GGSRDN
Educational Services Private Limited
NEET, JEE(Main/Advanced)
अभ्यास ही सबसे बड़ा गुरू है।

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Educational Services Private Limited

9th, 10th, NEET, JEE(Main/Advanced)

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CLASS : XI (MATHEMATICS)

DPP

DAILY PRACTICE PROBLEM

DPP-71 to 87

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 71

Total Marks : 48

Max. Time : 48 min.

Topic : Binomial Theorem

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.1, to12 and 15,16	(3 marks, 3 min.)	[42, 42]
Assertion and Reason (no negative marking) Q.13,14	(3 marks, 3 min.)	[6, 6]

SPECIAL DPP ON BINOMIAL THEOREM (QUESTION ASKED IN AIEEE)

1. The coefficient of x^5 in $(1 + 2x + 3x^2 + ...)^{-3/2}$ is :

(1) 21 (2) 25 (3) 26 (4) none of these

2. The number of integral terms in the expansion of $\left(\sqrt{3}+\sqrt[8]{5}\right)^{256}$ is :

(1) 32 (2) 33 (3) 34 (4) 35.

3. If x is positive, the first negative term in the expansion of $(1+x)^{\frac{27}{5}}$ is :

(1) 7th term (2) 5th term (3) 8th term (4) 6th term.

4. The coefficient of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is the same, if α equals :

(1) $-\frac{5}{3}$ (2) $\frac{10}{3}$ (3) $-\frac{3}{10}$ (4) $\frac{3}{5}$

5. The coefficient of x^n in the expansion of $(1 + x)(1 - x)^n$ is-

(1) $(n - 1)$ (2) $(-1)^n (1 - n)$ (3) $(-1)^{n-1}(n - 1)^2$ (4) $(-1)^{n-1} n$

6. If $s_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$, then $\frac{t_n}{s_n}$ is equal to-

(1) $\frac{n}{2}$ (2) $\frac{n}{2} - 1$ (3) $n - 1$ (4) $\frac{2n-1}{2}$

7. If the coefficients of r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms in the binomial expansion of $(1 + y)^m$ are in AP, then m and r satisfy the equation :

(1) $m^2 - m(4r - 1) + 4r^2 + 2 = 0.$ (2) $m^2 - m(4r + 1) + 4r^2 - 2 = 0.$

(3) $m^2 - m(4r+1) + 4r^2 + 2 = 0.$ (4) $m^2 - m(4r - 1) + 4r^2 - 2 = 0.$

8. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is :

(1) ${}^{56}C_4$ (2) ${}^{56}C_3$ (3) ${}^{55}C_3$ (4) ${}^{55}C_4$

9. If x is so small that x^3 and higher powers of x may be neglected, then $\dfrac{(1+x)^{3/2}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$ may be approximated as :

(1) $\dfrac{x}{2}-\dfrac{3}{8}x^2$ (2) $-\dfrac{3}{8}x^2$ (3) $3x+\dfrac{3}{8}x^2$ (4) $1-\dfrac{3}{8}x^2$

10. If the expansion in powers of x of the function $\dfrac{1}{(1-ax)(1-bx)}$ is $a_0 + a_1x + a_2x^2 + a_3x^3 + ...$, then a_n is :

(1) $\dfrac{a^n-b^n}{b-a}$ (2) $\dfrac{a^{n+1}-b^{n+1}}{b-a}$ (3) $\dfrac{b^{n+1}-a^{n+1}}{b-a}$ (4) $\dfrac{b^n-a^n}{b-a}$

11. For natural numbers m, n if $(1 - y)^m (1 + y)^n = 1 + a_1y + a_2y^2 + ...$ and $a_1 = a_2 = 10$, then (m, n) is :

(1) (35, 20) (2) (45, 35) (3) (35, 45) (4) (20, 45)

12. The sum of the series ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... + {}^{20}C_{10}$ is

(1) $-{}^{20}C_{10}$ (2) $\dfrac{1}{2}\,{}^{20}C_{10}$ (3) 0 (4) ${}^{20}C_{10}$

13. **Statement-1 :** $\sum_{r=0}^{n}(r+1)\,{}^nC_r = (n + 2)\,2^{n-1}$

Statement-2 : $\sum_{r=0}^{n}(r + 1)\,{}^nC_r\,x^r = (1 + x)^n + nx\,(1 + x)^{n-1}$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

14. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$.

Statement -1 : $S_3 = 55 \times 2^9$.
Statement -2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.
(1) Statement -1 is true, Statement-2 is true ; Statement -2 is not a correct explanation for Statement -1.
(2) Statement-1 is true, Statement-2 is false.
(3) Statement -1 is false, Statement -2 is true.
(4) Statement -1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1.

15. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is :
(1) 144 (2) – 132 (3) – 144 (4) 132

16. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$ is :

(1) an irrational number (2) an odd positive integer
(3) an even positive integer (4) a rational number other than positive integers

MATHEMATICS DPP

DAILY PRACTICE PROBLEMS

DPP No. 72

Total Marks : 25

Max. Time : 26 min.

Topics : Permutation & Combination, Binomial Theorem

Type of Questions		M.M., Min.
Comprehension (no negative marking) Q.1 to Q.3	(3 marks, 3 min.)	[9, 9]
Single choice Objective (no negative marking) Q.4	(3 marks, 3 min.)	[3, 3]
Multiple choice objective (no negative marking) Q.5	(5 marks, 4 min.)	[5, 4]
Subjective Questions (no negative marking) Q.6,7	(4 marks, 5 min.)	[8, 10]

COMPREHENSION # 1 (1 to 3)

Consider, sum of the series $\sum\sum_{0\le i<j\le n} f(i)\,f(j)$

In the given summation, i and j are not independent.

In the sum of series $\sum_{i=1}^{n}\sum_{j=1}^{n} f(i)\,f(j) = \sum_{i=1}^{n}\left(f(i)\left(\sum_{j=1}^{n} f(j)\right)\right)$ i and j are independent. In this summation, three types of terms occur, those when i < j, i > j and i = j.

Also, sum of terms when i < j is equal to the sum of the terms when i > j if f(i) and f(j) are symmetrical.

So, in that case

$$\sum_{i=1}^{n}\sum_{j=1}^{n} f(i)f(j) = \sum\sum_{0\le i<j\le n} f(i)f(j) + \sum\sum_{0\le i<j\le n} f(i)f(j) + \sum\sum_{i=j} f(i)f(j)$$

$$= 2\sum\sum_{0\le i<j\le n} f(i)f(j) + \sum\sum_{i=j} f(i)f(j)$$

$$\Rightarrow \quad \sum\sum_{0\le i<j\le n} f(i)f(j) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} f(i)f(j) - \sum\sum_{i=j} f(i)f(j)}{2}$$

When f(i) and f(j) are not symmetrical, we find the sum by listing all the terms.

1. $\sum\sum_{0\le i<j\le n} {}^{n}C_i\ {}^{n}C_j$ is equal to -

(A) $\frac{2^{2n} - {}^{2n}C_n}{2}$ (B) $\frac{2^{2n} + {}^{2n}C_n}{2}$ (C) $\frac{2^{2n} - {}^{n}C_n}{2}$ (D) $\frac{2^{2n} + {}^{n}C_n}{2}$

2. $\sum_{m=0}^{n}\sum_{p=0}^{m} {}^{n}C_m \cdot {}^{m}C_p$ is equal to -

(A) $2^n - 1$ (B) 3^n (C) $3^n - 1$ (D) 2^n

3. $\sum\sum_{0\le i\le j\le n}\left({}^nC_i + {}^nC_j\right)$ is equal to -

(A) $n2^n$ (B) $(n + 1)2^n$ (C) $(n - 1)2^n$ (D) $(n + 1)2^n - 1$

4. Find the three digit numbers in which the middle one is a perfect square are formed using the digits 1 to 9 is (repeatition of digits is allowed)

(A) 243 (B) 242 (C) 244 (D) 246

5. The no. of ways in which 5 different books to be distributed among 3 persons to that each person gets at least one book, is equal to the number of ways in which

(A) 5 persons are alloted 3 different residential flats such that each person is alloted at most one flat and no two persons are alloted the some flat.

(B) No. of parallelograms formed by one set of 6 parallel lines and other set of 5 parallel lines that goes in other direction.

(C) 5 different toys are to be distributed among 3 children, so that each child gets at least one toy.

6. In how many ways can 5 colours be selected out of 8 different colours including red, blue and green

(1) if blue and green are always to be included

(2) if red is always excluded

(3) if red & blue are always included but green excluded ?

7. How many numbers between 400 and 1000 (both exclusive) can be made with the digits 2,3,4,5,6,0 if

(1) repetition of digits not allowed

(2) repetition of digits is allowed

MATHEMATICS DPP
DAILY PRACTICE PROBLEMS

DPP No. 73

Total Marks : 27

Max. Time : 28 min.

Topic : Permutation & Combination

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.12,3,4,5	**(3 marks, 3 min.)**	**[15, 15]**
Subjective Questions (no negative marking) Q.6	**(4 marks, 5 min.)**	**[4, 5]**
Match the Following (no negative marking) Q.7	**(8 marks, 8 min.)**	**[8, 8]**

1. 10 IIT & 2 PET students sit in a row. If the number of ways in which exactly 3 IIT students sit between 2 PET students is K.10!, then the value of 'K' is :

(A) 16. 10!　(B) 2.10!　(C) 12!　(D) 16

2. Number of ways in which 7 people can occupy six seats, 3 seats on each side in a first class railway compartment if two specified persons are to be always included and occupy adjacent seats on the same side, is (k). 5 ! then k has the value equal to:

(A) 2　(B) 4　(C) 8　(D) none

3. Number of different ways in which 8 different books can be distributed among 3 students, if each student receives at least 2 books is

(A) 2940　(B) 2600　(C) 2409　(D) 2446

4. If letters of the word "PARKAR" are written down in all possible manner as they are in a dictionary, then the rank of the word 'PARKAR' is

(A) 98　(B) 99　(C) 100　(D) 101

5. 5 Indian & 5 American couples meet at a party & shake hands. If no wife shakes hands with her husband & no Indian wife shakes hands with a male, then the number of hand shakes that takes place in the party is :

(A) 95　(B) 110　(C) 135　(D) 150

6. The tamer of wild animals has to bring one by one 5 lions & 4 tigers to the circus arena. The number of ways this can be done if no two tigers immediately follow each other is ....

7. **Match the column**

Column - I	**Column - II**
(A) Six boys and six girls sit along a line alternately in x ways and along a circle (again alternately) in y ways, then x = ky, then k =	(p) 2.48!
(B) There are 50 persons among whom 2 are brothers. The number of ways they can be arranged in a circle, if there is exactly one person between the two brothers is	(q) 12
(C) The number of ways in which 10 boys can take positions around a circular table round table, if two particular boys must not be seated side by side is :	(r) 360
(D) The number of 5 digit numbers of the form xyzyx in which $x < y$ is :	(s) 7.8!

MATHEMATICS

DPP
DAILY PRACTICE PROBLEMS

DPP No. 74

Total Marks : 23

Max. Time : 24 min.

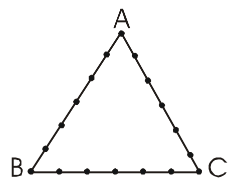
Topics : Permutation & Combination, Probability

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3,4,5	(3 marks, 3 min.)	[15, 15]
Fill in the Blanks (no negative marking) Q.6	(4 marks, 4 min.)	[4, 4]
Subjective Questions (no negative marking) Q.7	(4 marks, 5 min.)	[4, 5]

1. 6 chocolates out of 8 different brands available in the market are choosen, what is the probability that all the chocolates are of different brands.

(A) $\frac{{}^{8}C_6}{{}^{13}C_6}$ (B) $\frac{{}^{8}C_6}{{}^{13}C_8}$ (C) $\frac{{}^{8}C_6}{8^6}$ (D) None of these

2. 18 points are indicated on the perimeter of a triangle ABC (see figure).
If three points are choosen probability it will form a triangle.



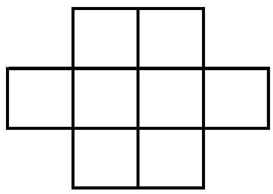
(A) $\frac{331}{816}$ (B) $\frac{1}{2}$

(C) $\frac{355}{408}$ (D) $\frac{711}{816}$

3. A five digits number of the form x y z y x is choosen, probability that x < y is :

(A) $\frac{35}{90}$ (B) $\frac{6}{15}$ (C) $\frac{19}{45}$ (D) $\frac{13}{30}$

4. Find the probability in which 5 X's can be placed in the squares of the figure so that no row remains empty is



(A) $\frac{11}{28}$ (B) $\frac{11}{14}$

(C) $\frac{9}{14}$ (D) $\frac{1}{2}$

5. The probability of choosing randomly a number which is from 1 to 90 divisible by 6 or 8 is

(A) $\frac{1}{6}$ (B) $\frac{11}{90}$ (C) $\frac{1}{30}$ (D) $\frac{23}{90}$

6. (i) The number of arrangements that can be made taking 4 letters, at a time, out of the letters of the word "PASSPORT" is _____
(ii) Probability that both S appear in such 4 letter words is ________
(iii) Probability that all letter are distinct in such 4 letter words is ________

7. A 10 digit numbers is choose with odd digits. Find the probability that no two consecutive digits are same.

MATHEMATICS
DPP
DAILY PRACTICE PROBLEMS

DPP No. 75

Total Marks : 22

Max. Time : 23 min.

