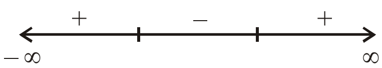
Hence, $a_{max} = 3$]

Q.3 The domain of definition of the function $f(x) = \log_2\left(1 - 3^{\frac{1}{x} - 1}\right)$, is

(A*) $(-\infty, 0) \cup (1, \infty)$ (B) $(-1, 1)$
(C) $(-\infty, \infty)$ (D) $(-\infty, 2) \cup (3, \infty)$

[Sol.$_{343/func/SC}$ Given, $1 - 3^{\frac{1}{x} - 1} > 0$

$\Rightarrow \quad 3^{\frac{1}{x} - 1} < 1 \Rightarrow \frac{1}{x} - 1 < 0 \Rightarrow \frac{x-1}{x} > 0$



Clearly, $x \in (\infty, 0) \cup (1, \infty)$. **Ans.**]

Q.4 The expression $\frac{x^2-y^2}{(x-y)^2}\cdot\frac{x^2-xy+y^2}{x^2-2xy+y^2}\div\frac{x^3+y^3}{(x-y)^4}$ simplifies to

(A) $\frac{1}{xy}$ (B*) $(x-y)$ (C) $\frac{x^2-xy+y^2}{x^2+2xy+y^2}$ (D) $(x+y)$

[Sol.$_{321/qe/SC}$ Use Bodmas Rule.]

Q.5 Given that $0\le x\le\frac{1}{2}$, the value of $\tan\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}+\sqrt{\frac{1-x^2}{2}}\right)-\sin^{-1}x\right)$ is

(A) – 1 (B*) 1 (C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{3}$

[Sol.$_{423/itf/SC}$ Put $x=\sin\theta\ \Rightarrow\theta\in\left[0,\frac{\pi}{6}\right]$

$$\tan\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}+\sqrt{\frac{1-x^2}{2}}\right)-\sin^{-1}x\right)=\tan\left(\sin^{-1}\left(\frac{\sin\theta}{\sqrt{2}}+\frac{\cos\theta}{\sqrt{2}}\right)-\theta\right)$$

$$=\tan\left(\frac{\pi}{4}+\theta-\theta\right)=\tan\left(\frac{\pi}{4}\right)=1.$$]

[INTEGER TYPE] **[1 × 5 = 5]**

Q.6 Let N denotes the number of odd integers between 550 and 800 using the digits 4, 5, 6,7, 8 and 9. Find the sum of the digits in N. [Ans. 0006]

[Sol.$_{50025/perm/OMR}$ **Case-1:** Consider the numbers between 550 – 599

fixed ↑ | 5 | | |

4, 5, 6, 7, 8, 9
1st place in one way (i.e. 5)
2nd place in 5 ways (i.e. 5, 6, 7, 8, 9)
3rd place in 3 ways (i.e. 5, 7, 9)
Number of numbers in this case = 1 · 5 · 3 = 15.

Case-2: 600 – 799 (both inclusive)
Number of numbers = 2 · 6 · = 36
Total = 15 + 36 = 51.
Number of digits in N = 6. **Ans.**]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-11

[SINGLE CORRECT CHOICE TYPE] **[4 × 3 = 12]**

Q.1 The value of $\sum\sum_{0 \le i < j \le 5} \left({}^5C_j\right)\left({}^jC_i\right)$ is equal to

(A*) $3^5 - 2^5$ (B) $3^5 - 1$ (C) $2^5 - 1$ (D) $3^5 \cdot 2^5$

[Sol.$_{157/bin/SC}$ Given expression becomes

$= {}^5C_1 \cdot {}^1C_0 + {}^5C_2\,({}^2C_0 + {}^2C_1) + \ldots\ldots + {}^5C_5\,({}^5C_0 + {}^5C_1 + \ldots\ldots\ {}^5C_4)$

