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9th, 10th, NEET, JEE(Main/Advanced)
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CLASS : XII (MATHS)

DPP

DAILY PRACTICE PROBLEM

DPP-61 to 76

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 61

Total Marks : 28

Max. Time : 29 min.

Topics : Three Dimensional Geometry, Vector, Indefinite Integration

Type of Questions		M.M., Min.
Comprehension (no negative marking) Q.1 to Q.3	(3 marks, 3 min.)	[9, 9]
Single choice Objective (no negative marking) Q. 4, 5	(3 marks, 3 min.)	[6, 6]
Multiple choice objective (no negative marking) Q.6	(5 marks, 4 min.)	[5, 4]
Subjective Questions (no negative marking) Q.7, 8	(4 marks, 5 min.)	[8, 10]

COMPREHENSION (Q. NO. 1 TO 3)

Let two planes $P_1 : 2x - y + z = 2$ and $P_2 : x + 2y - z = 3$ are given.

1. Equation of the plane which passes through the point (–1, 3, 2) and is perpendicular to each of the planes P_1 and P_2 is

(A) $x + 3y - 5z + 2 = 0$
(B) $x - 3y + 2z - 18 = 0$
(C) $x - 3y - 5z + 20 = 0$
(D) $x - 3y + 5z = 0$

2. The equation of the acute angle bisector of planes P_1 and P_2 is

(A) $x - 3y + 2z + 1 = 0$ (B) $3x + 3y - 2z + 1 = 0$ (C) $x + 3y - 2z + 1 = 0$ (D) $3x + y = 5$

3. The image of plane P_1 in the plane mirror P_2 is

(A) $x + 7y - 4x + 5 = 0$ (B) $3x + 4y - 5z + 9 = 0$ (C) $7x - y + 2z - 9 = 0$ (D) None of above

4. A mirror and a source of light are situated at the origin O and a point A on OX respectively. A ray of light from the source strikes the mirror and is reflected. If the direction ratios of the normal to the plane of mirror are 1, –1, 1, then direction cosines for the reflected ray are

(A) $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$ (B) $-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$ (C) $-\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}$ (D) $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

5. $\int \frac{\sin^3 x\, dx}{(\cos^3 x + 3\cos^2 x + 1)\tan^{-1}(\sec x + \cos x)} =$

(A) $\tan^{-1}(\sec x + \cos x) + c$
(B) $\ell n \tan^{-1}(\sec x + \cos x) + c$
(C) $\frac{1}{(\sec x + \cos x)^2} + c$
(D) none of these

6. If $\vec{b}$ is vector whose initial point divides the join of $5\hat{i}$ and $5\hat{j}$ in the ratio k : 1 and terminal point is origin and $|\vec{b}| \le \sqrt{37}$, then the set of exhaustive values of k is

(A) $\left[-6, -\frac{1}{6}\right]$ (B) $(-\infty, -6) \cup \left[-\frac{1}{6}, \infty\right)$ (C) [0, 6] (D) $\left[-\frac{1}{6}, \infty\right)$

7. Evaluate :

(i) $\int \frac{(\tan^{-1} x)^3}{1 + x^2} dx$ (ii) $\int \frac{3x^2 + 5}{x^2 + 4} dx$

8. Find $\int \left(\left(\frac{x}{e}\right)^x + \left(\frac{e}{x}\right)^x\right) \ln x\, dx$

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 62

Total Marks : 27

Max. Time : 27 min.

Topic : **Binomial Theorem**

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3,4,5,6	(3 marks, 3 min.)	[18, 18]
Multiple choice objective (no negative marking) Q.7	(5 marks, 4 min.)	[5, 4]
Subjective Questions (no negative marking) Q.8	(4 marks, 5 min.)	[4, 5]

1. The largest real value of 'x' such that $\sum_{k=0}^{4}\left(\frac{5^{4-k}}{(4-k)!}\cdot\frac{x^k}{k!}\right) = \frac{8}{3}$ is

(A) $2\sqrt{2} - 5$ (B) $2\sqrt{2} + 5$ (C) $-2\sqrt{2} - 5$ (D) $-2\sqrt{2} + 5$

2. The value of $\sum\sum_{0\le i<j\le n} j\,.\,{}^nC_i$ is equal to

(A) $n\,.\,2^{n-3}$ (B) $n(n + 3)\,.\,2^{n-3}$ (C) $(n + 3)\,.\,2^{n-3}$ (D) $n(3n + 1)\,.\,2^{n-3}$

3. The value of $\frac{1}{81^n} - \frac{10}{81^n}\,{}^{2n}C_1 + \frac{10^2}{81^n}\,{}^{2n}C_2 - \frac{10^3}{81^n}\,{}^{2n}C_3 + \ldots\ldots + \frac{10^{2n}}{81^n}$ is

(A) 2 (B) 0 (C) 1/2 (D) 1

4. If $(1!)^2 + (2!)^2 + (3!)^2 + \ldots.. + (99!)^2 + (100!)^2$ is divided by 100, the remainder is

(A) 27 (B) 28 (C) 17 (D) 14

5. $\sum_{r=1}^{n}\left(\sum_{p=0}^{r-1}{}^nC_r\,{}^rC_p\,2^p\right)$ is equal to

(A) $4^n - 3^n + 1$ (B) $4^n - 3^n - 1$ (C) $4^n - 3^n + 2$ (D) $4^n - 3^n$

6. If in the expansion of $\left(x^3 - \frac{2}{\sqrt{x}}\right)^n$ a term like x^2 exists and 'n' is a double digit number, then least value of 'n' is :

(A) 10 (B) 11 (C) 12 (D) 13

7. $\lim_{n\to\infty} {}^nC_x\left(\frac{m}{n}\right)^x\left(1-\frac{m}{n}\right)^{n-x}$ equals to

(A) $\frac{m^x}{x!}\,.\,e^{-m}$ (B) $\frac{m^x}{x!}\,.\,e^{m}$ (C) e^0 (D) $\frac{m^{x+1}}{me^m x!}$

8. Find the sum of the series $\sum_{r=0}^{n}\left(\frac{n-3r+1}{n-r+1}\right)\frac{{}^nC_r}{2^r}$.

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 63

Total Marks : 32

Max. Time : 34 min.

Topics : Indefinite Integration, Determinant, Vector

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1, 2	(3 marks, 3 min.)	[6, 6]
Multiple choice objective (no negative marking) Q.3, 4	(5 marks, 4 min.)	[10, 8]
Subjective Questions (no negative marking) Q.5,6,7,8	(4 marks, 5 min.)	[16, 20]

1. $\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}}\,dx$ is equal to

(A) $\frac{1}{2}\sqrt{1+x}+C$ (B) $\frac{2}{3}(1+x)^{3/2}+C$ (C) $\sqrt{1+x}+C$ (D) $2(1+x)^{3/2}+C$

2. Let m be a positive integer & $D_r = \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix}$ $(0 \le r \le m)$, then the value of $\sum_{r=0}^{m} D_r$ is given by :

(A) 0 (B) m^2-1 (C) 2^m (D) $2^m \sin^2(2^m)$

3. $\int \frac{\sin x \cos x}{\sqrt{1-\sin^4 x}}\,dx$ is equal to

(A) $\frac{1}{2}\sin^{-1}(\sin^2 x)+C$ (B) $-\frac{1}{2}\cos^{-1}(\sin^2 x)+C$ (C) $\tan^{-1}(\sin^2 x)+C$ (D) $\cot^{-1}(\sin x)+c$

4. The vector $\hat{i}+x\hat{j}+3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i}+(4x-2).\hat{j}+2\hat{k}$. Then values of x are

(A) $-\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) 2

5. Evaluate the following

(i) $\int \frac{(x+1)\,e^x}{\cos^2(xe^x)}\,dx$ (ii) $\int \frac{1}{\sqrt{2x^2+3x-2}}\,dx$

6. Evaluate the following

(i) $\int e^{2x}\left(\frac{1+\sin 2x}{1+\cos 2x}\right)dx$ (ii) $\int \frac{x\sin^{-1}x}{(1-x^2)^{3/2}}\,dx$

7. Evaluate the following

(i) $\int \frac{e^x}{x+2}[(1+(x+2)\ell n(x+2)]\,dx$ (ii) $\int \frac{x^5}{x^2+1}\,dx$

8. Evaluate the following

(i) $\int \frac{dx}{\sqrt{x}(x+9)}$ (ii) $\int e^x(1-\cot x+\cot^2 x)\,dx$

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 64

Total Marks : 34

Max. Time : 39 min.

Topics : **Indefinite Integration, Sequence & Series**

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1, 2	(3 marks, 3 min.)	[6, 6]
Subjective Questions (no negative marking) Q.3,4,5,6,7	(4 marks, 5 min.)	[20, 25]
Match the Following (no negative marking) Q.8	(8 marks, 8 min.)	[8, 8]

1. $\int \frac{(x+1)}{x(1+xe^x)^2}\,dx$ is equal to

(A) $\ell n \left|\frac{xe^x}{1+xe^x}\right| + C$ (B) $\ell n \left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x} + C$ (C) $\frac{1}{1+xe^x} + C$ (D) None of these

2. $\int \frac{dx}{\tan x + \cot x + \sec x + \operatorname{cosec} x}$ is equal to

(A) $\frac{1}{2}$(sinx + cosx + x) + c (B) $\frac{1}{2}$ (sinx – cosx – x) + c

(C) $\frac{1}{2}$ (cosx – x + sinx) + c (D) None of these

3. If a and b are the arithmetic means of $a_1, a_2, \ldots\ldots, a_n$ and $b_1, b_2, \ldots, b_n$ respectively, and $a_i + b_i = 1$ (i = 1, 2, ..., n), show that $\sum_{i=1}^{n} a_i b_i = nab - \sum_{i=1}^{n} (a_i - a)^2$.

4. Evaluate : (i) $\int \frac{dx}{\sin^6 x}$ (ii) $\int \frac{\cos x + \sin x}{\sqrt{\sin 2x}}\,dx$

5. Evaluate : $\int \frac{(x\cos x + 1)}{\sqrt{2x^3 e^{\sin x} + x^2}}\,dx$

6. Evaluate : $\int \frac{x}{x^4 + x^2 + 1}\,dx$

7. Evaluate : $\int \frac{1-x^7}{x\left(1+x^7\right)}\,dx$

8. **Column - I** | **Column - II**

Column - I	Column - II
(A) $\int \frac{x^4-1}{x^2\sqrt{x^4+x^2+1}}\,dx$	(p) $\ell n\left(\frac{(x^2+1)+\sqrt{x^4+1}}{x}\right)+C$
(B) $\int \frac{x^2-1}{x\sqrt{1+x^4}}\,dx$	(q) $-\frac{1}{\sqrt{2}}\ln\left(\frac{\sqrt{x^4+1}-\sqrt{2}x}{(x^2-1)}\right)+C$
(C) $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx$	(r) $-\tan^{-1}\left(\sqrt{\sqrt{1+\frac{1}{x^4}}-1}\right)+C$
(D) $\int \frac{1}{(1+x^4)\sqrt{\sqrt{1+x^4}-x^2}}\,dx$	(s) $\frac{\sqrt{x^4+x^2+1}}{x}+C$

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 65

Total Marks : 34

Max. Time : 39 min.