Topic : Permutation & Combination

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.1,2,3,4,5,6	**(3 marks, 3 min.)**	**[18, 18]**
Subjective Questions (no negative marking) Q.7	**(4 marks, 5 min.)**	**[4, 5]**

1. Number of ways in which four different toys and five indistinguishable marbles can be distributed between 3 boys, if each boy receives at least one toy and at least one marble

(A) 42 (B) 100 (C) 150 (D) 216

2. If 'm' denotes the number of 5 digit numbers when each successive digits are in their descending order of magnitude and 'n' is the corresponding figure when the digits are in their ascending order of magnitude, then (m – n) has the value

(A) $2.\ {}^{10}C_5$ (B) ${}^{10}C_4$ (C) ${}^{9}C_3$ (D) ${}^{9}C_5$

3. The number of non negative integral solution of the equation, $x + y + 3z = 33$ is:

(A) 120 (B) 135 (C) 210 (D) 520

4. The total number of divisors of the number $N = 2^5 . 3^4 . 5^{10} . 7^6$ that are of the form $4k + 2$, $K \in N$ is equal to

(A) 385 (B) 384 (C) 96 (D) 77

5. There are 9 st. lines of which 5 are concurrent at a point and other 4 are concurrent at another point and no two of these 9 lines are parallel then number points of intersection is

(A) 20 (B) 22 (C) 36 (D) 38

6. Number of natural numbers between 100 & 1000 such that at least one of their digits is 6, is

(A) 251 (B) 243 (C) 258 (D) 252

7. 5 boys & 4 girls sit in a straight line. Find the number of ways in which they can be seated if 2 girls are together & the other 2 are also together but separated from the first 2.

MATHEMATICS DPP
DAILY PRACTICE PROBLEMS

DPP No. 76

Total Marks : 26

Max. Time : 28 min.

Topic : Limit

Type of Questions **M.M., Min.**

Single choice Objective (no negative marking) Q.1,2,3 (3 marks, 3 min.) [9, 9]
Multiple choice objective (no negative marking) Q.4 (5 marks, 4 min.) [5, 4]
Subjective Questions (no negative marking) Q.5,6,7 (4 marks, 5 min.) [12, 15]

1. $\lim\limits_{x\to 0} \dfrac{2^x + 2^{-x} - 2}{x^2} = ?$

(A) $2\ \ell n\ 2$ (B) $(\ell n\ 2)^2$ (C) 0 (D) none

2. $\text{Limit}_{x \to 0} \dfrac{e^{\tan x} - e^x}{\tan x - x} =$

(A) $\dfrac{1}{2}$ (B) 0 (C) 1 (D) none

3. $\text{Limit}_{x \to 0} \dfrac{a^{bx} - b^{ax}}{x}$ where a > 0, b > 0, is equal to:

(A) $\ell n a + \ell n b$ (B) $\ell n a - \ell n b$ (C) $b\ \ell n a - a\ \ell n b$ (D) none

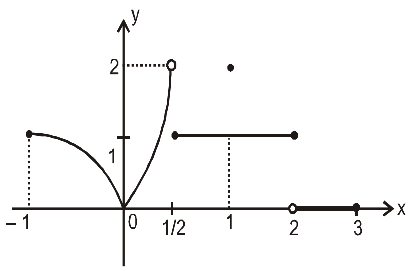
4. Which of the following statements are true of the function f defined for $-1 \le x \le 3$ in the figure shown.

(A) $\text{Limit}_{x\to -1^+} f(x) = 1$ (B) $\text{Limit}_{x\to 2} f(x)$ does not exist

(C) $\text{Limit}_{x\to 1^-} f(x) = 1$ (D) $\text{Limit}_{x\to 0^+} f(x) = \text{Limit}_{x\to 0^-} f(x)$

(E) $\text{Limit}_{x\to c} f(x)$ exists at every c between – 1 & 1

(F) $\text{Limit}_{x\to c} f(x)$ exists at every c between – 1 & 0 .



5. $\lim\limits_{x\to 3} \dfrac{x-3}{\sqrt{x-2} - \sqrt{4-x}}$ is equal to

6. $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x^2} - \sqrt{1-x^2}}{x^2}$ is equal to

7. $\lim\limits_{x\to 1} \left(\dfrac{1}{1-x} - \dfrac{3}{1-x^3}\right)$ is equal to

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 77

Total Marks : 40

Max. Time : 50 min.

Topic : Method of Differentiation

Type of Questions		**M.M.,**	**Min.**
Subjective Questions (no negative marking) Q.1,2,3,4,5,6,7,8,9,10	**(4 marks, 5 min.)**	**[40,**	**50]**

1. Find the derivative of x^2 from first principle.

2. Find the derivative of $\sqrt{\tan x}$ form first principle.

3. Find the derivative of cos(3x + 2) form first priciple.

4. If $g(t) = 1 - t^2$ then find $g'(1)$

5. For the function, given by $f(x) = x^2 - 6x + 8$, prove that $f'(5) - 3f'(2) = f'(8)$

6. If $y = x^3 \tan x$ then find $\frac{dy}{dx}$

7. Find the derivative of $5\sin x - 11\cos x + \frac{1}{x^2}$ w.r. to x

8. If y = xsinx then prove that $\frac{1}{y}\cdot\frac{dy}{dx} - \frac{1}{x} = \cot x$

9. If $y = \frac{\sin x + \cos x}{\sin x - \cos x}$ then find $\frac{dy}{dx}$

10. If $f(x) = \frac{x}{1 + \tan x}$ then find $f'(0)$

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 78

Total Marks : 19

Max. Time : 20 min.

Topic : Solution of Triangle

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.1,2,3,4,5	**(3 marks, 3 min.)**	**[15, 15]**
Subjective Questions (no negative marking) Q.6	**(4 marks, 5 min.)**	**[4, 5]**

1. In a ΔABC, A = $\frac{2\pi}{3}$, b – c = $3\sqrt{3}$ cm and area (ΔABC) = $\frac{9\sqrt{3}}{2}$ cm². Then 'a' is

(A) $6\sqrt{3}$ cm (B) 9 cm (C) 18 cm (D) none of these

2. In a ΔABC, if $\frac{s-a}{11} = \frac{s-b}{12} = \frac{s-c}{13}$, then $\tan^2 \frac{A}{2}$ is equal to

(A) $\frac{143}{342}$ (B) $\frac{13}{33}$ (C) $\frac{11}{39}$ (D) $\frac{12}{37}$

3. If the sides a, b, c of a triangle ABC are the roots of the equation $x^3 - 13x^2 + 54x - 72 = 0$, then the value of $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$ is equal to (with usual notation in ΔABC)

(A) $\frac{169}{144}$ (B) $\frac{61}{72}$ (C) $\frac{61}{144}$ (D) $\frac{169}{72}$

4. If p, q, r are the lengths of the internal bisectors of angles A, B, C of a ΔABC respectively, then

$\frac{1}{p}\cos\frac{A}{2} + \frac{1}{q}\cos\frac{B}{2} + \frac{1}{r}\cos\frac{C}{2} =$

(A) $\frac{1}{a} + \frac{1}{b} - \frac{1}{c}$ (B) $\frac{1}{a} + \frac{1}{c} - \frac{1}{b}$ (C) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ (D) $\frac{1}{b} + \frac{1}{c} - \frac{1}{a}$

5. The two adjacent sides of a cyclic quadrilateral are 2 and 5 and the angle between them is 60°. If the third side is 3, remaining fourth side is.

(A) 2 (B) 3 (C) 4 (D) 5

6. With usual rotation in ΔABC if 2b = 3a and $\tan^2 A = \frac{3}{5}$, prove that there are two values of third side, one of which is double the other.

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 79

Total Marks : 28

Max. Time : 30 min.

Topics : Solution of Triangle, Circle

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.1,2,3,4	**(3 marks, 3 min.)**	**[12, 12]**
Subjective Questions (no negative marking) Q.5,6	**(4 marks, 5 min.)**	**[8, 10]**
Match the Following (no negative marking) Q.7	**(8 marks, 8 min.)**	**[8, 8]**

1. If in a ΔABC, $\frac{r}{r_1} = \frac{1}{2}$, then the value of $\tan\frac{A}{2}\left(\tan\frac{B}{2}+\tan\frac{C}{2}\right)$ is equal to :

(A) 2 (B) $\frac{1}{2}$ (C) 1 (D) None of these

2. A triangle is inscribed in a circle. The vertices of the triangle divide the circle into three arcs of length 3, 4 and 5 units. Then area of the triangle is equal to:

(A) $\frac{9\sqrt{3}(1+\sqrt{3})}{\pi^2}$ (B) $\frac{9\sqrt{3}(\sqrt{3}-1)}{\pi^2}$ (C) $\frac{9\sqrt{3}(1+\sqrt{3})}{2\pi^2}$ (D) $\frac{9\sqrt{3}(\sqrt{3}-1)}{2\pi^2}$

3. Let PQR be a triangle of area Δ with a = 2, b = $\frac{7}{2}$ and c = $\frac{5}{2}$, where a, b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals

(A) $\frac{3}{4\Delta}$ (B) $\frac{45}{4\Delta}$ (C) $\left(\frac{3}{4\Delta}\right)^2$ (D) $\left(\frac{45}{4\Delta}\right)^2$

4. Orthocentre of an acute triangle ABC is at the origin and its circumcentre has the co-ordinates $\left(\frac{1}{2}, -\frac{1}{2}\right)$.

If the base BC has the equation 4x – 2y = 5, then the radius of the circle circumscribing the triangle ABC, is

(A) $\sqrt{5/2}$ (B) $\sqrt{3}$ (C) $\frac{3}{\sqrt{2}}$ (D) $\sqrt{6}$

5. In a triangle ABC, prove that the area of the incircle is to the area of triangle itself is,

$\pi : \cot\left(\frac{A}{2}\right) . \cot\left(\frac{B}{2}\right) . \cot\left(\frac{C}{2}\right)$.

6. In a triangle PQR, PL & QM are the medians. If PL = 6 cm, $\angle QPL = \pi/6$ and $\angle PQM = \pi/3$, then the area of triangle PQR is ________.

7.

Column – I		**Column – II**	
(A)	In a ΔABC, a = 4, b = 3 and the medians AA_1 and BB_1 are mutually perpendicular, then square of area of the ΔABC is equal to	(p)	3
(B)	If in an acute angled ΔABC, line joining the circumcentre and orthocentre is parallel to side AC, then value of tan A.tan C is equal to	(q)	7
(C)	In a ΔABC, a = 5, b = 4 and $\tan\frac{C}{2} = \sqrt{\frac{7}{9}}$, then side 'c' is equal to	(r)	6
(D)	In a ΔABC, $2a^2 + 4b^2 + c^2 = 4ab + 2ac$, then value of (8 cos B) is equal to	(s)	11

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 80

Total Marks : 25

Max. Time : 26 min.

Topic : Solution of Triangle

Type of Questions		**M.M., Min.**
Comprehension (no negative marking) Q.1 to Q.3	(3 marks, 3 min.)	[9, 9]
Single choice Objective (no negative marking) Q.4,5,6	(3 marks, 3 min.)	[12, 12]
Subjective Questions (no negative marking) Q.7	(4 marks, 5 min.)	[4, 5]

COMPREHENSION (Q. 1 to 3)

G is the centroid of triangle ABC. Perpendiculars from vertices A, B, C meet the sides BC, CA, AB at D, E, F respectively. P, Q, R are feet of the perpendiculars from G on sides BC, CA, AB respectively. L, M, N are the mid points of sides BC, CA, AB respectively, then

1. Length of the side PG is

(A) $\frac{1}{2}$ b sin C (B) $\frac{1}{2}$ c sin C (C) $\frac{2}{3}$ b sin C (D) $\frac{1}{3}$ c sin B

2. (Area of ΔGPL) to (Area of ΔALD) is equal to

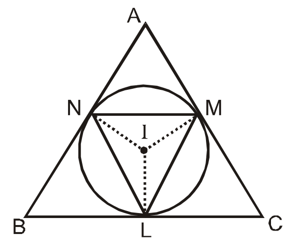
(A) $\frac{1}{3}$ (B) $\frac{1}{9}$ (C) $\frac{2}{3}$ (D) $\frac{4}{9}$

3. Area of ΔPQR is

(A) $\frac{1}{9}(a^2 + b^2 + c^2)$ sin A sin B sin C (B) $\frac{1}{18}(a^2 + b^2 + c^2)$ sin A sin B sin C

(C) $\frac{2}{9}(a^2 + b^2 + c^2)$ sin A sin B sin C (D) $\frac{1}{3}(a^2 + b^2 + c^2)$ sin A sin B sin C

4. If the incircle of the ΔABC touches its sides at L, M and N as shown in the figure and if x, y, z be the circumradii of the triangles MIN, NIL and LIM respectively, where I is the incentre, then the product xyz is equal to :



(A) Rr^2 (B) rR^2

(C) $\frac{1}{2}Rr^2$ (D) $\frac{1}{2}rR^2$

5. Given an isosceles triangle, whose one angle is 120° and radius of its incircle is $\sqrt{3}$ unit. Then the area of the triangle in sq. units is

(A) $7 + 12\sqrt{3}$ (B) $12 - 7\sqrt{3}$ (C) $12 + 7\sqrt{3}$ (D) 4π

6. If in triangle ABC, right angle at B, s – a = 3 and s – c = 2, then

(A) a = 2, c = 3 (B) a = 3, c = 4 (C) a = 4, c = 3 (D) a = 6, c = 8

7. Circles with radii 3, 4 and 5 touch each other externally. If P is the point of intersection of tangents to these circles at their points of contact, find the distance of P from the points of contact.

MATHEMATICS

DPP
DAILY PRACTICE PROBLEMS

DPP No. 81

Total Marks : 21

Max. Time : 21 min.