$= {}^5C_1\,(2^1 - 1) + {}^5C_2\,(2^2 - 1) + \ldots\ldots + {}^5C_5\,(2^5 - 1)$

$= ({}^5C_1 \cdot 2^1 + {}^5C_2 \cdot 2^2 + \ldots\ldots\ {}^5C_5 \cdot 2^5) - ({}^5C_1 + {}^5C_2 + \ldots\ldots + {}^5C_5)$

$= [(1 + 2)^5 - 1] - [2^5 - 1]$

$= 3^5 - 2^5$. **Ans.**]

Q.2 The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternatively positive and negative, then the first term is

(A) – 4 (B*) – 12 (C) 12 (D) 4

[Sol.$_{157/seq}$ Let $a, ar, ar^2, \ldots\ldots..$

Now, $a + ar = 12$ ....(1)

$ar^2 + ar^3 = 48$ ....(2)

Now, $\dfrac{\text{equation (2)}}{\text{equation (1)}} \Rightarrow \dfrac{ar^2(1+r)}{a(r+1)} = 4$

$\Rightarrow$ $r^2 = 4$, (As $r \ne -1$)

$\Rightarrow$ $r = -2$

Also, $a = -12$ (using (1)). **Ans.**]

Q.3 A function g has domain [0, 2] and range [– 1, 3]. The domain and range of the function f defined by $f(x) = 3 - 2g(9x^2 - 1)$ are equal to

(A) [0, 2] and [– 1, 3]

(B) $\left[\dfrac{-1}{3}, \dfrac{1}{3}\right]$ and [–3, 5]

(C*) $\left[\dfrac{-1}{\sqrt{3}}, \dfrac{-1}{3}\right] \cup \left[\dfrac{1}{3}, \dfrac{1}{\sqrt{3}}\right]$ and [–3, 5]

(D) $\left[\dfrac{-1}{\sqrt{3}}, \dfrac{-1}{3}\right]$ and [3, 5]

[Sol.$_{55/func/SC}$ We have $f(x) = 3 - 2g(9x^2 - 1)$

As domain of $g(x) = [0, 2]$ so, for domain of $f(x)$,

$0 \le 9x^2 - 1 \le 2 \Rightarrow \dfrac{1}{9} \le x^2 \le \dfrac{1}{3} \Rightarrow x \in \left[\dfrac{-1}{\sqrt{3}}, \dfrac{-1}{3}\right] \cup \left[\dfrac{1}{3}, \dfrac{1}{\sqrt{3}}\right]$ = Domain of f(x).

As range of $g(x) = [-1, 3]$

$\Rightarrow -1 \le g \le 3 \Rightarrow -6 \le -2g \le 2 \Rightarrow -3 \le 3 - 2g \le 5 \Rightarrow -3 \le f(x) \le 5$

So, range of $f(x) = [-3, 5]$. **Ans.**]

Q.4 If $\cos^{-1}(2x-1-x^2)+\sin^{-1}y=0$ then the value of x^2+y^2+xy is equal to
(A) 0 (B*) 1 (C) –1 (D) 3

[Sol.$_{431/itf/SC}$ $2x-1-x^2=-(x^2-2x+1)=-(x-1)^2\le 0$

$\therefore \frac{\pi}{2}\le\cos^{-1}(-(x-1)^2)\le\pi$

$\therefore$ for LHS to be zero

$\cos^{-1}\left(-(x-1)^2\right)=\frac{\pi}{2}$ and $\sin^{-1}y=\frac{-\pi}{2}$

$(x-1)^2=0 \Rightarrow x=1$ and $y=-1$

$\therefore x^2+y^2+xy=1.$]

[MULTIPLE CORRECT CHOICE TYPE] **[3 × 4 = 12]**

Q.5 If $f^2(x)\cdot f\left(\frac{1-x}{1+x}\right)=x^3\ \forall x\in R-\{-1,1\}$ and $f(x)\ne 0$, then which of the following is correct.
(A*) $f(2)\cdot f(-2)=16$ (B) $f(2)\,f(-2)=8$ (C*) $f(2)=-12$ (D*) $f(3)=-18$