Topic : Indefinite Integration

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1, 2	(3 marks, 3 min.)	[6, 6]
Subjective Questions (no negative marking) Q.3,4,5,6,7	(4 marks, 5 min.)	[20, 25]
Match the Following (no negative marking) Q.8	(8 marks, 8 min.)	[8, 8]

1. $\int \frac{dx}{(x^2+4x+5)^2}$ is equal to

(A) $\frac{1}{2}\left[\tan^{-1}(x+1)+\frac{x+2}{x^2+4x+5}\right]$ + c

(B) $\frac{1}{2}\left[\tan^{-1}(x+2)-\frac{x+2}{x^2+4x+5}\right]$ + c

(C) $\frac{1}{2}\left[\tan^{-1}(x+1)-\frac{x+2}{x^2+4x+5}\right]$ + c

(D) $\frac{1}{2}\left[\tan^{-1}(x+2)+\frac{x+2}{x^2+4x+5}\right]$ + c

2. $\int \frac{x\,dx}{\sqrt{(1+x^2)+\sqrt{(1+x^2)^3}}}$ is equal to

(A) $\frac{1}{2}\ \ell n\left(1+\sqrt{1+x^2}\right)$ + c

(B) $2\sqrt{1+\sqrt{1+x^2}}$ + c

(C) $2\left(1+\sqrt{1+x^2}\right)$ + c

(D) $4\sqrt{1+\sqrt{1+x^2}}$ + c

3. Integrate : $\int \frac{(x+\sqrt{1+x^2})^{2009}}{\sqrt{1+x^2}}\,dx$

4. Integrate : $x^{13/2} . (1 + x^{5/2})^{1/2}$ w. r. t. x

5. Evaluate : $\int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}}\,dx$

6. Evaluate : $\int \frac{(\sin^{3/2}\theta+\cos^{3/2}\theta)\ d\theta}{\sqrt{\sin^3\theta\ \ \cos^3\theta\sin(\theta+\alpha)}}$

7. Evaluate : $\int \frac{(x^2-4)}{(x^2+1)(x^2+2)(x^2+3)}\,dx.$

8.

Column - I		Column - II	
(A)	$\int\sqrt{1+\sec x}\ dx$ is equal to	(p)	$\tan^{-1}(\tan^2 x)$ + c
(B)	$\int\frac{dx}{(\sin x-2\cos x)(2\sin x+\cos x)}$ is equal to	(q)	$\tan^{-1}\left(\sqrt{\cos x+\sec x+1}\right)$ + c
(C)	$\int\frac{\sin 2x}{\sin^4 x+\cos^4 x}\,dx$ is equal to	(r)	$\frac{1}{5}\log_e\left\|\frac{\tan x-2}{2\tan x+1}\right\|+c$
(D)	$\int\frac{\sin^3(x/2)}{\cos(x/2).\sqrt{\cos^3 x+\cos^2 x+\cos x}}\,dx$ is equal to	(s)	$2\tan^{-1}\sqrt{\frac{1-\cos x}{\cos x}}$ + c

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 66

Total Marks : 29

Max. Time : 31 min.

Topics : Permutation & Combination, Binomial Theorem, Indefinite Integration

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1 to 4	(3 marks, 3 min.)	[12, 12]
Multiple choice objective (no negative marking) Q.5	(5 marks, 4 min.)	[5, 4]
Subjective Questions (no negative marking) Q.6 to Q.8	(4 marks, 5 min.)	[12, 15]

1. There are 50 persons among whom 2 are brothers. The number of ways they can be arranged in a circle, if there is exactly one person between the two brothers is

(A) 47! (B) 48! (C) 2.48! (D) 2.47!

2. The streets of a city are arranged like the lines of a chess board. There are 5 streets running North to South & '3' streets running East to West. The number of ways in which a man can travel from NW to SE corner going the shortest possible distance is:

(A) 34 (B) 64 (C) $\frac{8!}{5!.3!}$ (D) 15

3. The cofficient of x^n in the polynomial $(x + {}^{2n+1}C_0)(x + {}^{2n+1}C_1)(x + {}^{2n+1}C_2)\,........\,(x + {}^{2n+1}C_n)$ is

(A) 2^{n+1} (B) $2^{2n+1} - 1$ (C) 2^{2n} (D) None of these

4. $\int \sqrt{1+2\cot x(\cot x + \text{cosec}\, x)}$ dx is equal to

(A) $2 \ln \left(\cos\frac{x}{2}\right) + c$ (B) $2\, \ell n \left(\sin\frac{x}{2}\right) + c$ (C) $\frac{1}{2}\, \ell n \left(\cos\frac{x}{2}\right) + c$ (D) $\frac{1}{2}\, \ell n \left(\sin\frac{x}{2}\right) + c$

5. If $\int \frac{(x^{-7/6} - x^{5/6})}{x^{1/3}(x^2+x+1)^{1/2} - x^{1/2}(x^2+x+1)^{1/3}}\, dx = -\lambda\left(\frac{z^3}{3} + \frac{z^p}{2} + \frac{z^q}{r} + \ell n\,|z-1|\right) + k$, where

$z = \left(x + \frac{1}{x} + 1\right)^{1/6}$, then

(A) $\lambda = 6$ (B) $\lambda = 1$ (C) $p + q = 3$ (D) $q = r = 1$

6. Out of 50 consecutive natural numbers in how many ways two numbers can be chosen such that their sum is divisible by 2.

7. Integrate : $\int \frac{\cos 2x - 3}{\sin^4 x\sqrt{4 - \tan^2 x}}\, dx$

8. Evaluate : $\int \frac{(1+\log_e x)^2}{1+\log_e x^{x+1} + \left(\log_e x^{\sqrt{x}}\right)^2} dx$

MATHEMATICS DPP

DAILY PRACTICE PROBLEMS

DPP No. 67

Total Marks : 30

Max. Time : 30 min.

Topics : **Permutation & Combination, Indefinite Integration, Definite Integration**

Type of Questions			M.M., Min.
Single choice Objective (no negative marking) Q.1, 2	(3 marks, 3 min.)	[6,	6]
Subjective Questions (no negative marking) Q.3 to Q.8	(4 marks, 5 min.)	[24,	24]

1. How many maximum points of intersection can we get by arranging 8 straight lines and 4 circles in a plane ?

(A) 100 (B) 104 (C) 64 (D) 92

2. $\int \frac{x^2+2}{x^4+4}\,dx$ is equal to

(A) $\frac{1}{2}\tan^{-1}\left(\frac{x^2+2}{2x}\right) + c$ (B) $\frac{1}{2}\tan^{-1}\left(\frac{x^2-2}{2x}\right) + c$

(C) $\frac{1}{2}\tan^{-1}\left(\frac{2x}{x^2-2}\right) + c$ (D) $\frac{1}{2}\tan^{-1}(x^2+2) + c$

3. 20 points lie on a circle. Find the number of triangle that can be formed such that no two vertices are consecutive.

4. Evaluate :

(i) $\int_0^2 |x^2+2x-3|\,dx$ (ii) $\int_0^4 \{x\}\,dx$, where {.} denotes fractional part function

5. (i) $\int_1^4 \{x\}^{[x]}\,dx$ is equal to (where [.] and {.} represent greatest integer function and fractional part function respectively).

(ii) The value of $\int_0^2 [x+[x+[x]]]\,dx$ is (where [.] represent greatest integer function).

6. Evaluate :

(i) $\int_{-1}^{3} (|x|+|x-1|)\,dx$ (ii) $\int_0^{\pi} |\cos x|\,dx$

7. Evaluate :

(i) $\int_{-1}^{1} e^{|x|}\,dx$ (ii) $\int_0^1 |\sin 2\pi x|\,dx$ (iii) $\int_{-1}^{1} \frac{x\,dx}{\sqrt{5-4x}}$

8. Integrate :

(i) $\int \frac{x^4+1}{x(x^2+1)^2}\,dx$ (ii) $\int \frac{(3x^2-1)\cot^{-1}x}{2x\sqrt{x}}\,dx$

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 68

Total Marks : 32

Max. Time : 35 min.

Topics : Definite Integration , Indefinite Integration

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.1,2,3,4	(3 marks, 3 min.)	[12, 12]
Subjective Questions (no negative marking) Q.5,6,7	(4 marks, 5 min.)	[12, 15]
Match the Following (no negative marking) Q.8	(8 marks, 8 min.)	[8, 8]

1. The value of the integral $\int_{-\pi}^{\pi} (\cos px - \sin qx)^2\, dx$ where p, q are integers, is equal to :

(A) $-\pi$ (B) 0 (C) π (D) 2π

2. The value of the integral $\int_{\pi/3}^{\pi/2} x \sin(\pi[x] - x)\, dx$ is (where [x] denotes greater integer function)

(A) $\frac{1}{2}+\frac{\pi}{6}$ (B) $-\frac{1}{2}-\frac{\pi}{6}$ (C) $1-\frac{\sqrt{3}}{2}+\frac{\pi}{6}$ (D) $\frac{\sqrt{3}}{2}-1-\frac{\pi}{6}$

3. If $\int_{0}^{a} \frac{dx}{\sqrt{x+a}+\sqrt{x}} = \int_{0}^{\pi/8} \frac{2\tan\theta}{\sin 2\theta} d\theta$, then value of 'a' is equal to (a > 0)

(A) $\frac{3}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{3\pi}{4}$ (D) $\frac{9}{16}$

4. If the value of the integral $\int_{1}^{2} e^{x^2} dx$ is α then $\int_{e}^{e^4} \sqrt{\ell n x}\, dx$ is equal to :

(A) $e^4 - e - \alpha$ (B) $2e^4 - e - \alpha$ (C) $2(e^4 - e) - \alpha$ (D) none of these

5. Evaluate : $\int \frac{\sqrt{1+x^2}}{1-x^2} dx$

6. Evaluate :

(i) $\int_{0}^{\pi/2} \frac{\cos x\, dx}{(1+\sin x)(2+\sin x)}$ (ii) $\int_{1}^{2} \frac{dx}{x(x^4+1)}$ (iii) $\int_{0}^{5} (|x-3| + |1-x|)\, dx$

7. Evaluate :

(i) $\int_0^{3\pi} \sin^{-1}(\sin x)dx$

(ii) $\int_{-2}^{2} \min(x-[x], -x-[-x])dx$ (where ([x] represents greatest integer less than or equal to x)

(iii) $\int_0^{\infty} \frac{x\log x}{(1+x^2)^2}dx$

8. **Column – I** **Column – II**

(A) f(x) = min {x + 1, 2 sgn (|x|) }, $\forall$ x $\in$ R, then $\int_{-5}^{4} f(x)dx$ = (p) 3

(B) If f(x) is a continuous function for all real values of x and satisfies $\int_n^{n+1} f(|x|)dx = \frac{n^2}{2}$, $\forall$ n $\in$ I, then $\int_{-3}^{5} f(|x|)dx =$ (q) 0

(C) If $\int_n^{n+1} [x+[x+[x]]]dx = kn$, n $\in$ I (where [.] denotes the greatest integer function), then k is/are (r) 22

(D) If f(x) = $\int_1^x \frac{e^t}{t}dt$, x $\in$ R^+, then the number of solutions of $f'(x) = 1$ is (s) 1

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 69

Total Marks :

Max. Time : min.

Topic : Permutation and Combination

REVISION DPP ON PERMUTATION AND COMBINATION

1. (i) Find the number of four letter word that can be formed from the letters of the word HISTORY. (each letter to be used at most once)
(ii) How many of them contain only consonants?
(iii) How many of them begin & end in a consonant?
(iv) How many of them begin with a vowel?
(v) How many contain the letters Y?
(vi) How many begin with T & end in a vowel?
(vii) How many begin with T & also contain S?
(viii) How many contain both vowels?