Topic : Mathematical Reasoning

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3,4,5,6,7	(3 marks, 3 min.)	[21, 21]

1. $(p \vee q) \wedge \sim p$ is logically equivalent to

(A) $p \wedge q$ (B) $\sim p \wedge q$

(C) $\sim p \wedge \sim q$ (D) $\sim (p \wedge q)$

2. If Mumbai is in England then 2 + 2 = 5 is

(A) a true statement (B) a false statement

(C) not a statement (D) may be true or false

3. Negation of " If it is raining then game is cancelled" is

(A) It is raining and game is not cancelled

(B) It is not raining and game is cancelled

(C) It is not raining and game is not cancelled

(D) If it is raining then game is not cancelled

4. Converse of the statement : If a number n is even, then n^2 is even, is

(A) If a number n^2 is even, then n is even

(B) If a number n is not even, then n^2 is not even

(C) Neither number n nor n^2 is even

(D) None of these

5. Contrapositive of p : "If x and y are intergers such that xy is odd, then both x and y are odd" is

(A) If both integers x and y are odd, then xy is odd

(B) If both integers x and y are even, then xy is even

(C) If integer x or integer y is odd, then xy is odd

(D) If both x and y are not odd, then the product xy is not odd

6. Let p, q be the statements : p : X is a square, q : X is a rectangle, then which one of the following represents converse of $p \rightarrow q$.

(A) If X is a rectangle then X is a square

(B) If X is a rectangle then X is not a square

(C) X is rectangle but X is not a square

(D) none of these

7. Let p, q, r be three statements, then $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$, is a

(A) tautology (B) contradiction (C) fallacy (D) None of these

MATHEMATICS DPP
DAILY PRACTICE PROBLEMS

DPP No. 82

Total Marks : 21

Max. Time : 21 min.

Topic : Statistics

Type of Questions — **M.M., Min.**

Single choice Objective (no negative marking) Q.1,2,3,4,5,6,7 (3 marks, 3 min.) — **[21, 21]**

1 The mean of a set of numbers is $\bar{x}$. If each number is multiplied by λ, then mean of new set is

(A) $\bar{x}$ (B) $\lambda+\bar{x}$ (C) $\lambda\bar{x}$ (D) None of these

2 The mean of discrete observations $y_1, y_2, \ldots, y_n$ is given by

(A) $\dfrac{\sum_{i=1}^{n} y_i}{n}$ (B) $\dfrac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} i}$ (C) $\dfrac{\sum_{i=1}^{n} y_i f_i}{n}$ (D) $\dfrac{\sum_{i=1}^{n} y_i f_i}{\sum_{i=1}^{n} f_i}$

3 The reciprocal of the mean of the reciprocals of n observations is their

(A) A.M. (B) G.M. (C) H.M. (D) None of these

4 The weighted mean of first n natural numbers whose weights are equal to the squares of corresponding numbers is

(A) $\dfrac{n+1}{2}$ (B) $\dfrac{3n(n+1)}{2(2n+1)}$ (C) $\dfrac{(n+1)(2n+1)}{6}$ (D) $\dfrac{n(n+1)}{2}$

5 A student obtain 75%, 80% and 85% in three subjects. If the marks of another subject is added, then his average cannot be less than

(A) 60% (B) 65% (C) 80% (D) 90%

6 If the mean of the set of numbers $x_1, x_2, x_3, \ldots, x_n$ is $\bar{x}$, then the mean of the numbers $x_i + 2i, 1 \le i \le n$ is

(A) $\bar{x}+2n$ (B) $\bar{x}+n+1$ (C) $\bar{x}+2$ (D) $\bar{x}+n$

7 Mean of 100 items is 49. It was discovered that three items which should have been 60, 70, 80 were wrongly read as 40, 20, 50 respectively. The correct mean is

(A) 48 (B) $82\frac{1}{2}$ (C) 50 (D) 80

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MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 83

Total Marks : 28

Max. Time : 26 min.

Topics : Fundamentals of Mathematics, Binomial Theorem

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.1,2,3	**(3 marks, 3 min.)**	**[9, 9]**
Multiple choice objective (no negative marking) Q.4,5	**(5 marks, 4 min.)**	**[10, 8]**
Fill in the Blanks (no negative marking) Q.6	**(4 marks, 4 min.)**	**[4, 4]**
Subjective Questions (no negative marking) Q.7	**(4 marks, 5 min.)**	**[4, 5]**

1. If $|r - 6| = 11$ and $|2q - 12| = 8$ then, the minimum value of $\frac{q}{r}$:

(A) -2 (B) $\frac{17}{10}$ (C) $\frac{-1}{5}$ (D) $\frac{2}{5}$

2. If the number 397A is divisible by 6 and the number 2358B is divisible by 4 then the number of possible ordered pair of (A, B) is , (where A, B are digits)

(A) 2 (B) 5 (C) 6 (D) 3

3. If $z = \frac{2+i}{4i+(1+i)^2}$, then $\bar{z}$ is equal to

(A) $\frac{1}{6}+\frac{i}{3}$ (B) $-\frac{1}{6}+\frac{i}{3}$ (C) $\frac{1}{6}-\frac{i}{3}$ (D) $-\frac{1}{6}-\frac{i}{3}$

4. If 2576a456b is divisible by 15, then

(A) a may take the value 5
(B) b may take the value 0
(C) a may take the value 4
(D) a may take the value 6

5. In the expansion of $(x + y + z)^{25}$

(A) every term is of the form $^{25}C_r.\ ^{r}C_k.\ x^{25-r}.\ y^{r-k}.\ z^k$
(B) the coefficient of $x^8\ y^9\ z^9$ is 0
(C) the number of terms is 325
(D) none of these

6. The solution set of the equation $\sqrt[4]{|x-3|^{x+1}} = \sqrt[3]{|x-3|^{x-2}}$ is ______.

7. $\frac{(x-2)(x-4)(x-7)}{(x+2)(x+4)(x+7)} > 1$

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 84

Total Marks : 24

Max. Time : 24 min.

Topics : Fundamentals of Mathematics, Straight Line, Hyperbola, Ellipse

Type of Questions		**M.M.,**	**Min.**
Single choice Objective (no negative marking) Q.1,2,3,4,5	**(3 marks, 3 min.)**	**[15,**	**15]**
Fill in the Blanks (no negative marking) Q.6	**(4 marks, 4 min.)**	**[4,**	**4]**
Subjective Questions (no negative marking) Q.7	**(4 marks, 5 min.)**	**[4,**	**5]**

1. Number of possible ordered pairs of all positions of point P, so that area of rectangle PDOC is 30 sq. units is

(A) 3 (B) 2 (C) 1 (D) 0

2. Point P(–1, 4) is translated by $5\sqrt{2}$ units parallel to the line $2x + 2y + 3 = 0$ so that its ordinate increases. Let Q be its new position, then image of Q with respect to the line $2x + 2y + 3 = 0$ is

(A) (0, – 6) (B) (– 4, –2) (C) $\left(-\frac{21}{2}, \frac{9}{2}\right)$ (D) (– 6, 0)

3. If the point $(1 + \cos\theta, \sin\theta)$ lies between the region corresponding to the acute angle between the lines $3y = x$ & $6y = x$ and $a < \tan\frac{\theta}{2} < b$, then $[a + b]$ is equal to

(where [.] denotes the greatest integer function)

(A) 9 (B) 1 (C) 0 (D) none of these

4. The equation $(x - 2)^2 + (y + 4)^2 = 25\,\frac{(x + 2y - 4)^2}{5}$ represents

(A) parabola (B) ellipse (C) Hyperbola (D) Pair of lines

5. The equation, $9x^2 + 4y^2 - 18x - 16y - 11 = 0$ represents

(A) a parabola (B) an ellipse

(C) a hyperbola (D) a pair of straight lines

6. If $(a^2 + b^2)^3 = (a^3 + b^3)^2$ and $ab \neq 0$ then the numerical value of $\frac{a}{b} + \frac{b}{a}$ is equal to _______

7. Find the solution set of the inequality $||x| - 1| < 1 - x$

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 85

Total Marks : 31

Max. Time : 38 min.

Topic : Statistics

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1	(3 marks, 3 min.)	[3, 3]
Subjective Questions (no negative marking) Q.2,3,4,5,6,7,8	(4 marks, 5 min.)	[28, 35]

1. If the S.D. of a set of observations is 8 and if each observation is divided by –2, the S.D. of the new set of observations will be :

(A) –4 (B) –8 (C) 8 (D) 4

2. Find the mean marks of students from the following cumulative frequency distribution :

Marks	Number of students	Marks	Number of students
0 and above	80	60 and above	28
10 and above	77	70 and above	16
20 and above	72	80 and above	10
30 and above	65	90 and above	8
40 and above	55	100 and above	0
50 and above	43		

3. Compute the mode for the following frequency distribution :

Size of items	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20	20 – 24	24 – 28	28 – 32	32 – 36	36 – 40
Frequency	5	7	9	17	12	10	6	3	1	0

4. The mean and variance of 7 observations are 8 and 16 respecitvely. If 5 of the observations are 2, 4, 10, 12, 14 find the remaining two observations.

5. For a group of 200 candidates the mean and S.D. were found to be 40 and 15 respectively. Later on it was found that the score 43 was misread as 34. Find the correct mean and correct S.D.

6. Calculate the mean and standard deviation for the following data :

Wages upto (in Rs.)	15	30	45	60	75	90	105	120
No. of workers	12	30	65	107	157	202	222	230

7. The sum and sum of squares corresponding to length x (in cm) and weight y (in gm) of 50 plant products are given below :

$$\sum_{i=1}^{50} x_i = 212,\ \sum_{i=1}^{50} x_i^2 = 902.8,\ \sum_{i=1}^{50} y_i = 261,\ \sum_{i=1}^{50} y_i^2 = 1457.6$$

Which is more varying the length or weight ?

8. Coefficient of variation of two distributions are 60% and 70% and their standard deviations are 21 and 16 respectively. What are their arithmetic means ?

MATHEMATICS DPP

DAILY PRACTICE PROBLEMS

DPP No. 86

Total Marks : 38

Max. Time : 43 min.

Topic : Mathematical Induction

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3	(3 marks, 3 min.)	[9, 9]
Multiple choice objective (no negative marking) Q.4	(5 marks, 4 min.)	[5, 4]
Subjective Questions (no negative marking) Q.5,6,7,8,9,10	(4 marks, 5 min.)	[24, 30]

1. If p(n) : $n^2 > 100$ then

(A) p(1) is true
(B) p(4) is true
(C) p(k) is true $\forall$ $k \geq 5, k \in N$
(D) p(k + 1) is true whenever p(k) is true where $k \in N$

2. $1 + 2 + 3 + ... + n < \frac{(n+2)^2}{8}$, $n \in N$, is true for

(A) $n \geq 1$
(B) $n \geq 2$
(C) all n
(D) none of these

3. $n^3 + (n + 1)^3 + (n + 2)^3$ is divisible for all $n \in N$ by

(A) 3
(B) 9
(C) 27
(D) 81

4. By principle of mathematical induction, $3^{2n+2} - 8n - 9$ is divisible for every natural number n by

(A) 16
(B) 8
(C) 64
(D) 9

5. Let P(n) be the statement "$n^3 + n$ is divisible by 3". Write P(1), P(4)

6. Prove that $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ... + \frac{1}{2^n} = 1 - \frac{1}{2^n}$, $n \in N$.

7. By using PMI, prove that $2 + 4 + 6 + ... + 2n = n(n + 1)$, $n \in N$

8. By using PMI, prove that $1.3 + 2.3^2 + 3.3^3 + ... + n.3^n = \frac{(2n-1)3^{n+1} + 3}{4}$, $n \in N$

9. Prove that $2^n > n$, $n \in N$.

10. If 3^{2n}, where n is a natural number, is divided by 8, prove that the remainder is always 1.

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 87

Total Marks : 31

Max. Time : 32 min.

Topic : Parabola

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.1,2,3	**(3 marks, 3 min.)**	**[9, 9]**
Multiple choice objective (no negative marking) Q.4,5	**(5 marks, 4 min.)**	**[10, 8]**
Subjective Questions (no negative marking) Q.6,7,8	**(4 marks, 5 min.)**	**[12, 15]**

1. The parabola having its focus at (3, 2) and directrix along the y – axis has its vertex at–

(A) (2, 2) (B) $\left(\frac{3}{2}, 2\right)$ (C) $\left(\frac{1}{2}, 2\right)$ (D) $\left(\frac{2}{3}, 2\right)$

2. Through the vertex 'O' of the parabola $y^2 = 4ax$, variable chords OP and OQ are drawn at right angles. If the variable chord PQ intersects the axis of x at R, then distance OR:

(A) varies with different positions of P and Q

(B) equals the semi latus rectum of the parabola

(C) equals latus rectum of the parabola

(D) equals double the latus rectum of the parabola

3. Area of the triangle formed by the tangents at the points (4, 6), (10, 8) and (2, 4) on the parabola $y^2 - 2x = 8y - 20$, is (in sq. units)

(A) 4 (B) 2 (C) 1 (D) 8

4. The equation of tangents drawn to the parabola $y^2 + 12x = 0$ from the point (3, 8) is/are

(A) $3x - y - 1 = 0$ (B) $x - 2y + 13 = 0$ (C) $x + 3y - 27 = 0$ (D) none of these

5. The equation $y^2 + 3 = 2(2x + y)$ represents a parabola with the vertex at :

(A) $\left(\frac{1}{2}, 1\right)$ & axis parallel to x – axis (B) $\left(1, \frac{1}{2}\right)$ & axis parallel to x – axis

(C) $\left(\frac{1}{2}, 1\right)$ & focus at $\left(\frac{3}{2}, 1\right)$ (D) $\left(\frac{1}{2}, 1\right)$ & axis parallel to y – axis

6. The focal distance of a point on a parabola $y^2 = 8x$ is 8. Find it

7. Two tangents to the parabola $y^2 = 8x$ meet the tangent at its vertex in the points P and Q. If PQ = 4 units, find the locus of the point of intersection of the two tangents.