[Sol.$_{40019/func/MORE}$ $\because f^2(x)\cdot f\left(\frac{1-x}{1+x}\right)=x^3$ ……(1)

Replace x by $\left(\frac{1-x}{1+x}\right)$, we get

$$f^2\left(\frac{1-x}{1+x}\right)f(x)=\left(\frac{1-x}{1+x}\right)^3 \quad ……(2)$$

From $\frac{(1)^2}{(2)}$, we get

$$f^3(x)=x^6\left(\frac{1+x}{1-x}\right)^3$$

$\therefore$ $f(x)=x^2\left(\frac{1+x}{1-x}\right)$

$\therefore$ $f(2)=4\left(\frac{3}{-1}\right)=-12$

and $f(-2)=4\left(\frac{-1}{3}\right)=\frac{-4}{3}$

$f(2)\,f(-2)=16$

and $f(3)=\frac{9\times4}{-2}=-18$ **Ans.**]

Q.6 Let a, b $\in$ R satisfying $4a^2-5b^2+6a+1=0$ and the line $ax+by+1=0$ touches fixed circle, then

(A*) centre of circle is (3, 0) (B*) radius of circle is $\sqrt{5}$

(C) radius of circle is $\sqrt{3}$ (D*) circle passes through M(1, 1)

[Sol.$_{40542/cir/MORE}$ $\left|\dfrac{a\alpha + b\beta + 1}{\sqrt{a^2 + b^2}}\right| = r$

$\Rightarrow\ a^2(\alpha^2 - r^2) + b^2(\beta^2 - r^2) + 2ab\alpha\beta + 2a\alpha + 2b\beta + 1 = 0$

Now compare with the given equation

$\therefore\ \alpha = 3, \beta = 0, r^2 = 5$.]

Q.7 In acute angle triangle ABC with usual notations, $r = r_2 + r_3 - r_1$ and angle $B > \frac{\pi}{3}$ then the number of integer(s) in the range of $\left(\frac{a-c}{b}\right)$ is less than

(A) 0 (B*) 1 (C*) 2 (D*) 3

[Sol.$_{40554/sot/MORE}$ $r = r_2 + r_3 - r_1$

$$\frac{\Delta}{s} = \frac{\Delta}{s-b} + \frac{\Delta}{s-c} - \frac{\Delta}{s-a}$$

$$\frac{(s-a)(s-c)}{s(s-b)} = \frac{a-c}{b}$$

$$\tan^2\left(\frac{B}{2}\right) = \left(\frac{a-c}{b}\right)$$

Since $\frac{B}{2} \in \left(\frac{\pi}{6}, \frac{\pi}{4}\right)$

$\therefore\ \frac{a-c}{b} \in \left(\frac{1}{3}, 1\right)$]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

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[SINGLE CORRECT CHOICE TYPE] **[4 × 3 = 12]**

Q.1 Let F (x) = f (x) + g (x) + h (x) where $f(x) = \sqrt{\dfrac{\log_{0.3}(x-1)}{x^2-2x-8}}$, $g(x) = \sqrt{{}^{x^2+4x}C_{2x^2+3}}$ and $h(x) = \sqrt{1-\log_3(x^2-6x+11)}$. The number of positive integers in the domain of F (x), is

(A*) 2 (B) 3 (C) 9 (D) 11

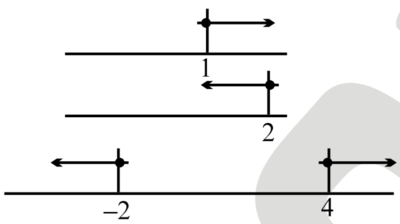
[Sol.$_{282/func/SC}$ For f (x) :