2. How many natural numbers are there from 1 to 1000 which have none of their digits repeated.

3. If all letters of the word " VARUN" are written in all possible ways and then are arranged as in a dictionary, then the rank of the word VARUN is :
(A) 98 (B) 99 (C) 100 (D) 101

4. In how many ways can 5 letters be mailed if there are 3 mailboxes available if each letter can be mailed in any mailbox. (Repetition allowed)

5. In how many ways four persons can be accommodated in 3 different chairs if each person can occupy only one chair. Also find number of ways in which three persons can accomodate in 4 chairs.

6. The number of ways in which 7 letters can be put in 7 envelopes such that exactly four letters are in wrong envelopes is
(A) 300 (B) 315 (C) 325 (D) 1035

7. In a telephone system four different letter P, R, S, T and the four digits 3, 5, 7, 8 are used. Find the maximum number of "telephone numbers" the system can have if each consists of a letter followed by a four-digit number in which the digit may be repeated.

8. It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

9. An ice cream parlour has ice creams in eight different varieties. Number of ways of choosing 3 ice creams taking atleast two ice creams of the different variety, is :
(A) 56 (B) 112 (C) 100 (D) none
(Assume that ice creams of the same variety are identical & available in unlimited supply)

10. A women has 11 close friends. Find the number of ways in which she can inivite 5 of them to dinner, if two particular of them are not on speaking terms & will not attend together.

11. Distinct 3 digit numbers are formed using only the digits 1, 2, 3 and 4 with each digit used at most once in each number thus formed. The sum of all possible numbers so formed is
(A) 6660 (B) 3330 (C) 2220 (D) none

12. There are 2 identical white balls, 3 identical red balls and 4 green balls of different shades. The number of ways in which they can be arranged in a row so that atleast one ball is separated from the balls of the same colour, is :
(A) $6\,(7\,! - 4!)$ (B) $7\,(6\,! - 4\,!)$ (C) $8\,! - 5\,!$ (D) none

13. The number of noncongruent rectangle that can be formed on a chessboard, is
(A) 30 (B) 32 (C) 33 (D) 36

14. There are 6 boxes numbered 1, 2,.................6. Each box is to be filled up either with a red or a green ball in such a way that at least 1 box contains a green ball and the boxes containing green balls are consecutive. The total number of ways in which this can be done, is
(A) 21 (B) 33 (C) 60 (D) 6

15. Find the number of different permutations of the letters of the word "BOMBAY" taken four at a time. How would the result be affected if the name is changed to "MUMBAI". Also find the number of combinations of the letters taken 3 at a time in both the cases.

16. Find the number of 10 digit numbers using the digits 0, 1, 2,................9 without repetition. How many of these are divisible by 4.

17. Six married couple are sitting in a room. Find the number of ways in which 4 people can be selected so that
(A) they do not form a couple (B) they form exactly one couple
(C) they form at least one couple (D) they form atmost one couple

18. The number of ways in which 14 men be partitioned into 6 committies where two of the committies contain 3 men & the other contain 2 men each is :

(A) $\frac{14!}{(3!)^2(2!)^4}$ (B) $\frac{14!}{(3!)^2(2!)^5}$ (C) $\frac{14!}{4!(3!)^2.(2!)^4}$ (D) $\frac{14!}{(2!)^5.(3!)^2.4!}$

19. The number of ways in which 8 non-identical apples can be distributed among 3 boys such that every boy should get atleast 1 apple & atmost 4 apples is K. $^7P_3$ where K has the value equal to :
(A) 88 (B) 66 (C) 44 (D) 22

20. Delegates of the five of the member countries of SAARC decide to hold a round table conference. There are 5 Indians, 4 Bangladeshis, 4 Pakistanis, 3 Sri Lankans and 3 Nepales. In how many ways can they be seated ? In how many ways can they be seated, if those of the same nationality sit together ?

21. How many necklace of 11 beads each can be made from 23 beads of various colours ?

(A) $\frac{1}{22}\left(\frac{23!}{12!}\right)$ (B) $\frac{23!}{12!}$ (C) $\left(\frac{23!}{2.12!}\right)$ (D) none of these

22. Given 11 points, of which 5 lie on one circle, other than these 5, no 4 lie on one circle. Then the maximum number of circles that can be drawn so that each contains atleast three of the given points is :
(A) 216 (B) 156 (C) 172 (D) none

23. How many ways are there to seat n married couples ($n \geq 3$) around a table such that men and women alternate and each women is not adjacent to her husband.

24. The number of positive integral solutions of the equation $x + y + z + w = 19$ is equal to
(A) the number of ways in which 15 identical things can be distributed among 4 persons.
(B) the number of ways in which 19 identical things can be distributed among 4 persons.
(C) coefficients of x^{19} in $(x^0 + x^1 + x^2 + ... + x^{19})^4$
(D) coefficients of x^{19} in $(x + x^2 + x^3 + ... + x^{19})^4$

25. A man wants to distribute 101 coins of a rupee each, among his 3 sons with the condition that no one receives more money than the combined total of other two. The number of ways of doing this is :

(A) $^{103}C_2 - 3\,^{52}C_2$ (B) $\frac{^{103}C_2}{3}$ (C) 1275 (D) $\frac{^{103}C_2}{6}$

26. The number of combination of 16 things, 8 of which are alike and the rest different, taken 8 at a time is___

27. Number of divisors of $N = 2^7 . 3^3 . 5^4$ divisible by 6 but not by 15 is
(A) 24 (B) 21 (C) 30 (D) 60

28. Number of divisors of 240 in the from $4n + 2$ ($n \geq 0$) is equal to
(A) 4 (B) 8 (C) 10 (D) 3

29. If $N = 2^{p-1}.(2^p - 1)$, where 2^p-1 is a prime, then the sum of the divisors of N expressed in terms of N is equal to ______

30. Exponent of 12 in 100! is
(A) 24 (B) 48 (C) 54 (D) 36

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 70

Total Marks : 34

Max. Time : 36 min.

Topic : Definite Integration

Type of Questions		M.M., Min.
Single choice Objective ('–1' negative marking) Q.2,3,4	(3 marks, 3 min.)	[9, 9]
Multiple choice objective ('–1' negative marking) Q.5	(5 marks, 4 min.)	[5, 4]
Subjective Questions ('–1' negative marking) Q.1,6,7	(4 marks, 5 min.)	[12, 15]
Match the Following (no negative marking) Q.8	(8 marks, 8 min.)	[8, 8]

1. Evaluate : $\int_{-1}^{3}\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx$

2. Given $\int_{0}^{\pi/2}\frac{dx}{1+\sin x+\cos x}=\ell n\,2$, then the value of $\int_{0}^{\pi/2}\frac{\sin x}{1+\sin x+\cos x}$ dx is equal to:

(A) $\frac{1}{2}\ell n\,2$ (B) $\frac{\pi}{2}-\ell n\,2$ (C) $\frac{\pi}{4}-\frac{1}{2}\ell n\,2$ (D) $\frac{\pi}{2}+\ell n\,2$

3. If $I=\int_{0}^{\pi/2}\ell n(\sin x)\,dx$ then $\int_{-\pi/4}^{\pi/4}\ell n(\sin x+\cos x)\,dx=$

(A) $\frac{I}{2}$ (B) $\frac{I}{4}$ (C) $\frac{I}{\sqrt{2}}$ (D) I

4. Let $u=\int_{0}^{1}\frac{\ell n(x+1)}{x^2+1}dx$ and $v=\int_{0}^{\pi/2}\ell n(\sin 2x)\,dx$, then

(A) u = 4v (B) 4u + v = 0 (C) u + 4v = 0 (D) 2u + v = 0

5. If $A_n=\int_{0}^{\pi/2}\frac{\sin(2n-1)x}{\sin x}dx$; $B_n=\int_{0}^{\pi/2}\left(\frac{\sin nx}{\sin x}\right)^2dx$; for $n\in N$, then :

(A) $A_{n+1}=A_n$ (B) $B_{n+1}=B_n$ (C) $A_{n+1}-A_n=B_{n+1}$ (D) $B_{n+1}-B_n=A_{n+1}$

6. Prove that $\int_{0}^{1}\tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx=2\int_{0}^{1}\tan^{-1}x\,dx$. Hence or otherwise,

evaluate the integral $\int_{0}^{1}\tan^{-1}\left(1-x+x^2\right)dx$

7. Given that, $F(x) = \frac{1}{x^2}\int_4^x (4t^2 - 2F'(t))\,dt$, find $F'(4)$.

8. **Column – 1** **Column – 2**

Column – 1		Column – 2	
(A)	The value of $\int_{-1}^{1} \max.\left(\lvert x\rvert, x^2, x^4\right)dx$ is equal to	(p)	-1
(B)	If for a continuous f(x), $\int_{-a}^{a} f(x)\,dx = \lvert k\rvert \int_0^a (f(x)+f(-x))\,dx$, then k is/are	(q)	$\frac{17}{3}$
(C)	If $\int_0^{\infty} e^{-2x}(\sin 2x + \cos 2x)\,dx = -\frac{k}{2}$, then value of k is/are	(r)	$\frac{5}{2}$
(D)	$\lim_{x\to 0}\frac{\int_0^x (\sin^2 4t + t^2)\,dt}{x^3}$ is equal to	(s)	1

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 71

Total Marks : 31

Max. Time : 33 min.

Topic : Definite Integration

Type of Questions			M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3,4,5	(3 marks, 3 min.)	[15,	15]
Subjective Questions (no negative marking) Q.6,7	(4 marks, 5 min.)	[8,	10]
Match the Following (no negative marking) Q.8	(8 marks, 8 min.)	[8,	8]

1. $\int_{2}^{4}\left(\log_x 2 - \frac{(\log_x 2)^2}{\ell n 2}\right) dx$ equals to :

(A) 0 (B) 1 (C) 2 (D) 4

2. The value of $\int_{1}^{2}\left([x^2] - [x]^2\right) dx$, where [.] denotes the greatest integer function, is equal to :

(A) $4 + \sqrt{2} - \sqrt{3}$ (B) $4 - \sqrt{2} + \sqrt{3}$ (C) $4 - \sqrt{3} - \sqrt{2}$ (D) none of these

3. The area of the closed figure bounded by x = – 1, x = 2 and $y = \begin{cases} -x^2 + 2, & x \le 1 \\ 2x - 1, & x > 1 \end{cases}$ and the abscissa axis is

(A) 16/3 sq. units (B) 10/3 sq. units (C) 13/3 sq. units (D) 7/3 sq. units

4. The area of the region for which $0 < y < 3 - 2x - x^2$ and $x > 0$ is

(A) $\int_{1}^{3}(3 - 2x - x^2)\, dx$ (B) $\int_{0}^{3}(3 - 2x - x^2)\, dx$

(C) $\int_{0}^{1}(3 - 2x - x^2)\, dx$ (D) $\int_{-1}^{3}(3 - 2x - x^2)\, dx$

5. The area bounded by the curves $\sqrt{x} + \sqrt{y} = 1$ and x + y = 1 is

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) none of these

6. Evaluate : $\displaystyle\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{\cos^{-1}\left(\frac{2x}{x^2+1}\right)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)}{e^x+1}dx$

7. For $\theta \in (0, \pi) \cup (\pi, 2\pi)$ show that $\displaystyle\int_0^{\infty} \frac{dx}{x^2+2x\cos\theta+1} = 2\int_0^1 \frac{dx}{x^2+2x\cos\theta+1}$

8. **Column – I** **Column – II**

Column – I		Column – II	
(A)	Let $f : R \to R$ be a differentiable function and $f(1) = 1$, $f'(1) = 3$. Then the value of $\displaystyle\lim_{x\to 1} \frac{\int_1^{x^2}(f(t)-t)\,dt}{(x-1)^2}$ is	(P)	2
(B)	If $\displaystyle\int_0^3 (3ax^2+2bx+c)\,dx = \int_1^3 (3ax^2+2bx+c)\,dx$ where a, b, c are constants, then a + b + c =	(Q)	0
(C)	Number of rational points $P \equiv (\alpha, \beta)$ lying on $\left(x-\sqrt{2}\right)^2 + \left(y-\sqrt{3}\right)^2 = 4$ is (rational point means x and y co-ordinate both are rational)	(R)	1
(D)	Number of integral values of 'a' for which the function $f(x) = x^3 + (a + 2)x^2 + 3ax + 5$ is monotonic in R is	(S)	4

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 72

Total Marks : 35

Max. Time : 36 min.