8. Find the equations of common tangents to the parabola $y^2 = 16x$ and the circle $x^2 + y^2 = 8$.

DPP 71 TO DPP 87 ANSWER KEY

DPP NO. - 71

1. (4) **2.** (2) **3.** (3) **4.** (3)

5. (2) **6.** (1) **7.** (2) **8.** (1)

9. (2) **10.** (3) **11.** (3) **12.** (2)

13. (1) **14.** (2) **15.** (3) **16.** (1)

DPP NO. - 72

1. (A) **2.** (B) **3.** (A) **4.** (A)

5. (B)(C) **6.** (1) 20 (2) 21 (3) 10

7. (1) 60 (2) 107

DPP NO. - 73

1. (D) **2.** (C) **3.** (A) **4.** (B)

5. (C) **6.** 43200

7. (A) $\rightarrow$ (q), (B) $\rightarrow$ (p), (C) $\rightarrow$ (s), (D) $\rightarrow$ (r)

DPP NO. - 74

1. (A) **2.** (D) **3.** (B) **4.** (B)

5. (D) **6.** (i) 606 (ii) $\frac{21}{101}$ (iii) $\frac{{}^{6}C_{4}.4!}{606}$

7. $\left(\frac{4}{5}\right)^{9}$

DPP NO. - 75

1. (D) **2.** (D) **3.** (C) **4.** (A)

5. (B) **6.** (D) **7.** 43200

DPP NO. - 76

1. (B) **2.** (C) **3.** (C) **4.** (A)(C)(F)

5. 1 **6.** 1 **7.** – 1

DPP NO. - 77

1. $f'(x) = 2x$ **2.** $\frac{\sec^2 x}{2\sqrt{\tan x}}$ **3.** $-3\sin(3x + 2)$

4. – 8 **6.** $x^3\sec^2x + 3x^2\tan x$

7. $5\cos x + 11\sin x - \frac{2}{x^3}$ **9.** $\frac{2}{\sin 2x - 1}$ **10.** 1

DPP NO. - 78

1. (B) **2.** (B) **3.** (C) **4.** (C)

5. (A)

DPP NO. - 79

1. (B) **2.** (A) **3.** (C) **4.** (A)

6. $8\sqrt{3}$ sq. unit

7. (A) $\rightarrow$ (s), (B) $\rightarrow$ (p), (C) $\rightarrow$ (r), (D) $\rightarrow$ (q)

DPP NO. - 80

1. (D) **2.** (B) **3.** (B) **4.** (C)

5. (C) **6.** (B) **7.** $\sqrt{5}$

DPP NO. - 81

1. (B) **2.** (A) **3.** (A) **4.** (A)

5. (D) **6.** (A) **7.** (A)

DPP NO. - 82

1 (C) **2** (A) **3** (C) **4** (B)

5 (A) **6** (B) **7** (C)

DPP NO. - 83

1. (A) **2.** (C) **3.** (A) **4.** (A)(B)(C)

5. (A)(B) **6.** 2, 4, 11 **7.** $(-\infty, -7) \cup (-4, -2)$

DPP NO. - 84

1. (B) **2.** (C) **3.** (C) **4.** (C)

5. (B) **6.** 2/3 **7.** $(-\infty, 0)$

DPP NO. - 85

1. (D) **2.** 51.75 Marks **3.** 32.66

4. x = 6, y = 8 **5.** 14.995 **6.** 25.883

7. 26.43 **8.** 35, 22.85

DPP NO. - 86

1. (D) **2.** (D) **3.** (B) **4.** (A)(B)(C)

5. P(1) : $1^3 + 1$ is divisible by 3,
P(4) : $4^3 + 4$ is divisible by 3

DPP NO. - 87

1. (B) **2.** (C) **3.** (B) **4.** (A)(C)

5. (A)(C) **6.** $(6, 4\sqrt{3})$, $(6, -4\sqrt{3})$ **7.** $y^2 = 8(x + 2)$]

8. $x \pm y + 4 = 0$

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CLASS : XI (MATHEMATICS)

DPP

DAILY PRACTICE PROBLEM

Solutions

DPP-71 to 87

DPP NO. - 71

1. $\because$ $(1 + 2x + 3x^2+.....)^{-3/2} = [(1 - x)^{-2}]^{-3/2} = (1 - x)^3$
So, coefficient of x^5 in $(1 + 2x + 3x^2+....)^{-3/2}$
= coefficient of x^5 in $(1 - x)^3 = 0$.

2. $(r + 1)^{th}$ term of $\left(\sqrt{3}+\sqrt[8]{5}\right)^{256}$

i.e., $T_{r+1} = {}^{256}C_r\,(3)^{(256-r)/2}\,(5)^{r/8}$

The terms are integral, if $\frac{256-r}{2}$ and $\frac{r}{8}$ are both positive integer.
$\Rightarrow r = 0, 8, 16, 24, 32,......, 256$
Hence total terms are 33.

3. $\because$ $(r + 1)^{th}$ term in the expansion of $(1 + x)^{27/5}$

$$= \frac{\frac{27}{5}\left(\frac{27}{5}-1\right)....\left(\frac{27}{5}-r+1\right)}{r!}x^r$$

Now this term will be negative, if the last factor in numerator is the only negative factor.

$\Rightarrow \frac{27}{5} - r + 1 < 0 \Rightarrow \frac{32}{5} < r$
$\Rightarrow 6.4 < r \Rightarrow$ least value of r is 7.
Thus first negative term will be 8th.

4. Coefficient of middle term in $(1 + \alpha x)^4 = {}^4C_2\,\alpha^2$
coefficient of middle term in $(1 - \alpha x)^6 = {}^6C_3\,(-\alpha)^3$
${}^4C_2\,\alpha^2 = -\,{}^6C_3\,\alpha^3$

$-\frac{6}{20} = \alpha$

$\alpha = \frac{-3}{10}$

5. $(1 - x)(1 - x)^n = (1 - x)^n + x(1 - x)^n$
Coefficient of $x^n = (-1)^n + (-1)^{n-1}.\,n$
$= (-1)^n(1 - n)$

6. $s_n = \sum_{r=0}^{n}\frac{1}{{}^nC_r} \quad\Rightarrow\quad t_n = \sum_{r=0}^{n}\frac{r}{{}^nC_r}$

$t_n = \sum_{r=0}^{n}\frac{n-r}{{}^nC_r} \quad\therefore\quad 2t_n = n\sum_{r=0}^{n}\frac{1}{{}^nC_r} = ns_n$

$\Rightarrow \frac{t_n}{s_n} = \frac{n}{2}$

7. $(1 + y)^m$
$T_r = {}^mC_{r-1}\,.\,y^{r-1}$
$T_{r+1} = {}^mC_r\,.\,y^r$
$T_{r+2} = {}^mC_{r+1}\,.\,y^{r+1}$
$\because$ ${}^mC_{r-1} + {}^mC_{r+1} = 2\,{}^mC_r$

$\Rightarrow \frac{{}^mC_{r-1}}{{}^mC_r} + \frac{{}^mC_{r+1}}{{}^mC_r} = 2$

$\Rightarrow m^2 - m(4r + 1) + 4r^2 - 2 = 0$

8. ${}^{50}C_4 + {}^{55}C_3 + {}^{54}C_3 + ... + {}^{50}C_3$
$= {}^{56}C_4$

9. $\frac{(1+x)^{3/2} - \left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$

$$\Rightarrow \frac{\left(1+\frac{3}{2}x+\frac{3}{2}.\frac{1}{2}.\frac{x^2}{2!}\right) - \left(1+\frac{3x}{2}+\frac{3.2}{2}.\frac{x^2}{4}\right)}{(1-x)^{1/2}}$$

$= \left(-\frac{3}{8}x^2\right)(1 - x)^{-1/2} = -\frac{3}{8}x^2$

10. $(1 - ax)^{-1}(1 - bx)^{-1} = (1 + ax + (ax)^2 + ...)(1 + bx + (bx)^2 +............)$
so, $a_n = a^n + a^{n-1}b + a^{n-2}b^2 +....... + b^n$

$$= a^n\,\frac{1-\left(\frac{b}{a}\right)^{n+1}}{1-\frac{b}{a}} = \frac{a^{n+1}-b^{n+1}}{b-a}$$

11. $(1 - y)^m(1 + y)^n = 1 + a_1y + a_2y^2 +........$
$(1 - my + {}^mC_2y^2........)(1 + ny + {}^nC_2y^2.........) = 1 + a_1y + a_2y^2 +........$
$a_1 = n - m = 10$ ..... (1)
$a_2 = {}^mC_2 + {}^nC_2 - mn = 10$ ..... (2)
solving (1) & (2), we get $(m, n) \equiv (35, 45)$

12. $S = {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2\ ..... + {}^{20}C_{10}$
We know,
${}^{20}C_0 - {}^{20}C_1 + ... + {}^{20}C_{20} = 0$
$2({}^{20}C_0 - {}^{20}C_1 + ... - {}^{20}C_9) + {}^{20}C_{10} = 0$

$\therefore$ ${}^{20}C_0 - {}^{20}C_1 + ... - {}^{20}C_9 = -\frac{1}{2}\,{}^{20}C_{10}$

so, $S = \frac{1}{2}\,{}^{20}C_{10}$

13. **Statement -1 :** $\sum_{r=0}^{n}(r+1)\,{}^nC_r = \sum_{r=0}^{n}r.{}^nC_r + \sum_{r=0}^{n}{}^nC_r$
$= n\,.\,2^{n-1} + 2^n = (n + 2)\,2^{n-1}$

Statement-2 : $\sum_{r=0}^{n}(r+1)\,{}^nC_rx^r = \sum_{r=0}^{n}r.\,{}^nC_r\,x^r + \sum_{r=0}^{n}{}^nC_r\,x^r$
$= xn(1 + x)^{n-1} + (1 + x)^n$

14. $S_1 = \sum_{j=1}^{10}j(j-1).\frac{10(10-1)}{j(j-1)}\,{}^8C_{j-2} \Rightarrow S_1 = 9 \times 10$

$\sum_{j=2}^{10}{}^8C_{j-2} \Rightarrow S_1 = 90\,.\,2^8$

$S_2 = \sum_{j=1}^{10}j.\frac{10}{j}\,{}^9C_{j-1} = 10.2^9$

$$S_3 = \sum_{j=1}^{10} (j(j-1)+j)\ {}^{10}C_j$$

$$= \sum_{j=1}^{10} j(j-1)\ {}^{10}C_j + \sum_{j=1}^{10} j\ {}^{10}C_j$$

$$= 90\sum_{j=2}^{10} {}^{8}C_{j-2} + 10\sum_{j=1}^{10} {}^{9}C_{j-1}$$

$= 90 \times 2^8 + 10 \times 2^9 = (45 + 10) . 2^9 = (45 + 10) . 2^9 = 55.2^9$
so statement-1 is true and statement 2 is false.
Hence correct option is (2)

15. $(1 - x - x^2 + x^3)^6$
$(1 - x)^6 (1 - x^2)^6$
$({}^6C_0 - {}^6C_1 x^1 + {}^6C_2 x^2 - {}^6C_3 x^3 + {}^6C_4 x^4 - {}^6C_5 x^5 + {}^6C_6 x^6)\ ({}^6C_0 - {}^6C_1 x^2 + {}^6C_2 x^4 - {}^6C_3 x^6 + {}^6C_4 x^8 + \ldots\ldots\ldots + {}^6C_6 x^{12})$
Now coefficient of $x^7 = {}^6C_1{}^6C_3 - {}^6C_3{}^6C_2 + {}^6C_5{}^6C_1$
$= 6 \times 20 - 20 \times 15 + 36$
$= 120 - 300 + 36$
$= 156 - 300$
$= -144$ **Ans.**

16. $(\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n}$
$= 2[{}^{2n}C_1(\sqrt{3})^{2n-1} + {}^{2n}C_3(\sqrt{3})^{2n-3} + {}^{2n}C_5(\sqrt{3})^{2n-5} + \ldots\ldots..]$
= which is an irrational number