$\dfrac{\log_{0.3}(x-1)}{x^2-2x-8} \ge 0$ and $x \ge 1$

Case-I: $\log_{0.3}(x-1) \ge 0$ $\Rightarrow$ $x - 1 \le 1$ $x \le 2$

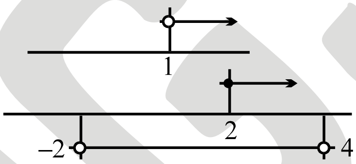
and $(x-4)(x+2) > 0$ $\Rightarrow$ $x > 4$ or $x < -2$

No solution



Case-II: $x > 1$ and $\log_{0.3}(x-1) \le 0$ $\Rightarrow$ $x - 1 \ge 1$ $\Rightarrow$ $x \ge 2$

and $(x-4)(x+2) < 0$



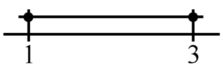
Hence domain [2, 4)

For g (x) :

$x^2 + 4x \ge 2x^2 + 3$ and $x \in N$

$x^2 - 4x + 3 \le 0$

$(x-3)(x-1) \le 0$



$x \in \{1, 2, 3\}$

For h (x) :

$1 \ge \log_3(x^2 - 6x + 11)$ [Note $x^2 - 6x + 11 > 0 \ \forall \ x \in R$]

$\Rightarrow$ $\log_3(x^2 - 6x + 11) \le 1$

$\Rightarrow$ $x^2 - 6x + 11 \le 3$

$\Rightarrow$ $x^2 - 6x + 8 \le 0$ $\Rightarrow$ $2 \le x \le 4$ **Ans.**]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Q.2 If $a^2 + b^2 + c^2 + ab + bc + ca \le 0$, where a, b, c $\in$ R, then domain of the function

$f(x) = \sqrt{\text{sgn}(x)\cdot(ax^2 + bx + c)}$ will be

(A) $(0, \infty)$ (B) $[0, \infty)$ (C) $(-\infty, 0]$ (D*) $(-\infty, \infty)$

[Sol.$_{311/func/SC}$ $\because\ a^2 + b^2 + c^2 + ab + bc + ca \le 0$

$\Rightarrow (a + b)^2 + (b + c)^2 + (c + a)^2 \le 0$

$\Rightarrow\ a + b = b + c = c + a = 0 \Rightarrow\ a = b = c = 0$

$\therefore\ D_f = R$]

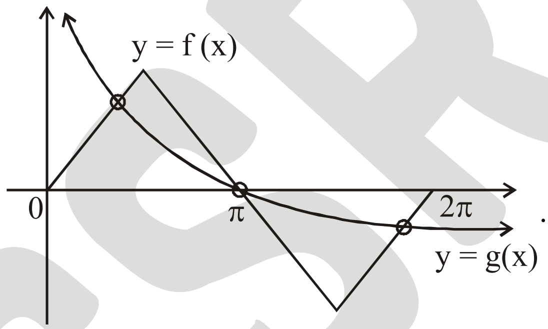
Q.3 The number of solutions of the equation $x \sin^{-1}(\sin x) + x = \pi$ in $[0, 2\pi]$ is

(A) 1 (B) 2 (C*) 3 (D) 4

[Sol.$_{425/itf/SC}$ $x \sin^{-1}(\sin x) + x = \pi$

$\sin^{-1}(\sin x) = \frac{\pi}{x} - 1$

$f(x) = \sin^{-1}\sin x$ and $g(x) = \frac{\pi}{x} - 1$



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Q.4 If $\theta_1 = \sin^{-1}\frac{4}{5} + \sin^{-1}\frac{1}{3}$ and $\theta_2 = \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{1}{3}$, then

(A) $\theta_1 > \theta_2$ (B) $\theta_1 = \theta_2$ (C*) $\theta_1 < \theta_2$ (D) none of these

[Sol.$_{415/itf/SC}$ $\theta_1 = \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{2\sqrt{2}} = \tan^{-1}\frac{8\sqrt{2}+3}{6\sqrt{2}-4} < \frac{\pi}{2}$,

$\theta_2 = \frac{\pi}{2} - \sin^{-1}\frac{4}{5} + \frac{\pi}{2} - \sin^{-1}\frac{1}{3} = \pi - \theta_1 > \frac{\pi}{2}$.]