Topic : Definite Integration

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3,4,5,6	(3 marks, 3 min.)	[18, 18]
Multiple choice objective (no negative marking) Q.7	(5 marks, 4 min.)	[5, 4]
Fill in the Blanks (no negative marking) Q.8	(4 marks, 4 min.)	[4, 4]
Subjective Questions (no negative marking) Q.9,10	(4 marks, 5 min.)	[8, 10]

1. The degree of the differential equation, $e^{(d^3y/dx^3)^2} + x\dfrac{d^2y}{dx^2} + y = 0$ is:

(A) 1 (B) 2 (C) 0 (D) not defined

2. If $\int_0^1 \tan^{-1} x\,dx = \alpha$, then $\int_0^{\pi/4} \tan^{-1}\left(\dfrac{2\cos^2\theta}{2-\sin 2\theta}\right)\sec^2\theta\,d\theta$ is equal to

(A) α (B) $\dfrac{\alpha}{2}$ (C) 3α (D) 2α

3. The line y = mx bisects the area enclosed by the curve $y = 1 + 4x - x^2$ and the lines x = 0, $x = \dfrac{3}{2}$ and y = 0. Then the value of m is

(A) $\dfrac{13}{6}$ (B) $\dfrac{6}{13}$ (C) $\dfrac{3}{2}$ (D) 4

4. The area of the closed figure bounded by y = x, y = – x and the tangent to the curve $y = \sqrt{x^2 - 5}$ at the point (3, 2) is:

(A) 5 (B) $\dfrac{15}{2}$ (C) 10 (D) $\dfrac{35}{2}$

5. The area $\{(x, y); x^2 \le y \le \sqrt{x}\}$ is equal to

(A) $\dfrac{1}{3}$ (B) $\dfrac{2}{3}$ (C) $\dfrac{1}{6}$ (D) none of these

6. The solution of the differential equation, $x(x^2 + 3y^2)\,dx + y(y^2 + 3x^2)\,dy = 0$ is

(A) $x^4 + y^4 + x^2y^2 = c$ (B) $x^4 + y^4 + 3x^2y^2 = c$

(C) $x^4 + y^4 + 6x^2y^2 = c$ (D) $x^4 + y^4 + 9x^2y^2 = c$

7. Identify the statement(s) which is/are True.

(A) $f(x, y) = e^{y/x} + \tan\frac{y}{x}$ is homogeneous of degree zero

(B) $x.\,\ell n\frac{y}{x}\,dx + \frac{y^2}{x}\sin^{-1}\frac{y}{x}\,dy = 0$ is homogeneous of degree one

(C) $f(x, y) = x^2 + \sin x.\cos y$ is not homogeneous

(D) $(x^2 + y^2)\,dx - (xy^2 - y^3)\,dy = 0$ is a homogeneous differential equation.

8. The order and degree of the differential equation $\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$ are ______ and ______ respectively.

9. Evaluate : $\int_{-(\pi/4)^{1/3}}^{(\pi/4)^{1/3}} \frac{x^2}{(1+\sin^2 x^3)(1+e^{x^7})}dx$

10. Find the order and degree of the following differential equations

(i) $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

(ii) $(x^2 + y^2)\,dx - 2xy\,dy = 0$

(ii) $\sqrt{1+\left(\frac{dy}{dx}\right)^2} = x^3$

(iv) $\sqrt{\frac{d^3y}{dx^3}} = \sqrt[4]{\left(\frac{dy}{dx}+5\right)}$

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 73

Total Marks : 29

Max. Time : 28 min.

Topic : **Definite Integration**

Type of Questions		**M.M., Min.**	
Single choice Objective ('–1' negative marking) Q.1,2,3,4,5,6,7,8	**(3 marks, 3 min.)**	**[24,**	**24]**
Multiple choice objective ('–1' negative marking) Q.9	**(5 marks, 4 min.)**	**[5,**	**4]**

1. A particular solution of $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$ is -

(A) y siny = x log x (B) y^2 siny = x logx (C) y sin y = x^2 log x (D) none of these

2. The solution of differential equation $(1 + y^2) + \left(x - 2e^{\tan^{-1}y}\right)\frac{dy}{dx} = 0$ is

(A) $(x - 2) = k\,e^{2\tan^{-1}y}$ (B) $x\,e^{\tan^{-1}y} = e^{2\tan^{-1}y} + k$

(C) $x\,e^{\tan^{-1}y} = \tan^{-1}y + k$ (D) $x\,e^{2\tan^{-1}y} = e^{2\tan^{-1}y} + k$

3. The solution of the differential equation $x^2\frac{dy}{dx}\cdot\cos\left(\frac{1}{x}\right) - y\sin\left(\frac{1}{x}\right) = -1$, where $y \to -1$ as $x \to \infty$ is.

(A) $y = \sin\frac{1}{x} + \cos\frac{1}{x}$ (B) $y = \frac{x+1}{x\sin(1/x)}$

(C) $y = \sin\frac{1}{x} - \cos\frac{1}{x}$ (D) $y = \frac{x+1}{x\cos(1/x)}$

4. Solution of the differential equation $(x^2 + y^3)(2x^2dx + 3ydy) = 12x\,dx + 18y^2\,dy$ is

(A) $\frac{2}{3}x^3 + \frac{3}{2}y^2 = 6\ln(x^2 + y^3) + c$ (B) $x^2 + y^3 = 9\ln(x^2 + y^3) + c$

(C) $\frac{2}{3}x^3 + \frac{3}{2}y^2 = 6\ln(x^3 + y^2) + c$ (D) $x^3 + y^2 = 6\ln(x^2 + y^3) + c$

5. Solution of differential equation $(2x\,\ell n\,y)\,dx + \left(\frac{x^2}{y} + 3y^2\right)dy = 0$ is

(A) $x^2\,\ell n\,y + y^3 = c$ (B) $x\,\ell n\,y + y^2 = c$ (C) $x^2\,\ell n\,y + y^2 = c$ (D) none of these

6. If solution of the differential equation $\frac{dy}{dx} = \frac{1}{x\cos y + \sin 2y}$ is $x = ce^{\sin y} - k(1 + \sin y)$, then k =

(A) 1 (B) 2 (C) 3 (D) 5

7. A curve passes through the point (2a, a) and is such that sum of subtangent and abscissa is equal to a. Its equation is

(A) $(x - a)y^2 = a^3$ (B) $(x - a)^2y = a^3$ (C) $(x - a)y = a^2$ (D) none of these

8. If [·] represents the gretest integer function, then $\int_4^{10}\frac{[x^2]}{[x^2 - 28x + 196] + [x^2]}dx$ is equal to -

(A) 0 (B) 1 (C) 3 (D) None of these

9. Which of the following equation(s) is/are linear differential equation.

(A) $\frac{dy}{dx} + \frac{y}{x} = \ln x$ (B) $y\left(\frac{dy}{dx}\right) + 4x = 0$ (C) dx + dy = 0 (D) $\frac{d^2y}{dx^2} = \cos x$

MATHEMATICS

DPP
DAILY PRACTICE PROBLEMS

DPP No. 74

Total Marks : 33

Max. Time : 36 min.

Topics : **Permutation & Combination, Probability**

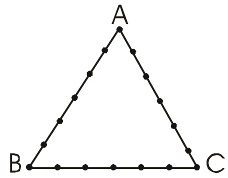
Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.1,2,3,4,5,6,7	**(3 marks, 3 min.)**	**[21, 21]**
Subjective Questions (no negative marking) Q.8,9,10	**(4 marks, 5 min.)**	**[12, 15]**

1. Number of natural number between 100 and 1000 such that at least one of their digits is 6, is
(A) 243 (B) 252 (C) 258 (D) 648

2. 6 chocolates out of 8 different brands available in the market are choosen, what is the probability that all the chocolates are of different brands.
(A) $\frac{^8C_6}{^{13}C_6}$ (B) $\frac{^8C_6}{^{13}C_8}$ (C) $\frac{^8C_6}{8^6}$ (D) None of these

3. If a, b, c are odd positive integer then number of positive integral solution of a + b + c = 13.
(A) 15 (B) 21 (C) 56 (D) 28

4. 18 points are indicated on the perimeter of a triangle ABC (see figure).
If three points are choosen probability that it will form a triangle :-



(A) $\frac{331}{816}$ (B) $\frac{1}{2}$
(C) $\frac{355}{408}$ (D) $\frac{711}{816}$

5. A natural number is selected at random from the set X = {x : 1 ≤ x ≤ 100}. Probability that the number satisfies the inequation $x^2 - 13x \le 30$ is
(A) $\frac{9}{50}$ (B) $\frac{3}{20}$ (C) $\frac{2}{11}$ (D) none of these

6. A five digits number of the form x y z y x is choosen, probability that x < y is :
(A) $\frac{35}{90}$ (B) $\frac{6}{15}$ (C) $\frac{19}{45}$ (D) $\frac{13}{30}$

7. The probability of choosing randomly a number which is from 1 to 90 divisible by 6 or 8 is
(A) $\frac{1}{6}$ (B) $\frac{11}{90}$ (C) $\frac{1}{30}$ (D) $\frac{23}{90}$

8. A seven digit number is choosen. What the probability that even number occupy even places ?

9. (i) A coin is tossed 20 times find the probability that number of tail obtained is more than number of heads.
(ii) From 52 playing card person A picks one card and then person B picks another cards randomly. Find the probability that these card are of different colours.

10. 4 people are selected randomly out of six married couple. Find the probability that
(i) exactly one married couple is formed (ii) exactly two married couple are formed
(iii) they do not form a married couple.

DPP No. 75

Total Marks : 34

Max. Time : 34 min.