DPP NO. - 72

1. $\displaystyle\sum\sum_{0\le i<j\le n} {}^nC_i\ {}^nC_j$

$$= \frac{\left(\sum_{i=0}^{n}\sum_{j=0}^{n} {}^nC_i\ {}^nC_j\right) - \sum_{i=0}^{n} ({}^nC_i)^2}{2}$$

$$= \frac{\left(\sum_{i=0}^{n} {}^nC_i\, 2^n\right) - \sum_{i=0}^{n} ({}^nC_i)^2}{2}$$

$$= \frac{2^n 2^n - \sum_{i=0}^{n} ({}^nC_i)^2}{2}$$

$$= \frac{2^{2n} - {}^{2n}C_n}{2}$$

2. $\displaystyle\sum_{m=0}^{n}\sum_{p=0}^{m} {}^nC_m . {}^mC_p = \sum_{m=0}^{n} {}^nC_m \left(\sum_{p=0}^{m} {}^mC_p\right)$

$$= \sum_{m=0}^{n} {}^nC_m \left(2^m\right)$$

$= 3^n$

3. $\displaystyle\sum\sum_{0\le i\le j\le n}\left({}^nC_i + {}^nC_j\right) = \frac{\left(\sum_{i=0}^{n}\sum_{j=0}^{n}\left({}^nC_i + {}^nC_j\right)\right) - \sum_{i=0}^{n} 2\,{}^nC_i}{2}$

$$= \frac{\left(\sum_{i=0}^{n}\left(\sum_{j=0}^{n} {}^nC_i + \sum_{j=0}^{n} {}^nC_j\right)\right) - 2\times 2^n}{2}$$

$$= \frac{\left(\sum_{i=0}^{n}\left({}^nC_i \sum_{j=0}^{n} 1 + 2^n\right)\right) - 2^{n+1}}{2}$$

$$= \frac{\left(\sum_{i=0}^{n}\left({}^nC_i (n+1) + 2^n\right)\right) - 2^{n+1}}{2}$$

$$= \frac{(n+1)\sum_{i=1}^{n} {}^nC_i + 2^n \sum_{i=0}^{n} 1 - 2^{n+1}}{2}$$

$$= \frac{(n+1)2^n + 2^n(n+1) - 2^{n+1}}{2}$$

$\mathbf{= (n + 1)2^n - 2^n = n2^n}$

4. Required permutation = 9 × 3 × 9 = 243

5. (A) $\dfrac{5!}{1!\,1!\,3!}\ \dfrac{3!}{2!} + \dfrac{5!}{1!\,2!\,2!}\ \dfrac{1}{2!}\ 3! = \dfrac{720}{12} + \dfrac{720}{8}$

(B) ${}^6C_2 . {}^5C_2 = 15 \times 10 = 150$

(C) ${}^5C_3$

6. (1) Total ways = ${}^6C_3 = 20$
(2) Total ways = ${}^7C_5 = 21$
(3) Total ways = ${}^5C_3 = 10$

7. (1) Total ways = [3 | 5 | 4] = 3 × 5 × 4 = 60

(2) Total ways = 3 × 6 × 6 – 1 = 107

DPP NO. - 73

1. ${}^{10}C_3\ 3!\ 2! = k\ 10!$

$\dfrac{10}{3!\ 7!}\ 3!\ 2! = k\ 10!$

k = 16

2. ${}^5C_3\ {}^2C_1\ 3!\ 2 \times 2 \times 2 = k\ 5!$
$8\ 5! = k\ 5! \Rightarrow k = 8$

3. $\dfrac{8!}{2!\,2!\,4!}\ \dfrac{3!}{2!} + \dfrac{8!}{2!\,3!\,3!}\ \dfrac{3!}{2!} + \dfrac{8\times7\times720\times6}{4}\left[\dfrac{1}{48} + \dfrac{1}{36}\right]$

$14 \times \dfrac{720\times6}{12}\left[\dfrac{7}{12}\right] = 210 \times 14 = 2940$

4. AA KP RR

Word begin A $\dfrac{5!}{2}$

Word begin K $\dfrac{5!}{2}$

Word begin PAA $= \frac{3!}{2!}$

PAK $= \frac{3!}{2!}$

PARA $= 2!$

PARKAR $= 1$

Rank $60 + 30 + 3 + 3 + 2 + 1 = 99$

5. Total number of shaker & hands = $^{20}C_2$
number of shakes hands between indian wife and male
$= 5 \times 10$
number of shakes hands between American wifes and husbands = 5
then number of shakes hands = 190 – 50 – 5 = 135

6. $5!\ {}^6P_4 \Rightarrow 120 \times 360 = 43200$

7. (A) $x = 2.6!\ 6!$
$y = 5!\ 5!$
$x = 12y$

(B) $^{48}C_1\ 47!\ 2 \Rightarrow 2 \,.\, 48!$

(C) $7!\ {}^8P_2 = 7! \frac{8!}{6!} \Rightarrow 7\,(8!)$

(D) $1 \,.\, [8 + 7 + ... + 1] \,.\, 10 = 360$

DPP NO. - 74

1. Prob. $= \frac{^8C_6}{^{13}C_6}$

2. Number of triangles $= {}^{18}C_3 - 3 \cdot ({}^7C_3)$

$= \frac{18.17.16}{3.2.1} - 3 \cdot \frac{76.5}{3.2.1} = 816 - 105 = 711$

3. Probability $= \frac{360}{9 \times 10 \times 10}$

4. Total case $= {}^8C_5 = 56$

Probability $= \frac{44}{56} = \frac{11}{14}$

5. Total case = 90
Now divisible by 6 = 15
divisible by 8 = 11
common number = 3
Favarable case = 15 + 11 – 3 = 23

Probability $= \frac{23}{90}$

6. (i) $^6C_4 \times 4! = 15 \times 2\,4 = 360$

and $^2C_1 . {}^5C_2 . \frac{4!}{2!} = 20 \times 12 = 240$

and $^2C_2 . \frac{4!}{2!2!} = 6$

Total = 606

(ii) when both S appear in the 4 letter word ways

$= {}^5C_2 . \frac{4!}{2!} + {}^2C_2 . \frac{4!}{2!2!} = 120 + 6 = 126$

Probability $= \frac{126}{606} = \frac{21}{101}$

(iii) Probability $= \frac{^6C_4 . 4!}{606}$

7.

5	4	4	4	4	4	4	4	4	4

Favourable case $= 5 \times 4^9$
Total case $= 5^{10}$

Probability $= \frac{5.4^9}{5^{10}} = \left(\frac{4}{5}\right)^9$

DPP NO. - 75

1. $^{5-1}C_{3-1} \cdot \frac{4!}{1!\,1!\,2!} \frac{3!}{2!} \Rightarrow 6 \,.\, \frac{24}{4} \times 6 = 216$

2. $m = {}^{10}C_5$
$n = {}^9C_5$
$m - n = {}^{10}C_5 - {}^9C_5$
$n = {}^9C_5$

$\frac{10!}{5!\,5!} - \frac{9!}{5!\,4!}$

$\frac{9!}{5!\,4!}\left[\frac{10}{5} - 1\right] \Rightarrow {}^9C_5$

3. $x + y + 3z = 33$
$x + y = 33 - 3z$

$\sum_{x=0}^{11} {}^{34-3z}C_1$

$^{34}C_1 + 31 + 28 \,.........\, 1$
$6[1 + 34] = 210$

4. $N = 2^5 \,.\, 3^4 \,.\, 5^{10} \,.\, 7^6$
Number of required divisor = 1.5 . 11 . 7 – 1 = 385 – 1
= 384

5. $^9C_2 - {}^5C_2 - {}^4C_2 + 2 = 36 - 10 - 6 + 2 = 22$

6. Total number – numbers in which no digit is 6
$899 - 8 \times 9 \times 9 + 1$
$899 - 648 + 1 \Rightarrow 252$

7. $\frac{4!}{2!\,2!} \cdot \frac{1}{2!} . 5!\ {}^6P_2$
$128 \times 36 \times 12$
$360 \times 120 = 43200$

DPP NO. - 76

1. $\lim_{x \to 0} \frac{(2^x - 1)^2}{x^2} = (\ln 2)^2$

2. $\lim_{x\to 0} \frac{e^{\tan x} - e^x}{\tan x - x} = \lim_{x\to 0} \frac{e^x(e^{\tan x - x} - 1)}{\tan x - x}$

Appliy limit

= 1

3. $\lim_{x\to 0} \frac{a^{bx} - b^{ax}}{x}$

$\lim_{x\to 0} \frac{b(a^{bx} - 1)}{bx} - \frac{(b^{ax} - 1)a}{ax}$

$= b\log_e b - a\log_e b$

5. $\lim_{x\to 3} \frac{(x-3)\left(\sqrt{x-2} + \sqrt{4-x}\right)}{(2x-6)} = \frac{1+1}{2} = 1.$

6. $\lim_{x\to 0} \frac{2x^2}{x^2\left(\sqrt{1+x^2} + \sqrt{1-x^2}\right)} = \frac{2}{1+1} = 1$

7. $\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3}\right) = \lim_{x\to 1} \frac{1+x^2+x-3}{(1-x)(x^2+x+1)}$

$= \lim_{x\to 1} \frac{(x-1)(x+2)}{-(x-1)(x^2+x+1)} = \frac{-3}{3} = -1$

DPP NO. - 76

1. $f(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$

$= \lim_{h\to 0} \frac{x^2 + h^2 + 2hx - x^2}{h}$

$= \lim_{h\to 0} h + 2x$

$= 2x$

2. $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{\sqrt{\tan(x+h)} - \sqrt{\tan x}}{h}$

$= \lim_{h\to 0} \frac{(\sqrt{\tan(x+h)} - \sqrt{\tan x}) \times (\sqrt{\tan(x+h)} + \sqrt{\tan x})}{h\,(\sqrt{\tan(x+h)} + \sqrt{\tan x})}$

$= \lim_{h\to 0} \frac{\tan(x+h) - \tan x}{h(\sqrt{\tan(x+h)} + \sqrt{\tan x})}$

$= \lim_{h\to 0} \frac{\tan x + \tanh - \tan x + \tan^2 x \tanh}{h\,(\sqrt{\tan(x+h)} + \sqrt{\tan x})}$

$= \lim_{h\to 0} \frac{\tanh}{h} \frac{1 + \tan^2 x}{\sqrt{\tan(x+h)} + \sqrt{\tan x}}$

$\Rightarrow$ Appling limit

$\frac{\sec^2 x}{2\sqrt{\tan x}}$

3. $f'(x) = \lim_{h\to 0} \frac{\cos(3(x+h)+2) - \cos(3x+2)}{h}$

$= \lim_{h\to 0} \frac{-2\sin\left(\frac{(6x+4+3h)}{2}\right)\sin\left(\frac{3h}{2}\right)}{\frac{3h}{2}x^2} \times 3$

$= -3\sin(3x + 2)$

4. $g'(t) = \lim_{h\to 0} \frac{1-(t+h)^2 - 1 + t^2}{h}$

$= \lim_{h\to 0} \frac{1 - t^2 - h^2 - t^4 - 1 + t^2}{h}$

$= \lim_{h\to 0} -h - 2t$

$g'(t) = -2t$

$g'(1) = -2 \times 1 = -2$

5. $f'(x) = \lim_{h\to 0} \frac{(x+h)^2 - 6(x+h) + 8 - x^2 + 6x - 8}{h}$

$= \lim_{h\to 0} \frac{x^2 + h^2 + 2xh - 6x - 6h + 8 - x^2 + 6x - 8}{h}$

$= \lim_{h\to 0} h + 2x - 6$

$= 2x - 6$

$f'(5) = 4 \quad f'(2) = -2 \quad f'(8) = 10$

$\Rightarrow f'(5) - 3f'(2) = f'8$

6. $y = x^3 \tan x$

$\frac{dy}{dx} = \lim_{h\to 0} \frac{(x+h)^3 \tan(x+h) - x^3 \tan x}{h}$

$= \lim_{h\to 0} \frac{(x^3 + h^3 + 3x^2h + 3h^2x)(\tan x + \tanh) - x^3.\tan x - x^3 \tan^2 x \tanh}{(1 - \tan x \tanh)\, h}$

Applying limit

$3x^2 \tan x + x^3 \sec^2 x$

7. $f(x) = 5\sin x - 11 \cos x + \frac{1}{x^2}$

$f'(x) = 5\cos x + 11 \sin x - \frac{2}{x^3}$

8. $y = x\sin x$

$\frac{dy}{dx} = x\cos x + \sin x$

divided by y on both sides

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \cot x + \frac{1}{x} \Rightarrow \frac{1}{y}\frac{dy}{dx} - \frac{1}{x} = \cot x$

9. $\frac{dy}{dx} = \frac{(\sin x - \cos x)(\cos x - \sin x) - (\sin x + \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^2}$

$= \frac{-\sin x - \cos x + 2\sin x \cos x - \sin^2 x - \cos^2 x - 2\sin x \cos x}{1 - \sin 2x}$

$= \frac{2}{\sin 2x - 1}$

10. $f(x) = \dfrac{x}{1+\tan x}$

$f'(x) = \dfrac{(1+\tan x) - x(\sec^2 x)}{(1+\tan x)^2}$

$f'(0) = \dfrac{1-0}{1}$

$f'(0) = 1$

DPP NO. - 78

1. $\Delta = \dfrac{1}{2}$ bc sin A $= \dfrac{9\sqrt{3}}{2}$

$\Rightarrow$ bc = 18

$\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

$\Rightarrow \cos\dfrac{2\pi}{3} = \dfrac{(b-c)^2 + 2bc - a^2}{2bc}$

$\Rightarrow$ a = 9 cm

2. $\dfrac{s-a}{11} = \dfrac{s-b}{12} = \dfrac{s-c}{13} = \dfrac{3s-(a+b+c)}{11+12+13} = \dfrac{s}{36}$

Now $\tan^2\left(\dfrac{A}{2}\right) = \dfrac{(s-b)(s-c)}{s(s-a)} = \dfrac{(12)(13)}{(11)\,(36)} = \dfrac{13}{33}$

3. a + b + c = 13

$\sum ab = 54$

abc = 72

Now $\dfrac{b^2+c^2-a^2}{2abc} + \dfrac{a^2+c^2-b^2}{2abc} + \dfrac{b^2+a^2-c^2}{2abc}$

$= \dfrac{a^2+b^2+c^2}{2abc} = \dfrac{(a+b+c)^2 - 2\sum ab}{2ab}$

$= \dfrac{169-2(54)}{2\times 72} = \dfrac{61}{144}$

4. $\dfrac{1}{p}\cos\dfrac{A}{2} + \dfrac{1}{q}\cos\dfrac{B}{2} + \dfrac{1}{r}\cos\dfrac{C}{2}$

$p = \dfrac{2bc}{b+c}\cos\dfrac{A}{2}$

$= \dfrac{b+c}{2bc} + \dfrac{c+a}{2ac} + \dfrac{a+b}{2ac}$

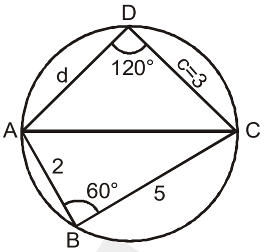
$= \dfrac{ab+ac+bc+ab+ac+bc}{2abc}$

$= \dfrac{2(ab+bc+ca)}{2abc}$

$= \dfrac{1}{b} + \dfrac{1}{c} + \dfrac{1}{a}$

5. Let AB = 2 and BC = 5

$\angle$ABC = 60° (given).