[COMPREHENSION TYPE] [2 × 3 = 6]

Paragraph for question nos. 5 & 6

Consider $f(x) = (a - 1)x^2 - 8x + 1$, $a \in R$, $a \ne 1$, $x \in R$ and $g(x) = x^2 - 8x + 13$, $x \in R$

Q.5 The largest integral value of 'a' for which maximum finite value of f(x) > minimum value of g(x) is

(A*) 0 (B) 1 (C) 5 (D) 6

Q.6 The least integral value of 'a' for which minimum finite value of f(x) > minimum value of g(x) is

(A) 0 (B) 1 (C) 5 (D*) 6

[Sol.$_{30019\text{-}20/qe}$ $f(x) = (a - 1)x^2 - 8x + 1$, $g(x) = x^2 - 8x + 13$

$-\frac{64-4(a-1)}{4(a-1)} > -3 \Rightarrow \frac{a-1-16}{a-1} > -3 \Rightarrow \frac{a-17+3a-3}{a-1} > 0$

$\Rightarrow \frac{a-5}{a-1} > 0 \Rightarrow a \in (-\infty, 1) \cup (5, \infty)$

(i) For the existence of maximum finite value of f(x)

$a-1<0 \Rightarrow a<1$

$\therefore$ largest possible integral value of 'a' is zero.

(ii) For the existance of minimum finite value of f(x)

$a-1>0 \Rightarrow a>1$

$\therefore$ least possible integral value of 'a' is 6.]

[MATRIX TYPE] **[2+2+2+2=8]**

Q.7 The circles $x^2+y^2=4$ and $x^2+y^2-2x-2y+1=0$ intersect at A and B, then

Column-I	Column-II
(A) if $ax+2by=5$ is common chord of given circles, then $\left(\frac{b}{a}\right)$ is equal to	(P) $\sqrt{\frac{7}{2}}$
	(Q) $\frac{3}{4}$
(B) if θ is acute angle between given circles then the value of $\cos\theta$ is equal to	(R) $\sqrt{\frac{5}{3}}$
(C) length of common tangent of given circles is equal to	(S) $\frac{1}{2}$
(D) diameter of smallest circle which is passing through A and B is equal to	(T) 1

[Ans. (A) S; (B) Q; (C) T; (D) P]

[Sol.$_{92009/cir}$

$x^2+y^2=4$; centre (0, 0) and radius = 2

$x^2+y^2-2x-2y+1=0$

centre (1, 1) and radius = 1

(A) The equation of common chord is $2x+2y=5$

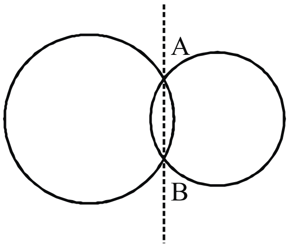
$\therefore$ a = 2, b = 1

$\Rightarrow \left(\frac{b}{a}\right)=\frac{1}{2}$ **Ans.**

(B) $\cos\theta = \left(\frac{r_1^2+r_2^2-d^2}{2r_1r_2}\right) = \frac{4+1-2}{2(1)(2)} = \frac{3}{4}$

(C) Length of common tangent $=\sqrt{d^2-(r_1-r_2)^2} = \sqrt{\left(\sqrt{2}\right)^2-(2-1)^2} = \sqrt{2-1}=1$ **Ans.**

(D)



$= 2\sqrt{4-\frac{25}{8}} = \frac{2\sqrt{7}}{2\sqrt{2}} = \sqrt{\frac{7}{2}}$. **Ans.**]