Topics : **Permutation & Combination, Probability**

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3,4,5,6,7	(3 marks, 3 min.)	[21, 21]
Multiple choice objective (no negative marking) Q.8	(5 marks, 4 min.)	[5, 4]
Fill in the Blanks (no negative marking) Q.9	(4 marks, 4 min.)	[4, 4]
Subjective Questions (no negative marking) Q.10	(4 marks, 5 min.)	[4, 5]

1. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is

(A) $\frac{1}{729}$ (B) $\frac{8}{9}$ (C) $\frac{8}{729}$ (D) $\frac{8}{243}$

2. If P(A) = 0.59, P(B) = 0.30, $P(A \cap B) = 0.21$, then $P(A' \cap B')$ is equal to

(A) 0.79 (B) 0.11 (C) 0.32 (D) 0.38

3. Two non-negative integers are chosen at random, then the probability that the sum of their squares is divisible by 5 is

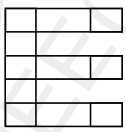
(A) $\frac{7}{25}$ (B) $\frac{8}{25}$ (C) $\frac{9}{25}$ (D) $\frac{5}{25}$

4. Suppose A and B shoot independently until each hits his target. They have probabilities $\frac{3}{5}$ and $\frac{5}{7}$ of hitting the targets at each shot. The probability that B will require more shots than A is

(A) $\frac{6}{31}$ (B) $\frac{7}{31}$ (C) $\frac{8}{31}$ (D) $\frac{1}{2}$

5. Number of ways in which A A B B B C can be placed in the squares of the figure as shown so that no row remains empty, is :

(A) 9720
(B) 4860
(C) 2160
(D) 1620



6. A person throws dice, one the common cube and the other regular terahedron, the number on the lowest face being taken in the case of a tetrahedron. The chance that the sum of numbers thrown is not less than 5 is

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) $\frac{4}{5}$ (D) $\frac{5}{6}$

7. If two events A and B are such that $P(A^c) = 0.3$. P(B) = 0.4 and $P(A \cap B^c) = 0.5$, then $P(B/A \cup B^c) =$

(A) 0.9 (B) 0.5 (C) 0.6 (D) 0.25

8. The letters of the word PROBABILITY are written down at random in a row. Let E_1 denotes the event that two I's are together and E_2 denotes the event that two B's are together, then

(A) $P(E_1) = P(E_2) = \frac{3}{11}$ (B) $P(E_1 \cap E_2) = \frac{2}{55}$ (C) $P(E_1 \cup E_2) = \frac{18}{55}$ (D) $P(E_1/E_2) = \frac{1}{5}$

9. (i) The number of arrangements that can be made taking 4 letters, at a time, out of the letters of the word “PASSPORT” is _____
(ii) Probability that both S appear in such 4 letter words is _______
(iii) Probability that all letter are distinct in such 4 letter words is _______

10. Find the last digit of $(73)^{75^{64^{76}}}$.

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 76

Total Marks : 32

Max. Time : 35 min.

Topics : **Probability, Permutation & Combination, Vector, Definite Integration**

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1 to 4	(3 marks, 3 min.)	[12, 12]
Subjective Questions (no negative marking) Q.5 to 7	(4 marks, 5 min.)	[12, 15]
Match the Following (no negative marking) Q.8	(8 marks, 8 min.)	[8, 8]

1. Two fair dice are rolled together, one of the dice showing 4, then the probability that the other is showing 6 is

(A) $\frac{2}{11}$ (B) $\frac{1}{18}$ (C) $\frac{1}{6}$ (D) $\frac{1}{36}$

2. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ & $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$, then the vector $\vec{c}$ such that $\vec{a} \cdot \vec{c} = 2$ & $\vec{a} \times \vec{c} = \vec{b}$ is

(A) $\frac{1}{3}\left(\hat{i} - 2\hat{j} + \hat{k}\right)$ (B) $\frac{1}{3}\left(-\hat{i} + 2\hat{j} + 5\hat{k}\right)$ (C) $\frac{1}{3}\left(\hat{i} + 2\hat{j} - 5\hat{k}\right)$ (D) $\frac{1}{3}\left(-\hat{i} + 2\hat{j} - 5\hat{k}\right)$

3. Number of permutations of alphabets a,b,c,d,e,f,g,h,i taken all at a time, such that the alphabet
'a' appearing some where to the left of 'b',
'c' appearing to the left of 'd', and
'e' somewhere to the left of 'f', is (Example - h a e g b c i d f would be one such permutation)

(A) 5 ! 4 ! (B) 8 ! (C) 8 ! 4 ! (D) 9 . 7 !

4. Number of ways in which 5A's and 6B's can be arranged in a row which reads the same backwards and forwards,is

(A) 12 (B) 10 (C) 8 (D) 6

5. Let E = {1, 2, 3, 4} and F = {1, 2} A function is defined from E to F

(i) Find the probability that it is onto

(ii) Find the probability that it is one one

6. There are two groups of subjects one of which consists of 5 science subjects and 3 engineering subjects and the other consists of 3 science and 5 engineering subjects. An unbiased die is cast. If number 3 or number 5 turns up, a subject is selected at random from the first group, otherwise the subject is selected at random from the second group. Find the probability that an engineering subject is selected ultimately.

7. Three shots are fired independently at a target in succession. The probabilities of a hit in the first shot is $\frac{1}{2}$, in the second $\frac{2}{3}$ and in the third shot is $\frac{3}{4}$. In case of one hit, the probability of destroying the target is $\frac{2}{3}$ and in the case of two hits $\frac{7}{11}$ and in the case of three hits 1.0. Find the probability of destroying the target in three shots.

8. Match the column

	Column-I		Column-II
(A)	$\int_{-2}^{2} \lvert 1-x^2 \rvert dx$	(p)	2
(B)	$\frac{20}{\pi}\int_{0}^{\pi/2} \frac{dx}{1+\tan^3 x}$	(q)	3
(C)	$\frac{2}{\pi}\int_{0}^{3} \sqrt{\frac{x}{3-x}}\, dx$	(r)	4
(D)	$\frac{8}{\pi^2}\int_{0}^{1} \frac{\sin^{-1}(\sqrt{x})dx}{\sqrt{x(1-x)}}$	(s)	5

MATHEMATICS

DPP
DAILY PRACTICE PROBLEMS

DPP No. 77

Total Marks : 26

Max. Time : 28 min.

Topics : **Probability, Definite Integration**

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1 to 6	(3 marks, 3 min.)	[18, 18]
Subjective Questions (no negative marking) Q.7 to 8	(4 marks, 5 min.)	[8, 10]

1. A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained. Then the probability that 5 comes before 7,is

(A) $\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

2. The sum of the terms of an infinite G.P. is equal to the greatest value of the function, $f(x) = x^3 + 3x - 9$ in the interval $[-2, 3]$ and the difference between the first and the second term is $f'(0)$. Then the first term of the G.P. can be

(A) -9 (B) 27 (C) 9 (D) $\frac{2}{3}$

3. One hundred identical coins, each with probability p, of showing up heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads on 51 coins, then the value of p is

(A) $\frac{1}{2}$ (B) $\frac{49}{101}$ (C) $\frac{50}{101}$ (D) $\frac{51}{101}$

4. If all the letters of the word "SUCCESS" are written down at random in a row, then the probability that no two C's and no two S's occur together is

(A) $\frac{2}{35}$ (B) $\frac{8}{35}$ (C) $\frac{2}{7}$ (D) none of these

5. For $U_n = \int_0^1 x^n (2-x)^n \, dx$; $V_n = \int_0^1 x^n (1-x)^n \, dx$ $n \in N$, which of the following statement(s) is/are true?

(A) $U_n = 2^n V_n$ (B) $U_n = 2^{-n} V_n$ (C) $U_n = 2^{2n} V_n$ (D) $U_n = 2^{-2n} V_n$

6. $\int_0^1 x(1-x)^{99} dx$ is equal to

(A) $\frac{1}{10100}$ (B) $\frac{1}{5050}$ (C) $\frac{1}{5051}$ (D) none of these

7. The probability of a shooter hitting a target is $\frac{3}{4}$. How many minimum number of times must he fire so that probability of hitting the target at least once is more than 0.99 ?

8. 'A' writes a letter to his friend 'B' and gives it to his son to post it in a letter box, the reliability of his son being $\frac{3}{4}$. The probability that a letter posted will get delivered is $\frac{8}{9}$. At a later date, 'A' hears from 'B' that the letter has not reached him. Find the probability that the son did not post the letter at all.

DPP 61 TO 76 (ANSWER KEY)

DPP NO. - 61

1. (C) **2.** (A) **3.** (C) **4.** (D)

5. (B) **6.** (B)(D)

7. (i) $\frac{(\tan^{-1}x)^4}{4}+C$ (ii) $3x-\frac{7}{2}\tan^{-1}\left(\frac{x}{2}\right)+C$

8. $\left(\frac{x}{e}\right)^x-\left(\frac{e}{x}\right)^x+c$

DPP NO. - 62

1. (A) **2.** (D) **3.** (D) **4.** (C)

5. (D) **6.** (A) **7.** (A)(D) **8.** $\frac{1}{2^n}$

DPP NO. - 63

1. (B) **2.** (A) **3.** (A)(B) **4.** (A)(D)

5. (i) $\tan(x e^x)+c$

(ii) $\frac{1}{\sqrt{2}}\log\left|\left(x+\frac{3}{4}\right)+\sqrt{x^2+\frac{3}{2}x-1}\right|+C$

6. (i) $\frac{e^{2x}}{2}\tan x+c$ (ii) $\frac{\sin^{-1}x}{\sqrt{1-x^2}}-\frac{1}{2}\log\left|\frac{1+x}{1-x}\right|+c$

7. (i) $e^x \ell n(x+2)+c$ (ii) $\frac{x^4}{4}-\frac{x^2}{2}+\frac{1}{2}\ell n(x^2+1)+c$

8. (i) $\frac{2}{3}\tan^{-1}\sqrt{x}+c$ (ii) $-e^x\cot x+c$

DPP NO. - 64

1. (B) **2.** (B)

4. (i) $c-\cot x-\frac{2}{3}\cot^3 x-\frac{1}{5}\cot^5 x$

(ii) $\sin^{-1}(\sin x-\cos x)+c$

5. $\ell n\left\{\frac{\sqrt{2xe^{\sin x}+1}-1}{\sqrt{2xe^{\sin x}+1}+1}\right\}+c$

6. $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+c$

7. $\ell n x-(2/7)\,\ell n(1+x^7)+c$

8. (A) $\to$ s ; (B) $\to$ p ; (C) $\to$ q ; (D) $\to$ r

DPP NO. - 65

1. (D) **2.** (B) **3.** $\frac{(x+\sqrt{1+x^2})^{2009}}{2009}+c$

4. [$\frac{4}{5}\left[\frac{t^7}{7}-\frac{2t^5}{5}+\frac{t^3}{3}\right]+c$ where $t^2=1+x^{5/2}$]s

5. $\frac{2}{\sqrt{3}}\tan^{-1}\left\{\frac{x}{\sqrt{3(x+1)}}\right\}+C$

6. $\frac{2}{\cos\alpha}\sqrt{(\cos\alpha\tan\theta+\sin\alpha)}$

$-\frac{2}{\sin\alpha}\sqrt{(\cos\alpha+\cot\theta\sin\alpha)}+c$

7. $-\frac{5}{2}\tan^{-1}x+\frac{6}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{7}{2\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+c$