Since the quadrilateral is cyclic,

$\angle$ CDA = 180° – 60° = 120°,

Let CD = c and DA = d

Also $AB^2 + BC^2 - 2AB \,.\, BC \cos 60° = AC^2$

$= CD^2 + DA^2 - 2CD \,.\, DA \cos 120°$

by cosine rule.

or $4 + 25 - 2.2\,.\,5\,.\,\dfrac{1}{2} = c^2 + d^2 + cd$

$19 = c^2 + d^2 + cd = 9 + d^2 + 3d$

$\therefore$ $d^2 + 3d - 10 = 0$

or $(d + 5)(d - 2) = 0$ $\quad\therefore$ d = 2

6. $\sec^2 A = 1 + \tan^2 A = \dfrac{8}{5}$

$\therefore$ $\cos A = \sqrt{\left(\dfrac{5}{8}\right)}$

or $b^2 + c^2 - a^2 = 2bc.\dfrac{\sqrt{5}}{2\sqrt{2}}$.

put $a = \dfrac{2b}{3}$

$\therefore$ $b^2 + c^2 - \dfrac{4b^2}{9} = bc\dfrac{\sqrt{5}}{\sqrt{2}}$

$c^2 - bc\sqrt{\left(\dfrac{5}{2}\right)} + \dfrac{5b^2}{9} = 0$

$\therefore$ $c_1 + c_2 = b\sqrt{\left(\dfrac{5}{2}\right)}$, $c_1c_2 = \dfrac{5b^2}{9}$

$\therefore$ $(c_1 - c_2)^2 = (c_1 + c_2)^2 - 4\,c_1c_2$

$= \dfrac{5b^2}{2} - \dfrac{20}{9}b^2 = \dfrac{5}{18}b^2$

$\therefore$ $c_1 - c_2 = \dfrac{b}{3}\sqrt{\left(\dfrac{5}{2}\right)}$

$\therefore$ $c_1 = \dfrac{2b}{3}\sqrt{\left(\dfrac{5}{2}\right)}$, $c_2 = \dfrac{b}{3}\sqrt{\left(\dfrac{5}{2}\right)}$

$\therefore$ $c_1 = 2c_2$

DPP NO. - 79

1. $\frac{r}{r_1} = \frac{1}{2} \Rightarrow \frac{\Delta/s}{\frac{\Delta}{s-a}} = \frac{1}{2}$

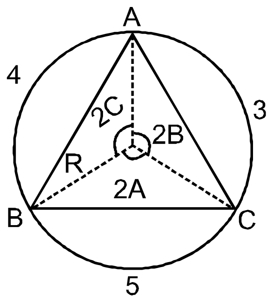
$\Rightarrow \frac{s-a}{s} = \frac{1}{2} \qquad \Rightarrow s = 2a$

Now, $\tan\frac{A}{2}\left(\tan\frac{B}{2}+\tan\frac{C}{2}\right)$

$= \frac{\Delta}{s(s-a)}\left(\frac{\Delta}{s(s-b)}+\frac{\Delta}{s(s-c)}\right)$

$\Rightarrow \frac{\Delta}{s(s-a)}\cdot\frac{\Delta}{s}\cdot\frac{a}{(s-b)(s-c)} = \frac{a}{s} = \frac{1}{2}$

2.



angle = $\frac{\text{arc}}{\text{radius}}$ ....(1)

$\because$ 4 + 5 + 3 = $2\pi R \Rightarrow R = 6/\pi$

$\because$ $2A = \frac{5}{R} = \frac{5\pi}{6}$,

$2B = \frac{3}{R} = \frac{\pi}{2}$ and

$2C = \frac{4}{R} = \frac{2\pi}{3}$

Area of Δ ABC = $\frac{1}{2}R^2\left[\sin\frac{2\pi}{3}+\sin\frac{5\pi}{6}+\sin\frac{\pi}{2}\right]$

$= \frac{R^2}{2}\left[\frac{\sqrt{3}}{2}+\frac{1}{2}+1\right] = \frac{R^2}{2}\left[\frac{\sqrt{3}+3}{2}\right]$

$= \frac{\sqrt{3}(\sqrt{3}+1)}{4} \times \frac{36}{\pi^2} = \frac{9\sqrt{3}(\sqrt{3}+1)}{\pi^2}$

3. a = 2 = QR

$b = \frac{7}{2}$ = PR

$c = \frac{5}{2}$ = PQ

$s = \frac{a+b+c}{2} = \frac{8}{4} = 4$

$\frac{2\sin P - 2\sin P\cos P}{2\sin P + 2\sin P\cos P} = \frac{2\sin P(1-\cos P)}{2\sin P(1+\cos P)}$

$= \frac{1-\cos P}{1+\cos P} = \frac{2\sin^2\frac{P}{2}}{2\cos^2\frac{P}{2}} = \tan^2\frac{P}{2}$

$= \frac{(s-b)(s-c)}{s(s-a)} = \frac{(s-b)^2(s-c)^2}{\Delta^2}$

$= \frac{\left(4-\frac{7}{2}\right)^2\left(4-\frac{5}{2}\right)^2}{\Delta^2} = \left(\frac{3}{4\Delta}\right)^2$

4.

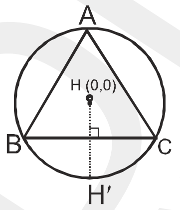


Image of H(0, 0) w.r.t
the line 4x – 2y = 5 is H′ (2, – 1)
$\therefore$ radius of circumcircle

$= \sqrt{\left(2-\frac{1}{2}\right)^2+\left(-1+\frac{1}{2}\right)^2} = \sqrt{5/2}$

5. $\because$ $\frac{\text{Area of incircle}}{\text{Area of }\Delta ABC} = \frac{\pi r^2}{\frac{1}{2}bc\sin A}$

$= \frac{\pi\times 16R^2\times\sin^2\frac{A}{2}\sin^2\frac{B}{2}\sin^2\frac{C}{2}}{\frac{1}{2}(2R\sin B)(2R\sin C)\left(2\sin\frac{A}{2}\cos\frac{A}{2}\right)}$

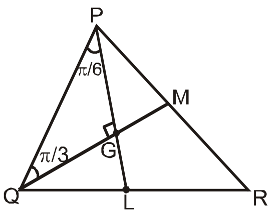
$= \frac{4\pi\sin\frac{A}{2}\sin^2\frac{B}{2}\sin^2\frac{C}{2}}{\left(2\sin\frac{B}{2}\cos\frac{B}{2}\right)\left(2\sin\frac{C}{2}\cos\frac{C}{2}\right)\cos\frac{A}{2}}$

$= \frac{\pi\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$

$= \frac{\pi}{\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}}$

$= \pi : \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$

6.



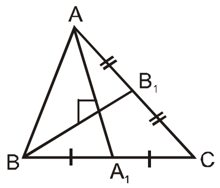
PG = $\frac{2}{3}$PL = 4cm

$\tan\frac{\pi}{3} = \frac{PG}{QG}$ ⇒ QG = $\frac{4}{\sqrt{3}}$ cm

Area of ΔPQR = 3.Area of ΔGPQ

$= 3.\frac{1}{2}.4.\frac{4}{\sqrt{3}} = 8\sqrt{3}$ sq. units

7.



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(A) ∵ AA_1 and BB_1 are perpendicular
∴ $a^2 + b^2 = 5c^2$

∴ $c^2 = \frac{a^2+b^2}{5} = 5$ ⇒ $c = \sqrt{5}$

∵ $\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{16+9-5}{2\times4\times3} = \frac{5}{6}$

∵ $\sin C = \frac{\sqrt{11}}{6}$

∴ $\Delta = \frac{1}{2}ab\sin C = \sqrt{11}$

∴ $\Delta^2 = 11$

(B) ∵ line joining the circumcentre and orthocentre is parallel to side AC

⇒ R cos B = 2R cos A cos C

⇒ –cos (A+C) = 2 cos A cos C

⇒ sin A sin C = 3 cos A cos C

⇒ tan A tan C = 3

(C) ∵ $\tan^2\frac{C}{2} = \frac{(s-a)(s-b)}{s(s-c)}$ ∵ a = 5, b = 4

∵ 2s = 9 + c

$= \frac{(9+c-10)(9+c-8)}{(9+c)(9-c)} = \frac{c^2-1}{81-c^2}$

⇒ $\frac{7}{9} = \frac{c^2-1}{81-c^2}$ ⇒ $c^2 = 36$ ⇒ c = 6

(D) ∵ $2a^2 + 4b^2 + c^2 = 4ab + 2ac$.

⇒ $(a - 2b)^2 + (a - c)^2 = 0$

⇒ a = 2b = c

DPP NO. - 80

1. ∵ PG = $\frac{1}{3}$AD

$= \frac{1}{3}.\frac{2\Delta}{a} = \frac{2}{3a}.\frac{1}{2}$.ab sin C or

$= \frac{1}{3}$b sin C (∵ $\Delta = \frac{1}{2}$ac sin B)

∴ PG = $\frac{2}{3a}.\frac{1}{2}$ac sin B

$= \frac{1}{3}$c sin B

2. ∵ Area of ΔGPL = $\frac{1}{2}$(PL) (PG)

and Area of ΔALD = $\frac{1}{2}$(DL) (AD)

∵ PL = $\frac{1}{3}$ DL and PG = $\frac{1}{3}$ AD

∴ $\frac{\text{Area of }\Delta GPL}{\text{Area of }\Delta ALD} = \frac{\frac{1}{2}(PL)(PG)}{\frac{1}{2}(DL)(AD)}$

$= \frac{\frac{1}{3}(DL)\times\frac{1}{3}(AD)}{(DL)(AD)} = \frac{1}{9}$

3. ∵ Area of ΔPQR = Area of ΔPGQ + Area of ΔQGR + Area of ΔRGP ...(1)

∵ Area of ΔPGQ = $\frac{1}{2}$ PG.GQ.sin(∠ PGQ)

$= \frac{1}{2} \times \frac{1}{3}$ AD × $\frac{1}{3}$BE sin (π – C)

$= \frac{1}{18} \times \frac{2\Delta}{a} \times \frac{2\Delta}{b}$ sin C

$= \frac{2}{9ab} \times \frac{1}{2}$bc sin A × $\frac{1}{2}$ac sin B × sin C

$= \frac{c^2}{18}$ sin A.sin B.sin C

Similarly Area of ΔQGR = $\frac{a^2}{18}$ sin A.sin B .sin C and

Area of ΔRGP = $\frac{b^2}{18}$ sin A.sin B.sin C

∴ From equation (1), we get

Area of ΔPQR = $\frac{1}{18}(a^2 + b^2 + c^2)$sin A.sin B.sin C

4. MINA is a cyclic quadrilatral

∴ $\frac{MN}{\sin A} = AI$

$\Rightarrow$ MN = r cosec $\frac{A}{2}$ sinA = 2r cos $\frac{A}{2}$

$\because$ IM = IN = r

$\therefore$ x = $\dfrac{\left(2r\cos\frac{A}{2}\right)(r)(r)}{4\times\frac{1}{2}r\times r\sin A}$ = $\dfrac{2r^3\cos\frac{A}{2}}{2r^2\sin A}$

$\dfrac{r\cos\frac{A}{2}}{\sin A} = \dfrac{r}{2\sin\frac{A}{2}}$

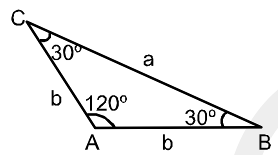
similarly y = $\dfrac{r}{2\sin\frac{B}{2}}$ and z = $\dfrac{r}{2\sin\frac{C}{2}}$

$\therefore$ xyz = $\dfrac{r^3}{8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}} = \dfrac{r^3}{2\frac{r}{R}} = \dfrac{1}{2}r^2 R$

5. $\Delta = \frac{1}{2}$. b . b . sin 120° = $\frac{\sqrt{3}}{4}b^2$ ....(1)

Also $\dfrac{\sin 120^\circ}{a} = \dfrac{\sin 30^\circ}{b}$ $\Rightarrow$ a = $\sqrt{3}$b ....(2)

and $\Delta = \sqrt{3}s$ and s = $\frac{1}{2}$(a + 2b)



$\Rightarrow$ $\Delta = \dfrac{\sqrt{3}}{2}$(a + 2b) ....(3)

From (1), (2) and (3), we get $\Delta = \left(12+7\sqrt{3}\right)$

6. $\because$ b cos² $\frac{A}{2}$ + a cos² $\frac{B}{2}$ = $\frac{3}{2}$c.

$\Rightarrow$ b $\dfrac{s(s-a)}{bc}$ + a $\dfrac{s(s-b)}{ac}$ = $\frac{3}{2}$c.

$\Rightarrow$ $\frac{s}{c}$ [s – a + s – b] = $\frac{3}{2}$ c

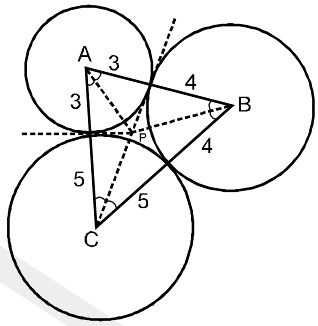
$\Rightarrow$ $\frac{s}{c}$ × c = $\frac{3}{2}$ c

$\Rightarrow$ $\dfrac{a+b+c}{2} = \dfrac{3c}{2}$ $\Rightarrow$ a + b = 2c

$\Rightarrow$ a, c, b are in A.P.

7. Clearly P is the incentre of triangle ABC.

r = $\dfrac{\Delta}{s} = \sqrt{\dfrac{(s-a)(s-b)(s-c)}{s}}$



Here 2s = 7 + 8 + 9 $\Rightarrow$ s = 12

Here r = $\sqrt{\dfrac{5.4.3}{12}} = \sqrt{5}$

DPP NO. - 81

1. (p v q) ∧ ~ p = (p ∧ ~ p) v (q ∧ ~ p) = fv (q ∧ ~ p)
= q ∧ ~ p = ~ p ∧ q

2. p : Mumbai is in England
q : 2 + 2 = 5
If p, then q $\Rightarrow$ p → q $\Rightarrow$ F → F $\Rightarrow$ T

3. ~ (p → q) ≡ (p ∧ ~ q)

4. Converse of p → q is q → p.

5. Contrapositive of p → q is ~ q → ~p.

6. Converse of p → q is q → p.

7. p → (q → r) ≡ ~ p v (q → r) ≡ ~ p v (~ q v r) ≡ [(~p v (~ q)] v r ≡ ~ (p ∧ q) v r ≡ p ∧ q → r
$\therefore$ given statement is a tautology.
p → (q → r) ≡ ~ p v (q → r) ≡ ~ p v (~ q v r) ≡ [(~p v (~ q)] v r ≡ ~ (p ∧ q) v r ≡ p ∧ q → r

DPP NO. - 82

1 $\bar{x} = \dfrac{\Sigma x_i}{n}, \Sigma x_i = n\bar{x}$

New mean = $\dfrac{\Sigma \lambda x_i}{n} = \lambda\dfrac{\Sigma x_i}{n} = \lambda\bar{x}$.

2 It is obvious.

3 It is a fundamental property.

4 Weighted mean = $\dfrac{1.1^2+2.2^2+......+n.n^2}{1^2+2^2+........+n^2}$

$= \dfrac{\Sigma n^3}{\Sigma n^2} = \dfrac{\frac{n(n+1)}{2}\frac{n(n+1)}{2}}{\frac{n(n+1)(2n+1)}{6}} = \dfrac{3n(n+1)}{2(2n+1)}$

5 Marks obtained from 3 subjects out of 300
= 75 + 80 + 85 = 240
If the marks of another subject is added, then the marks will be ≥ 240 out of 400

∴ Minimum average marks = $\frac{240}{4} = 60\%$,

[When marks in the fourth subject = 0]

6 We know that $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ i.e., $\sum_{i=1}^{n} x_i = n\bar{x}$

∴ $\frac{\sum_{i=1}^{n}(x_i + 2i)}{n} = \frac{\sum_{i=1}^{n} x_i + 2\sum_{i=1}^{n} i}{n}$

$= \frac{n\bar{x} + 2(1+2+....n)}{n} = \frac{n\bar{x} + 2\frac{n(n+1)}{2}}{n} = \bar{x} + n + 1$

7 Sum of 100 items = 49 × 100 = 4900
Sum of items added = 60 + 70 + 80 = 210
Sum of items replaced = 40 + 20 + 50 = 110
New sum = 4900 + 210 – 110 = 5000

∴ Correct mean = $\frac{5000}{100} = 50$

DPP NO. - 83

1. |r–6| = 11 and |2q–12| = 8
r = –5 or 17 q = 2 or 10

$\frac{q}{r} = -\frac{2}{5}, -2, \frac{2}{17}, \frac{10}{17}$ ∴ min $\frac{q}{r} = -2$

2. If 397 A is divisible by 2 then A = 0,2,4,6,8
If 397 A is divisible by 3 then A = 2,5,8
So If 397 A is divisible by 6 then A = 2,8
If 2358 B is divisible by 4 then B = 0,4,8
So ordered pair of (A,B) are
{(2,0), (2,4), (2,8), (8, 0), (8, 4), (8,8)}

3. $\bar{z} = \frac{2-i}{-4i+(1-i)^2} = \frac{2-i}{-6i} = \frac{1}{6} + \frac{i}{3}$

4. Since the number is divisible by 5
∴ b = 0, 5
number is divisible by 3 iff a + b + 2 is divisible by 3
(i) If b = 0, then a = 1, 4, 7
(ii) If b = 5, then a = 2, 5, 8

5. $(x + y + z)^{25}$

General term = $\frac{25!}{r_1!\,r_2!\,r_3!}\; x^{r_1} y^{r_2} z^{r_3}$

Putting $r_3 = k$, $r_2 = r - k$ and $r_1 = 25 - r$

$= \frac{25!}{(25-r)!(r-k)!(k)!} \times \frac{r!}{r!} \times x^{25-r}\, y^{r-k}\, z^k$

$= {}^{25}C_r \cdot {}^{r}C_k \cdot x^{25-r}\, y^{r-k}\, z^k$

$r_1 + r_2 + r_3 = 25$

∴ coefficient of $x^8 y^9 z^9$ is 0

∴ $x^8 y^9 z^9 = 0$

DPP NO. - 84

1. Let P (α, β)
Area αβ = 30
a(16 – 2a) = 30
$2\alpha^2 - 16\alpha + 30 = 0$
$\alpha^2 - 8\alpha + 15 = 0$
α = 3 or α = 5

2. $Q \equiv \left(-1 + 5\sqrt{2}\left(\frac{-1}{2}\right), 4 + 5\sqrt{2}\frac{1}{\sqrt{2}}\right) \equiv (-6, 9)$

⇒ for image of Q in 2x + 2y + 3 = 0

$\frac{x+6}{2} = \frac{y-9}{2} = -2\left(\frac{-12+18+3}{8}\right) = \frac{-9}{4}$

$\Rightarrow x = \frac{-21}{2}, \; y = \frac{9}{2}$

3. slope of OP = $\frac{\sin\theta}{1+\cos\theta} \Rightarrow \frac{1}{6} < \frac{\sin\theta}{1+\cos\theta} < \frac{1}{3}$

$\frac{1}{6} < \tan\frac{\theta}{2} < \frac{1}{3}$

$\Rightarrow a = \frac{1}{6}, \; b = \frac{1}{3} \Rightarrow$ [a + b] = 0

4. $(x-2)^2 + (y+4)^2 = 25\left(\frac{x+2y-4}{5}\right)^2$

focus (2, –4) directrix x + 2y – 4 = 0
e = 5
focus does not lies on directrix and e > 1
∴ it represents hyperbola

5. $9x^2 + 4y^2 - 16x - 16y - 11 = 0$
Δ ≠ 0 a = 9, b = 4, c = – 11
f = – 8, g = – 8, h = 0 and $h^2 - ab < 0$
∴ It represents an ellipse

6. $(a^2 + b^2)^2 = (a^3 + b^3)^2$
⇒ $a^6 + b^6 + 3a^4b^2 + 3b^4a^2 = a^6 + b^6 + 2a^3b^3$
⇒ $3a^2 + 3b^2 = 2ab$

$\frac{a}{b} + \frac{b}{a} = \frac{2}{3}$

7. ||x| – 1| < 1 – x

|x| – 1 < 1 – x	&	|x| – 1 > x – 1
when x ≥ 0		when x ≥ 0
x – 1 < 1 – x		x – 1 > x – 1
2x < 2		x – 1 > x – 1
x < 1 x ∈ [0, 1) ..(1)		x ∈ 0 ....(3) x ∈ (–∞, 0)
when x < 0		when x < 0
–x – 1 < 1 – x		– x – 1 > x – 1
x ∈ (–∞, 0) .....(2)		2x < 0
		x < 0
		x ∈ (–∞, 0) ....(4)

DPP NO. - 85

2. Here we have, the cumulative frequency distribution. So, first we convert it into an ordinary frequency distribution. We observe that there are 80 students getting marks greater than or equal to 0 and 77 students have secured 10 and more marks. Therefore, the number of students getting marks between 0 and 10 is 80 – 77 = 3. Similarly, the number of students getting marks between 10 and 20 is 77 – 72 = 5 and so on. Thus, we obtain the following frequency distribution.

Marks	Number of students	Marks	Number of students
0 – 10	3	50 – 60	15
10 – 20	5	60 – 70	12
20 – 30	7	70 – 80	6
30 – 40	10	80 – 90	2
40 – 50	12	90 – 100	8

Now, we compute arithmetic mean by taking 55 as the assumed mean.
Computation of Mean

Marks	Mid-value (x_i)	Frequency (f_i)	$u_i = \frac{x_i - 55}{10}$	f_iu_i
0 – 10	5	3	–5	–15
10 – 20	15	5	–4	–20
20 – 30	25	7	–3	–21
30 – 40	35	10	–2	–20
40 – 50	45	12	–1	–12
50 – 60	55	15	0	0
60 – 70	65	12	1	12
70 – 80	75	6	2	12
80 – 90	85	2	3	6
90 – 100	95	8	4	32
Total		$\Sigma f_i = 80$		$\Sigma f_iu_i = -26$

We have,

$N = \Sigma f_i = 80$, $\Sigma f_iu_i = -26$, A = 55 and h = 10

$$\therefore \quad \overline{X} = A + h\left\{\frac{1}{N}\Sigma f_iu_i\right\}$$

$$\Rightarrow \quad \overline{X} = 55 + 10 \times \frac{-26}{80} = 55 - 3.25 = 51.75 \text{ Marks}$$

3. Here, the maximum frequency is 17 and the corresponding class is 12-16 So 12-16 is the modal class.

We have, $\ell = 12$, h = 4, $f = 17$, $f_1 = 9$ and $f_2 = 12$

$$\therefore \quad \text{Mode} = \ell + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$\Rightarrow \quad \text{Mode} = 12 + \frac{17 - 9}{34 - 9 - 12} \times 4$$

$$\Rightarrow \quad \text{Mode} = 12 + \frac{8}{13} \times 4 = 12 + \frac{32}{13}$$

$= 12 + 10.66 = 32.66$

4. Let x and y be the remaining two observations, then Mean = 8

$$\Rightarrow \quad \frac{2+4+10+12+14+x+y}{7} = 8$$

$\Rightarrow$ 42 + x + y = 56

$\Rightarrow$ x + y = 14

$\Rightarrow$ variance = 16 ...(i)

$$\Rightarrow \quad \frac{1}{7}(2^2 + 4^2 + 10^2 + 12^2 + 14^2 + x^2 + y^2) - (\text{Mean})^2 = 16$$

$$\Rightarrow \quad \frac{1}{7}(4 + 16 + 100 + 144 + 196 + x^2 + y^2) - 64 = 16$$

$\Rightarrow$ $460 + x^2 + y^2 = 7 \times 80$

$\Rightarrow$ $x^2 + y^2 = 100$

Now $(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$ ...(ii)

$\Rightarrow$ $196 + (x - y)^2 = 2 \times 100$

$\Rightarrow$ $(x - y)^2 = 4$

$\Rightarrow$ $x - y = \pm 2$

If x – y = 2, then x + y = 14 and x – y = 2

$\Rightarrow$ x = 8, y = 6

If x – y = –2, then x + y = 14 and x – y = –2

$\Rightarrow$ x = 6, y = 8

Hence the remaining two observations are 6 and 8.