8. (A) $\to$ s ; (B) $\to$ r ; (C) $\to$ p ; (D) $\to$ q

DPP NO. - 66

1. C **2.** D **3.** C **4.** B

5. ACD **6.** $2\,{}^{25}C_2$

7. $\frac{1}{8}\left(\frac{(4\cot^2x-1)^{3/2}}{3}+9\sqrt{4\cot^2x-1}\right)+c$

8. $\ell n(1+x\,\ell n x)+c$

DPP NO. - 67

1. B **2.** B **3.** 800 **4.** (i) 4 (ii) 2

5. (i) $\frac{13}{12}$ (ii) 3 6. (i) 9 (ii) 2

7. (i) 2 e - 2 (ii) $\frac{1}{6}$ (iii) $\frac{2}{\pi}$

8. (i) $\ln x + \frac{1}{1+x^2} + c$

(ii) $\frac{(x^2+1)\cot^{-1}x}{\sqrt{x}} + 2\sqrt{x} + c$

DPP NO. - 68

1. (D) 2. (C) 3. (D) 4. (B)

5. $-\frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{1+x^2}-\sqrt{2}\,x}{\sqrt{1+x^2}+\sqrt{2}\,x}\right| - \log\left|x+\sqrt{1+x^2}\right| + c$

6. **(i)** $\ell n\frac{4}{3}$ **(ii)** $\frac{1}{4}\ln\frac{32}{17}$ **(iii)** 15

7. **(i)** $\frac{\pi^2}{4}$ **(ii)** 1 **(iii)** 0

8. (A) → (q), (B) → (r), (C) → (p), (D) → (q)

DPP NO. - 69

1. (i) 840 ; (ii) 120 ; (iii) 400 ; (iv) 240 ; (v) 480 ; (vi) 40 ; (vii) 60 ; (viii) 240 ;

2. 738 3. (C) 4. 243 ways 5. 24, 24

6. (B) 7. 1024 8. 2880 9. (B)

10. 378 11. (A) 12. (A) 13. (D)

14. (A) 15. 192; no change ; 14 ; 14

16. 9! × 9, (20) . 8 ! 17. 240, 240, 255, 480

18. (D) 19. (D) 20. 18! ; $(3!)^2 (4!)^3 (5!)$

21. (A) 22. (B) 23. n!(n–1)! – 2(n–1)!

24. (A)(D) 25. (A)(C) 26. 256 27. (B)

28. (A) 29. 2N 30. (B)

DPP NO. - 70

1. π 2. (C) 3. (A) 4. (B)

5. (A)(D) 6. ℓn 2 7. 32/9

8. (A) → (s), (B) → (p, s), (C) → (p), (D) → (q)

DPP NO. - 71

1. (A) 2. (C) 3. (A) 4. (C)

5. (A) 6. $\frac{\pi}{2\sqrt{3}}$

8. (A) → (S),(B) → (Q),(C) → (Q),(D) → (S)

DPP NO. - 72

1. (D) 2. (D) 3. (A) 4. (A)
5. (A) 6. (C) 7. (A)(B)(C)

8. 1, 2 9. $\frac{1}{3\sqrt{2}}\tan^{-1}\sqrt{2}$

10. (i) 2, 1 (ii) 1, 1 (iii) 1, 2 (iv) 3, 2

DPP NO. - 73

1. (C) 2. (B) 3. (C) 4. (A)
5. (A) 6. (B) 7. (C) 8. (C)
9. (A)(C) (D)

DPP NO. - 74

1. (B) 2. (A) 3. (B) 4. (D)

5. (B) 6. (B) 7. (D) 8. $\frac{9\times5^3}{9\times10^6}$

9. (i) $\frac{1}{2}\left(1-\frac{^{20}C_{10}}{2^{20}}\right)$ (ii) $\frac{26}{51}$

10. (i) $\frac{240}{^{12}C_4}$ (ii) $\frac{15}{^{12}C_4}$ (iii) $\frac{240}{^{12}C_4}$

DPP NO. - 75

1. (D) 2. (C) 3. (C) 4. (A)
5. (B) 6. (B) 7. (D) 8. (B)(C)(D)

9. (i) 606 (ii) $\frac{21}{101}$ (iii) $\frac{^6C_4 . 4!}{606}$ 10. 3

DPP NO. - 76

1. (A) 2. (B) 3. (D) 4. (B)

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अभ्यास ही सबसे बड़ा गुरू है।

CLASS : XII (MATHS)

DPP

DAILY PRACTICE PROBLEM

DPP-61 to 76

DPP NO. - 61

1. The equation of any plane through (–1, 3, 2) is
$a(x + 1) + b(y – 3) + c(z – 2) = 0$ ...(ii)
If this plane (ii) is perpendicular to P_1,
then $2a – b + c = 0$ ...(ii)
and If the plane (ii) is perpendicular to P_2
then $a + 2b – c = 0$ ...(iii)

From Eqs. (ii) and (iii), we get $\frac{a}{-1} = \frac{b}{3} = \frac{c}{5}$

Substituting these proportionate values of a, b, c in Eq. (ii), we get the required equation as
$–(x + 1) + 3(y – 3) + 5(z – 2) = 0$
or $x – 3y – 5z + 20 = 0$

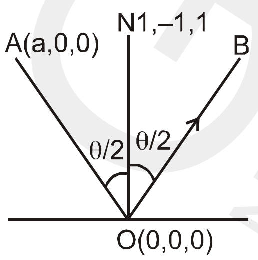
2. The given planes can be written as $–2x + y – z + 2 = 0$
and $–x – 2y + z + 3 = 0$
Here, $(–2)(–1) + (1)(–2) + (–1)(1) = –1 < 0$
Eaution of bisectors

$$\frac{(-2x+y-z+2)}{\sqrt{(4+1+1)}} = \pm \frac{(-x-2y+z+3)}{\sqrt{1+4+1}}$$

∴ Acute angle bisector is
$(–2x + y – z + 2) = (–x – 2y + z + 3)$
$\Rightarrow x – 3y + 2z + 1 = 0$

3. The image of plane P_1 in the plane mirror P_2, then
$2(2\cdot1 + (–1)\cdot2 + 1\cdot(–1))(x + 2y – z – 3)$
$= (1 + 4 + 1)(2x – y + z – 2)$
$\Rightarrow –(x + 2y – z – 3) = 3(2x – y + z – 2)$
$\Rightarrow 7x – y + 2z – 9 = 0.$

4. Let the source of light be situated at A(a 0, 0), where $a \neq 0$. Let OA be the incident ray and OB the reflected ray ON is the normal to th mirror at O



$\angle AON = \angle NOB = \frac{\theta}{2}$ (say)

DR's of OA are a, 0, 0 and so its DC's are 1, 0, 0

DC's of ON are $\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

$\therefore \cos\frac{\theta}{2} = \frac{1}{\sqrt{3}}$

Let ℓ, m, n be the DC's of the reflected ray OB.

then $\frac{n+0}{2\cos\frac{\theta}{2}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \ell = \frac{2}{3} - 1,\ m = -\frac{2}{3},\ n = \frac{2}{3}$

$\Rightarrow \ell = -\frac{1}{3},\ m = -\frac{2}{3},\ n = \frac{2}{3}$

Hence, DC's of the reflected ray are

$-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

5. Let $\tan^{-1}(\text{se } x + \cos x) = t$

$\Rightarrow \frac{1}{1+(\sec x+\cos x)^2}(\sec x \tan x - \sin x)\,dx = dt$

or $\frac{\sin^3 x\,dx}{\cos^4 x + 3\cos^2 x + 1} = dt$

$\therefore I = \int \frac{dt}{t} = \ell n|t| + c = \ell n \tan^{-1}(\sec x + \cos x) + c$

6. The point that divides $5\hat{i}$ and $5\hat{j}$ in the ratio of k : 1 is $\Rightarrow \frac{(5\hat{j})k + (5\hat{i}).1}{k+1}$

$\therefore \vec{b} = \frac{5\hat{i} + 5k\hat{j}}{k+1}$

also $|\vec{b}| \leq \sqrt{37} \Rightarrow \frac{1}{k+1}\sqrt{25+25k^2} \leq \sqrt{37}$

$\Rightarrow 5\sqrt{1+k^2} \leq \sqrt{37}(k + 1)$

Squaring both sides $25(1 + k^2) \leq 37(k^2 + 2k + 1)$ or $6k^2 + 37k + 6 \geq 0$

$\Rightarrow (6k + 1)(k + 6) \geq 0$

$k \in (-\infty, -6) \cup \left[-\frac{1}{6}, \infty\right)$

7. (i) $\int \frac{(\tan^{-1}x)^3}{1+x^2}dx$

$= \int t^3\,dt = \frac{t^4}{4} + C$

$= \frac{(\tan^{-1}x)^4}{4} + C$

(ii) $\int \frac{3x^2+5}{x^2+4}\,dx$

$= \int \frac{3(x^2+4)-7}{(x^2+4)}\,dx$

$= 3x - \frac{7}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$

8. $\int\left(\left(\frac{x}{e}\right)^x + \left(\frac{e}{x}\right)^x\right)\ell nx\,dx$

$\int\left(\frac{x}{e}\right)^x \ell nx\,dx + \int\left(\frac{e}{x}\right)^x \ell nx\,dx$

put $\left(\frac{x}{e}\right)^x = t$

$\Rightarrow x(\ell nx - \ell ne) = \ell nt \Rightarrow \ell nx\,dx = \frac{1}{t}dt$

$\Rightarrow I = \int t\left(\frac{1}{t}\right)dt + \int\frac{1}{t}\left(\frac{1}{t}dt\right)$

$= t - \frac{1}{t} + c = \left(\frac{x}{e}\right)^x - \left(\frac{e}{x}\right)^x + c$

DPP NO. - 63

1. $\int \frac{\left(\sqrt{x+1}\right)^2 + \sqrt{x}\sqrt{1+x}}{\sqrt{x}+\sqrt{1+x}}\,dx$

$\int \frac{\sqrt{x+1}\left(\sqrt{1+x}+\sqrt{x}\right)}{\left(\sqrt{x}+\sqrt{1+x}\right)}\,dx$

$= \int\sqrt{x+1}\,dx = \frac{2}{3}(1+x)^{3/2} + C$

2. $D_r = \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix}$

$\sum_{r=0}^{m} D_r = \begin{vmatrix} \sum_{r=0}^{m}(2r-1) & \sum_{r=0}^{m}{}^mC_r & \sum_{r=0}^{m}1 \\ m^2-1 & 2^m & m+1 \\ \sin(m^2) & \text{Sin}^2(m) & \sin^2(m+1) \end{vmatrix}$

$= \begin{vmatrix} m^2-1 & 2^m & m+1 \\ m^2-1 & 2^m & m+1 \\ \sin(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix} = 0$

3. $\sin^2 x = t \qquad 2\sin x \cos x\,dx = dt$

$\frac{1}{2}\int\frac{dt}{\sqrt{1-t^2}} \Rightarrow \frac{1}{2}\sin^{-1}(t) + c$

4. $|\vec{b}| = 2|\vec{a}|$

$\sqrt{16x^2-16x+24} = 2\sqrt{10+x^2}$

Solving, $(x-2)(3x+2) = 0$

$\therefore\ x = 2, -\frac{2}{3}$.

5. (i) $xe^x = t$

$(x+1)e^x\,dx = dt$

$\int\frac{1}{\cos^2 t}dt \Rightarrow \tan t + c$

(ii) $\sqrt{2}\int\frac{1}{\sqrt{x^2+\frac{3}{2}x-1}}\,dx$

$= \frac{1}{\sqrt{2}}\int\frac{1}{\sqrt{\left(x+\frac{3}{4}\right)^2-\left(\frac{5}{4}\right)^2}}\,dx$

6. (i) $\int e^{2x}\left(\frac{1+2\sin x\cos x}{2\cos^2 x}\right)dx$

$\frac{1}{2}\int e^{2x}\left(\sec^2 x + \tan x\right)dx$

$\frac{1}{2}e^{2x}\tan x + e$

(ii) $\int\frac{x\sin^{-1}x}{(1-x^2)\sqrt{1-x^2}}\,dx$

$\int\frac{\sin t.t.dt}{\left(1-\sin^2 t\right)}dx = \int t(\tan t \sec t)dt$

I II

7. (i) $\int e^x\left(\frac{1}{x+2} + \ell n(x+2)\right)dx = e^x\,\ell n\,(x+2) + c$

DPP NO. - 64

1. $\int \frac{(x+1)e^x}{e^x.x(1+e^x)^2}dx$

Put $1 + xe^x = t \Rightarrow e^x(x + 1)dx = dt$

$I = \int \frac{dt}{(t-1)\,t^2}$

$= \int \left(\frac{1}{t-1} + \frac{-1}{t} + \frac{-1}{t^2}\right)dt$

$= \ell n(t - 1) - \ell nt + \frac{1}{t} + C$

$= \ell n(xe^x) - \ell n(1 + xe^x) + \frac{1}{1+xe^x} + C$

$= \ell n\left(\frac{xe^x}{1+xe^x}\right) + \frac{1}{1+xe^x} + C$

2. $\int \frac{\sin x \cos x dx}{1+(\sin x+\cos x)} \cdot \frac{1-(\sin x+\cos x)}{1-(\sin x+\cos x)}$

$1 + \sin x + \cos x = t$

$= \int \frac{(1-\sin x-\cos x)\sin x \cos x dx}{1-1-2\sin x \cos x}$

$= -\frac{1}{2}\int(1 - \sin x - \cos x)\,dx$

3. (i) $\int \frac{dx}{\sin^6 x} = \int \operatorname{cosec}^4 x.\operatorname{cosec}^2 x\,dx$

put $\cot x = t$

$\Rightarrow -\operatorname{cosec}^2 x\,dx = dt$

$= \int (\cot^2 x+1)^2 \operatorname{cosec}^2 x\,dx$

$= \int (t^2+1)^2(-dt)$

$= -\int (t^4+2t^2+1)\,dt$

$= -\frac{t^5}{5} - 2\frac{t^3}{3} - t + c$ where $t = \cot x$

(ii) $I = \int \frac{\cos x + \sin x}{\sqrt{\sin 2x + 1 - 1}}dx$

$= \int \frac{(\cos x + \sin x)}{\sqrt{1-(\sin x - \cos x)^2}}dx$

Put $\sin x - \cos x = t$]

5. $\int \frac{x\cos x+1}{x\sqrt{2xe^{\sin x}+1}} \cdot \frac{e^{\sin x}}{e^{\sin x}}.dx$

$2xe^{\sin x} + 1 = t^2$

$(2x.e^{\sin x}.\cos x + 2e^{\sin x})\,dx = 2tdt$

$e^{\sin x}(x \cos x + 1)\,dx = tdt$

$\Rightarrow 2\int \frac{tdt}{(t^2-1)t}$

$= 2\int \frac{1}{t^2-1}dt$

6. $x^2 = t$

$\frac{1}{2}\int \frac{dt}{t^2+t+1}$

7. $I = \int \frac{1-x^7}{x^7(1+x^7)}x^6\,dx$ Put $x^7 = t$

$I = \frac{1}{7}\int \frac{1-t}{t(1+t)}dt = \frac{1}{7}\left(\int \frac{1}{t} - \frac{2}{1+t}\right)dt$

$I = \frac{1}{7}\ell n\,t - \frac{2}{7}\ell n(1 + t) + c$

8. (A) $\int \frac{x^4-1}{x^2\sqrt{x^4+x^2+1}}dx = \int \frac{\left(x-\frac{1}{x^3}\right)dx}{\sqrt{x^2+\frac{1}{x^2}+1}}dx$

put $x^2 + \frac{1}{x^2} + 1 = t^2$

$2\left(x-\frac{1}{x^3}\right)dx = 2t\,dt$

$I = \int \frac{t\,dt}{t} = t + c = \frac{\sqrt{x^4+x^2+1}}{x} + C$

(B) $\int \frac{x^2-1}{x\sqrt{1+x^4}}dx = \int \frac{\left(1-\frac{1}{x^2}\right)}{\sqrt{x^2+\frac{1}{x^2}}}dx$

put $x+\frac{1}{x} = t \Rightarrow \left(1-\frac{1}{x^2}\right)dx = dt$

$I = \int \frac{dt}{\sqrt{t^2-(\sqrt{2})^2}}$

$= \ell n\left|t+\sqrt{t^2-2}\right| + C$

$= \ell n\left|\left(x+\frac{1}{x}\right)+\sqrt{x^2+\frac{1}{x^2}}\right| + C$

(C) $\int \frac{\left(\frac{1}{x^2}+1\right)}{\left(\frac{1}{x}-x\right)\sqrt{\frac{1}{x^2}+x^2}}dx$

$I = \int \frac{\left(\frac{1}{x^2}+1\right)}{\left(\frac{1}{x}-x\right)\sqrt{\left(x-\frac{1}{x}\right)^2+2}}dx$

Let $x - \frac{1}{x} = t$

$I = \int \frac{-dt}{t\sqrt{t^2+2}} = \int \frac{-t\,dt}{t^2\sqrt{t^2+2}}$

Let $\sqrt{t^2+2} = u \Rightarrow t^2 = u^2 - 2$

$\Rightarrow$ 2t dt = 2 u du

$\therefore\ I = \int \frac{-u\,du}{(u^2-2)u} = \int \frac{-du}{u^2-(\sqrt{2})^2}$

$= \frac{-1}{2\sqrt{2}}\ell n\left|\frac{u-\sqrt{2}}{u+\sqrt{2}}\right| = \frac{-1}{2\sqrt{2}}\ell n\left(\frac{(u-\sqrt{2})^2}{u^2-2}\right)$

$= \frac{-1}{\sqrt{2}}\ell n\left(\frac{\sqrt{t^2+2}-\sqrt{2}}{t}\right)$

$= \frac{-1}{\sqrt{2}}\ell n\left(\frac{\sqrt{\left(x-\frac{1}{x}\right)^2+2}-\sqrt{2}}{\left(x-\frac{1}{x}\right)}\right)$

(D) $I = \int \frac{\frac{1}{x^5}}{\left(1+\frac{1}{x^4}\right)\sqrt{\left(\sqrt{1+\frac{1}{x^4}}\right)-1}}dx$

put $\sqrt{\left(\sqrt{1+\frac{1}{x^4}}\right)-1} = t$

$\Rightarrow\ 1 + \frac{1}{x^4} = (1 + t^2)^2$

$\frac{-4}{x^5}\,dx = 2(1 + t^2)\,2t\,dt$

$\therefore\ I = \int \frac{-(1+t^2)t\,dt}{(1+t^2)^2\,t} = -\int \frac{1}{1+t^2}dt$

$= -\tan^{-1}(t) + c = -\tan^{-1}\left(\sqrt{\left(\sqrt{1+\frac{1}{x^4}}\right)}\right) + c$

DPP NO. - 65

1. $I = \int \frac{dx}{((x+2)^2+1)^2}$

$x + 2 = \tan\theta$

$dx = \sec^2\theta$

$\int \frac{\sec^2\theta}{(1+\tan^2\theta)^2}d\theta$

$\int \cos^2\theta\,d\theta$

$= \int \frac{1+\cos 2\theta}{2}\,d\theta$

$= \frac{1}{2}\left[\theta+\frac{\sin 2\theta}{2}\right]+c$

$= \frac{1}{2}[\theta+\sin\theta\cos\theta]+c$

$= \frac{1}{2}\left[\tan^{-1}(x+2)+\frac{x+2}{x^2+4x+5}\right] + c$

2. $\int \frac{x\,dx}{\sqrt{(1+x^2)\left[1+\sqrt{1+x^2}\right]}}$

$1 + \sqrt{1+x^2} = t$

$\frac{2x}{2\sqrt{1+x^2}}dx = dt$

$\int \frac{dt}{\sqrt{t}}$

3. $x + \sqrt{1+x^2} = t$

$1 + \frac{2x}{2\sqrt{1+x^2}}dx = dt$

$$\frac{(\sqrt{1+x^2}+x)}{\sqrt{1+x^2}}dx = dt$$

$$\int \frac{t^{2009}}{t}dt = \frac{t^{2009}}{2009}tc$$

4. $\int x^{5/2}.x^{5/2}.x^{3/2}\sqrt{(1+x^{5/2})}dx$

$1+x^{5/2} = t^2$

$\frac{5}{2}x^{3/2}dx = 2tdt$

$\int (t^2-1)\frac{4}{5}t.t\,dt$

$\frac{4}{5}\int (t^4-2t^2+1)t^2\,dt$

5. $x + 1 = t^2$

6. $\int \left(\frac{\sin^{3/2}\theta}{\sqrt{\sin^3\theta\cos^3\theta(\sin\theta\cos\alpha+\cos\theta\sin\alpha)}} + \frac{\cos^{3/2}\theta}{\sqrt{\sin^3\theta\cos^3\theta(\sin\theta\cos\alpha+\cos\theta\sin\alpha)}}\right)d\theta$

$$\int \frac{d\theta}{\sqrt{\cos^4\theta(\cos\alpha.\tan\theta+\sin\alpha)}}d\theta + \int \frac{\cos^{3/2}\theta}{\sqrt{\sin^4\theta(\cos\alpha+\cot\theta.\sin\alpha)}}$$

let $\tan\theta = u$
let $\cot\theta = v$

7. Partial fraction

$$\frac{x^2-4}{(x^2+1)(x^2+2)(x^2+3)}$$

$= \frac{A}{x^2+1} + \frac{B}{x^2+2} + \frac{C}{x^2+3}$

$x^2 - 4 = A(x^2+2)(x^2+3) + B(x^2+1)(x^2+3) + C(x^2+1)(x^2+2)$

put $X^2 = -1, -2, -3$

find A, B, C

8. **(A)** $\int \sqrt{1+\sec x}\,dx$

$= \int \frac{\sqrt{\sec^2 x-1}}{\sqrt{\sec x-1}}dx$

$= \int \frac{\tan x}{\sqrt{\sec x-1}}dx$

Put $\sec x - 1 = t^2$

$\sec x \tan x\,dx = 2t\,dt$

$\tan x\,dx = \frac{2t\,dt}{t^2+1}$

$= \int \frac{1}{t}\left(\frac{2t}{t^2+1}\right)dt$

$= 2\tan^{-1}(t) + c$

$= 2\tan^{-1}(\sqrt{\sec x-1}) + c$

(B) $\int \frac{\sec^2 x\,dx}{(\tan x-2)(2\tan x+1)}$

$= \int \frac{dt}{(\tan x-2)(2\tan x+1)}$ Put $\tan x = t$

$= \frac{1}{5}\int \frac{(2\tan x+1)-2(\tan x-2)}{(\tan x-2)(2\tan x+1)}dt$

$= \frac{1}{5}\left(\int \frac{1}{\tan x-2}dt - 2\int \frac{1}{2\tan x+1}dt\right)$