5. We have,

n = 200, $\overline{X} = 40$, $\sigma = 15$

$$\therefore \quad \overline{X} = \frac{1}{n}\Sigma x_i \Rightarrow \Sigma x_i = n\overline{X} = 200 \times 40 = 8000$$

Correct Σx_i = incorrect Σx_i – (sum of incorrect values) + (sum of correct values)

= 8000 – 34 + 43 = 8009

$$\therefore \quad \text{Corrected mean} = \frac{\text{corrected} \sum x_i}{n} = \frac{8009}{200}$$

= 40.045

Now $\sigma = 15$

$$\Rightarrow \quad 15^2 = \frac{1}{200}\left(\sum x_i^2\right) - \left(\frac{1}{200}\sum x_i\right)^2$$

$$\Rightarrow \quad 225 = \frac{1}{200}\left(\sum x_i^2\right) - \left(\frac{8000}{200}\right)^2$$

$$\Rightarrow \quad 225 = \frac{1}{200}\left(\sum x_i^2\right) - 1600$$

$$\Rightarrow \quad \sum x_i^2 = 200 \times 1825 = 365000$$

$$\Rightarrow \quad \text{Incorrect} \sum x_i^2 = 365000$$

Corrected $\sum x_i^2$ = (incorrect $\sum x_i^2$) – (sum of squares of incorrect values)+(sum of squares of correct values)

$= 365000 - (34)^2 + (43)^2 = 365693$

so corrected σ = $\sqrt{\frac{1}{n}\sum x_i^2-\left(\frac{1}{n}\sum x_i\right)^2}$

$= \sqrt{\frac{365693}{200}-\left(\frac{8009}{200}\right)^2}$

$\sqrt{1828.465-1603.602}$ = 14.995

6. We are given the cummulative frequency distribution. So first we will prepare the frequency distribution as given below :

Class Interval	Cummlative frequency	Mid – values	Frequency	$u_i=\frac{x_i-67.5}{15}$	$f_i u_i$	$f_i u_i^2$
0 – 15	12	7.5	12	– 4	– 48	192
15 – 30	30	22.5	18	– 3	– 54	162
30 – 45	65	37.5	35	– 2	– 70	140
45 – 60	107	52.5	42	– 1	– 42	42
60 – 75	157	67.5	50	0	0	0
75 – 90	202	82.5	45	1	45	45
90 – 105	222	97.5	20	2	40	80
105 – 120	230	112.5	8	3	24	72
			$\sum f_i=230$		$\sum f_i u_i=-105$	$\sum f_i u_i^2=733$

Here A = 67.5, h = 15, N = 230, $\sum f_i u_i = -105$

and $\sum f_i u_i^2 = 733$

∴ Mean = $A + h\left(\frac{1}{N}\sum f_i u_i\right) = 67.5 + 15\left(\frac{-105}{230}\right)$

= 67.5 – 6.85 = 60.65

and $Var(X) = h^2\left[\frac{1}{N}\sum f_i u_i^2-\left(\frac{1}{N}\sum f_i u_i\right)^2\right]$

$\Rightarrow Var(X) = 225\left[\frac{733}{230}-\left(\frac{-105}{230}\right)^2\right]$

= 225[3.18 – 0.2025] = 669.9375

∴ S.D. = $\sqrt{Var(X)}$

= $\sqrt{669.9375}$ = 25.883

7. We have

$\sum_{i=1}^{50} x_i = 212$ and $\sum_{i=1}^{50} x_i^2 = 902.80$

$\therefore \bar{X} = \frac{\sum_{i=1}^{50} x_i}{50}$

and $\sigma_x^2 = \frac{1}{50}\left(\sum_{i=1}^{50} x_i^2\right)-\left(\frac{1}{50}\sum_{i=1}^{50} x_i\right)^2$

$\Rightarrow \bar{X} = \frac{212}{50}$ and $\sigma_x^2 = \frac{902.80}{50}-\left(\frac{212}{50}\right)^2$

$\Rightarrow \bar{X} = 4.24$ and $\sigma_x^2 = \frac{902.80}{50}-\left(\frac{212}{50}\right)^2$

$= 18.056 - (4.24)^2$

$\Rightarrow \bar{X} = 4.24$ and $\sigma_x = \sqrt{0.0784} = 0.28$

Also, $\sum_{i=1}^{50} y_i = 261$ and $\sum_{i=1}^{50} y_i^2 = 1457.6$

$\therefore \bar{Y} = \frac{\sum_{i=1}^{50} y_i}{50}$ and $\sigma_y^2 = \frac{1}{50}\left(\sum_{i=1}^{50} y_i^2\right)-\left(\frac{1}{50}\sum_{i=1}^{50} y_i\right)^2$

$\Rightarrow \bar{Y} = \frac{261}{50}$ and $\sigma_y^2 = \frac{1457.6}{50}-\left(\frac{261}{50}\right)^2$

$\Rightarrow \bar{Y} = 5.22$ and $\sigma_y^2 = 29.152 - (5.22)^2 = 1.9036$

$\Rightarrow \bar{Y} = 5.22$ and $\sigma_y = 1.3797$

In order to determine the variability of length and weight, we will have to compute the coefficients of variations in lengths and weights.

We have coefficient of variation in lengths = $\frac{\sigma_x}{\bar{X}} \times 100 = \frac{0.28}{4.24}\times 100 = 6.60$

coefficient of variation in weights = $\frac{\sigma_y}{\bar{Y}}\times 100 = \frac{1.3797}{5.22} \times 100 = 26.43$

Clearly coefficient of variation is weights is greater than the coefficient of variation in lengths. Hence weights have more variability than heights.

DPP NO. - 86

1. P(n) ; $n^2 > 100$

Let P(k) is true then $k^2 > 100$

$k^2 + 2k + 1 > 100 + 2k + 1$

$(k + 1)^2 > 100 + 2k + 1$

$(k + 1)^2 > 100$

∴ P(k + 1) is true

2. Let P(n) ; $1 + 2 + 3 + \ldots\ldots + n < \frac{(n+2)^2}{8}$, $n \in N$

Clearly ; P(A) is true

Let P(k) is true

then P(k) ; $1 + 2 + 3 + \ldots\ldots\ldots + k < \frac{1}{8}(k + 2)^2$

adding K + 1 both sides

$1 + 2 + 3 + \ldots\ldots K + (k + 1) < \frac{(k+2)^2}{8} + k + 1$

$< \frac{k^2+12k+12}{8}$

$< \frac{k^2+6k+9}{8}+\frac{6k+3}{8}$

$< \frac{(k+3)^2}{8}+\frac{6k+3}{8}$

Now not necessary P(k + 1) is true

3. Let $p(n) = n^3 + (n + 1)^3 + (n + 2)^3$, $p(A) = 36$, $p(B) = 99$ both are divisible by 99

Let it is true for n = k

$k^3 + (k + 1)^3 + (k + 2)^3 = 9q$; $q \in I$

adding $9k^2 + 27k + 27$ both sides

$k^3 + (k + 1)^3 + (k + 2)^3 + 9k^2 + 27k + 27 = 9q + 9k^2 + 27k + 27$

$(k + 1)^3 + (k + 2)^3 + (k + 3)^3 = 9r$; $r \in I$

4. Let n = 1 then p(A) = 64

Let p(k) is divisible by 64

$3^{2k+2} - 8k - 9$ is divisible by 64

Now,

$P(k + 1) = 3^{2(k+1)+2} - 8(k + 1) - 9$

$= 3^{2k+2} \times 9 - 8 \times k \times 9 - 9 \times 9 - 8 + 72 + 64k$

$= 9(3^{2k+2} - 8k - 9) + 64(k + 1)$

Which is divisible by 64

5. $P(1) : 1^3 + 1$ is divisible by 3

$P(4) : 4^3 + 4$ is divisible by 3

6. (i) Given statement is true for n = 1

(ii) Let us assume that the statement is true for n = k

i.e. $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ... + \frac{1}{2^k} = 1 - \frac{1}{2^k}$

(iii) For n = k + 1,

L.H.S. $= \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ... + \frac{1}{2^k} + \frac{1}{2^{k+1}}$

$= 1 - \frac{1}{2^k} + \frac{1}{2^{k+1}} = 1 - \frac{1}{2^{k+1}}$ = R.H.S.

so by principle of mathematical induction the statement is true for all $n \in N$

7. (i) Given statement is true for n = 1

(ii) Let us assume that the statement is true for n = k

i.e. $2 + 4 + 6 + ... + 2k = k(k + 1)$

(iii) For n = k + 1,

L.H.S. $= 2 + 4 + 6 + ... + 2k + 2(k + 1)$

$= k(k + 1) + 2k + 2 = (k + 1)(k + 2)$ = R.H.S.

so by principle of mathematical induction the statement is true for all $n \in N$

8. (i) Given statement is true for n = 1

(ii) Let us assume that the statement is true for n = k

i.e. $1.3 + 2.3^2 + 3.3^3 + ... + k.3^k$

$= \frac{(2k-1)3^{k+1} + 3}{4}$

(iii) For n = k + 1,

L.H.S.

$= 1.3 + 2.3^2 + 3.3^3 + ... + k.3^k + (k + 1)\,3^{k+1}$

$= \frac{(2k-1)3^{k+1} + 3}{4} + (k + 1)\,3^{k+1}$

$= \frac{(2k+1)3^{k+2} + 3}{4}$ = R.H.S.

so by principle of mathematical induction the statement is true for all $n \in N$

9. (i) Given statement is true for n = 1

(ii) Let us assume that the statement is true for n = k

i.e. $2^k > k \Rightarrow 2^k = k + \lambda$ where $\lambda \in R^+$

(iii) For n = k + 1,

$2^{k+1} = 2(2^k) = 2(k + \lambda) = k + k + 2\lambda > k + 1$

so by principle of mathematical induction the statement is true for all $n \in N$

10. (i) Given statement is true for n = 1

(ii) Let us assume that the statement is true for n = k

i.e. $3^{2k} = 8\lambda + 1$ where $\lambda \in N$

(iii) For n = k + 1,

$3^{2(k+1)} = 9(3^{2k}) = 9(8\lambda + 1)$

$= 72\lambda + 9$ = (multiple of 8) + 1

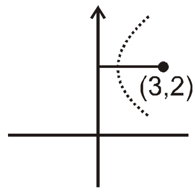
so by principle of mathematical induction the statement is true for all $n \in N$

DPP NO. - 87

1. Focus (3, 2)

Directirx y-axis

vertex $\left(\frac{3}{2}, 2\right)$

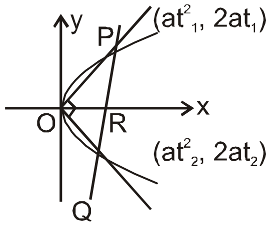


2. $M_{OP} = \frac{2}{t_1}$

$M_{OQ} = \frac{2}{t_2}$

$M_{OP} \times M_{OQ} = -1$

$t_1 t_2 = -4$



equation of PQ $y - 2at_1 = \frac{2}{t_1 + t_2}(x - at_1^2)$

Now at R, y = 0

$\Rightarrow -at_1^2 - at_1t_2 = x - at_1^2 \Rightarrow x = -at_1t_2$

$x = 4a \Rightarrow R(4a, 0)$

$\therefore$ OR = 4a = length of L/R

3. $y^2 - 2x = 8y - 20$

Points (4, 5), (10, 8), (2, 4)

Area of triangle whose vertices are (4, 6), (10, 8), (2, 4)

$$A = \frac{1}{2}\begin{vmatrix} 4 & 6 \\ 10 & 8 \\ 2 & 4 \\ 4 & 6 \end{vmatrix}$$

$= \frac{1}{2}|(32 + 40 + 12 - 60 - 16 - 16)| = \frac{1}{2}|-8| = 4$

$\therefore$ Required area of Δ which formed by tangents at these point = $\frac{1}{2}$ A = $\frac{1}{2}$ × 4 = 2

4. (3, 8) $y^2 = -12x$
$a = 3$

Tangents $y = mx + \frac{a}{m}$ ⇒ $y = mx + \frac{(3)}{m}$

pass (3, 8)

$8 = 3m - \frac{3}{m}$ ⇒ $3m^2 - 8m - 3 = 0$

$3m^2 - 9m + m - z = 0$ ⇒ $3m(m-3)+1(m-3) = 0$

$(3m + 1)(m - 3) = 0$ ⇒ $m = 3,\ m = \frac{1}{3}$

Required tangents are $y = 3x - 1$ and $y = -\frac{1}{3}x + 9$

⇒ $x = 3y - 27 = 0$

5. $y^2 + 3 = 2(2x + y)$ ⇒ $y^2 - 2y = 4x - 3$

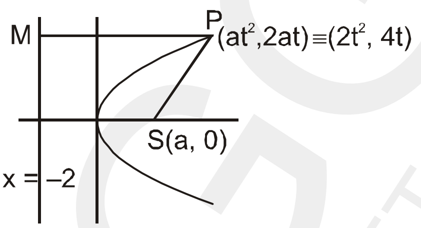
$(y - 1)^2 = 4\left(x - \frac{1}{2}\right)$ ⇒ Vertex $\left(\frac{1}{2}, 1\right)$

$x - \frac{1}{2} = 0,\ y - 1 = 0$ ⇒ $x = \frac{1}{2},\ y = 1$

foucs $\left(\frac{3}{2}, 1\right)$

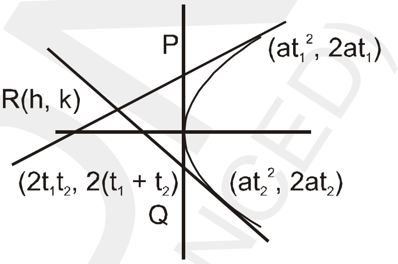
$x - \frac{1}{2} = 1,\ y - 1 = 0$ ⇒ $x = \frac{3}{2},\ y = 1$

6. $y^2 = 8x$ ⇒ $y^2 = 4(2)x$
$a = 2$
$S(2, 0)$
$SP = PM = 2 + 2t^2$



$8 = 2(1 + t^2)$
$t^2 + 1 = 4$
$t^2 = 3$
$t = \pm\sqrt{3}$
Points are
$(6, 4\sqrt{3})$ and $(6, -4\sqrt{3})$ $(6, 4\sqrt{3})$ $(6, -4\sqrt{3})$

7. Tangent at P ⇒ $t_1 y = x + 2t_1^2$
$P \equiv (0, 2t_1)$
by $Q \equiv (0, 2t_2)$ ⇒ $2|t_1 - t_2| = 4$
$|t_1 - t_2| = 2$ ⇒ $(t_1 + t_2)^2 = 4 + 4t_1t_2$



$\left(\frac{k}{2}\right)^2 = 4\left(\frac{h}{2}\right) + 4$

$k^2 = 4(2h + 4)$ ⇒ $k^2 = 8(h + 2)$
locus $y^2 = 8(x + 2)$

8. $y^2 = 16x,\ x^2 + y^2 = 8$

$r = 2\sqrt{2}$

equation of tangents $y = mx + \frac{4}{m}$

$2\sqrt{2} = \left|\frac{0 + 0 + \frac{4}{m}}{\sqrt{1+m^2}}\right|$ ⇒ $8(1 + m^2) = \frac{16}{m^2}$

$m^2(m^2 + 1) = 2$ ⇒ $m^4 + m^2 - 2 = 0$
$m^4 + 2m^2 - m^2 - 2 = 0$ ⇒ $(m^2 + 2)(m^2 - 1) = 0$
$m = \pm 1,\ m^2 + 2 \neq 0$
$y = x + 4$ and $x + y + 4 = 0$
$x \pm y + 4 = 0$