$= \frac{1}{5}\left(\ell n\left(\frac{(\tan x-2)}{2\tan x+1}\right)\right) + C$

$= \frac{1}{5}\ell n\left|\frac{\tan x-2}{2\tan x+1}\right| + C$

(C) $\int \frac{2\tan x\sec^2 x}{\tan^4 x+1}dx$

$= \int \frac{dt}{t^2+1} = \tan^{-1}t + c = \tan^{-1}(\tan^2 x) + C$

(D) $\int \frac{\sin^3\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)\sqrt{\cos^2 x(\cos x+1+\sec x)}}dx$

$$\int \frac{\sin^3\frac{x}{2}}{\cos\frac{x}{2}\cos x\sqrt{\cos x+\sec x+1}}dx$$

put $\cos x + \sec x + 1 = t^2$

$\Rightarrow$ $-\sin x + \sec x\tan x = 2t\,dt$

$\sin x\left(\frac{-\cos^2 x+1}{\cos^2 x}\right) = 2t\,dt$

$\Rightarrow$ $\frac{\sin^3 x}{\cos^2 x}dx = 2t\,dt$

$$= \int \frac{\sin^3 x}{8\cos^3\frac{x}{2}.\cos\frac{x}{2}.\cos x\sqrt{\cos x+\sec x+1}}dx$$

$$= \int \frac{\cos x}{2\left(2\cos^2\frac{x}{2}\right)^2}\frac{\sin^3 x}{\cos^2 x}\frac{1}{\sqrt{\cos x+\sec x+1}}dx$$

$$= \int \frac{\cos x}{2(1+\cos x)^2}\frac{\sin^3 x}{\cos^2 x}\frac{1}{\sqrt{\cos x+\sec x+1}}dx$$

$$= \int \frac{\cos x}{2(\sec x+\cos x+1+1)}\frac{\sin^3 x}{\cos^2 x}\frac{1}{\sqrt{\cos x+\sec x+1}}dx$$

$$= \frac{1}{2}\int \frac{1}{(1+t^2)}\frac{2t\,dt}{t} = \tan^{-1}(t) + c$$

$$= \tan^{-1}\left(\sqrt{\cos x+\sec x+1}\right) + c$$

DPP NO. - 66

1. taking two togther and one person together then total person 48 person.

Now arrange in circle in (48 – 1)! = 47! ways

But one person can be choose by 48 ways and two brother can be arranged in 2 ways.

there for total ways = 2.48. 47! = 2. 48

2. Here we should go (3 –1) steps to east and (5 –1) steps to south so total steps which we have to go are (5 + 3 – 2) ways.
Hence total no. of ways
$= {}^{5+3-2}C_{5-1} . {}^{3-1}C_{3-1} = 15$

5. $$\int \frac{x^{-7/6}-x^{5/6}}{x^{1/3}x^{1/2}\left(x+1+\frac{1}{x}\right)^{1/2}-x^{1/2}x^{1/3}\left(x+1+\frac{1}{x}\right)^{1/3}}dx$$

$$\int \frac{x^{-7/6}-x^{5/6}}{x^{5/6}\left[\left(x+1+\frac{1}{x}\right)^{1/2}-\left(x+1+\frac{1}{x}\right)^{1/3}\right]}dx$$

Put $x + \frac{1}{x} + 1 = z^6$

6. 25 numbers are even
and 25 numbers are odd
ways = ${}^{25}C_2 + {}^{25}C_2 = 2\,{}^{25}C_2$

7. $$\int \frac{(\cos 2x-3)\cot x}{\sin^4 x\sqrt{4\cot^2 x-1}}dx$$

$$\int \frac{(2\cos^2 x-1-3)\cot x}{\sin^4 x.\sqrt{4\cot^2 x-1}}dx$$

$$\int\left(\frac{2\cos^2 x.\cot x}{\sin^4 x.\sqrt{4\cot^2 x-1}}-\frac{4\cot x}{\sin^4 x.\sqrt{4\cot^2 x-1}}\right)dx$$

$$\int \frac{2\cot^2 x\cot x\,\mathrm{cosec}^2 x}{\sqrt{4\cot^2 x-1}}dx-\int\frac{4\cot x\,\mathrm{cosec}^2 x\,\mathrm{cosec}^2 x dx}{\sqrt{4\cot^2 x-1}}dx$$

Let $4\cot^2 x - 1 = t^2$

DPP NO. - 67

1. Two circle intersect at two points = ${}^4C_2 \times 2 = 12$

two lines interset at one point = ${}^8C_2 = 28$

one circle and one line intersect at two point

$= {}^4C_1 \times {}^8C_1 \times 2 = 64$

Total = 12 + 28 + 64 = 104

3. Required ways

$= {}^nC_3 - n(n-3) = {}^{20}C_3 - 20(20-3) = {}^{20}C_3 - 20(17)$

= 57 × 20 – 20(17) = 20 × 40 = 800

4. (i) $I = \int_0^1 -(x^2+2x-3)dx + \int_1^2 (x^2+2x-3)dx$

$$= -\left[\frac{x^3}{3}+x^2-3x\right]_0^1 + \left[\frac{x^3}{3}+x^2-3x\right]_1^2 = 4$$

(ii) $\int_0^4 (x-[x])dx = \int_0^4 x\,dx - \int_0^4 [x]dx$

$$= \left(\frac{x^2}{2}\right)_0^4 - \left(\int_0^1 0\,dx+\int_1^2 1dx+\int_2^3 2dx+\int_3^4 3dx\right)$$

= 8 – (1 + 2 + 3) = 2

5. **(i)** $I = \int_1^4 (x-[x])^{[x]}dx$

$$= \int_1^2 (x-1)dx + \int_2^3 (x-2)^2 dx + \int_3^4 (x-3)^3 dx$$

$$= \left(\frac{(x-1)^2}{2}\right)_1^2 + \left(\frac{(x-2)^3}{3}\right)_2^3 + \left(\frac{(x-3)^3}{4}\right)_3^4$$

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12}$$

(ii) $\int_0^2 3[x]dx = 3\left(\int_0^1 0\,dx + \int_1^2 1\,dx\right)$

= 3(0 + (2 – 1)) = 3

6. **(i)** $I = \int_{-1}^{0} -x\,dx + \int_0^3 x\,dx$

$$+ \int_{-1}^{1} -(x-1)\,dx + \int_1^3 (x-1)\,dx$$

$$= \left(\frac{-x^2}{2}\right)_{-1}^0 + \left(\frac{x^2}{2}\right)_0^3 - \left(\frac{(x-1)^2}{2}\right)_{-1}^1 + \left(\frac{(x-1)^2}{2}\right)_1^3$$

$$= \frac{1}{2} + \frac{9}{2} + 2 + 2 = 9$$

(ii) $\int_0^{\pi/2} |\cos x|\,dx + \int_{\pi/2}^{\pi} |\cos x|\,dx$

$$= \int_0^{\pi/2} \cos x\,dx - \int_{\pi/2}^{\pi} \cos x\,dx = (\sin x)_0^{\pi/2} - (\sin x)_{\pi/2}^{\pi}$$

= (1 – 0) – (0 – 1) = 2

7. **(i)** $2\int_0^1 e^x\,dx = 2(e^1 - e^0) = 2e - 2$

(ii) $2\int_0^{1/2} \sin 2\pi x\,dx = 2\left(\frac{-\cos 2\pi x}{2\pi}\right)_0^{1/2}$

$$= \left(\frac{-1}{\pi}(-1-1)\right) = \frac{2}{\pi}$$

(iii) $\int_{-1}^{1} \frac{x\,dx}{\sqrt{5-4x}}$

put $5 - 4x = t^2 \Rightarrow x = \frac{5-t^2}{4}$, $-4\,dx = 2t\,dt$

$$I = \int_3^1 \frac{\left(\frac{5-t^2}{4}\right)\left(\frac{-t}{2}\right)dt}{t}$$

$$= \frac{1}{8}\left(5t - \frac{t^3}{3}\right)_1^3 = \frac{1}{8}((15-9)-(5-1/3))$$

$$= \frac{1}{8}\left(\frac{4}{3}\right) = \frac{1}{6}$$

DPP NO. - 68

1. $I = \int_{-\pi}^{\pi} (\cos px - \sin qx)^2 dx$ ...(1)

using property

$I = \int_{-\pi}^{\pi} (\cos px + \sin qx)^2 dx$ ...(2)

$$2I = 2\int_{-\pi}^{\pi} (\cos^2 px + \sin^2 px)dx$$

$$I = \left(2x + \frac{\sin 2px}{2p} - \frac{\sin 2qx}{2q}\right)_0^{\pi}$$

$\Rightarrow I = 2\pi$

2. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} x \sin(\pi[x] - x)\,dx$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} x \sin x\,dx = (-x\cos x)_{\pi/3}^{\pi/2} + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 1.\cos x\,dx$$

$$= 0 + \frac{\pi}{3}\left(\frac{1}{2}\right) + (\sin x)_{\pi/3}^{\pi/2} = \frac{\pi}{6} + 1 - \frac{\sqrt{3}}{2}$$

$\because \frac{\pi}{3} < x < \frac{\pi}{2}$

1.04 < x < 1.57

$\Rightarrow [x] = 1$

3. $= \frac{1}{a}\int_0^a \left(\sqrt{x+a} - \sqrt{x}\right)dx = \frac{1}{a}\left(\frac{2(x+a)^{3/2}}{3} - \frac{2x^{3/2}}{3}\right)_0^a$

$$= \frac{1}{a}\left(\frac{2}{3}\right)\left(\left((2a)^{3/2} - a^{3/2}\right) - a^{3/2}\right)$$

$$= \frac{2}{3a}\left(2\sqrt{2}a^{3/2} - 2a^{3/2}\right) = \frac{4}{3}(\sqrt{2}-1)\sqrt{a}$$

$$\text{R.H.S.} = \int_0^{\pi/8} \frac{2\tan\theta}{2\sin\theta\cos\theta} d\theta = \int_0^{\pi/8} \sec^2\theta\, d\theta$$

$$= (\tan\theta)_0^{\pi/8} = \tan\frac{\pi}{8} = (\sqrt{2}-1)$$

$$\Rightarrow \frac{4}{3}(\sqrt{2}-1)\sqrt{a} = (\sqrt{2}-1) \Rightarrow a = \frac{9}{16}$$

4. $\int_1^2 e^{x^2} dx = \alpha$

Now $I = \int_e^{e^4} \sqrt{\ell nx}\, dx$,

put $\ell n\, x = t^2$, $x = e^{t^2}$, $dx = e^{t^2} .2t\, dt$

$$I = \int_1^2 t.e^{t^2} 2t\, dt$$

$$I = \left(t.e^{t^2}\right)_1^2 - \int_1^2 1.e^{t^2} dt$$

$I = (2e^4 - e^1) - \alpha = 2e^4 - e - \alpha$

5. $I = \int \frac{\sqrt{1+x^2}}{1-x^2} dx = \int \frac{\sqrt{1+x^2}}{1-x^2} \times \frac{\sqrt{1+x^2}}{\sqrt{1+x^2}} dx$

$$= \int \frac{1}{\sqrt{1+x^2}\,(1-x^2)} dx - \int \frac{(1-x^2)-1}{(1-x^2)\sqrt{1+x^2}} dx$$

$$= \int \frac{2}{(1-x^2)\sqrt{1+x^2}} dx - \int \frac{1}{\sqrt{1+x^2}} dx = I_1 - I_2 \quad \text{Now}$$

in I_1 put $x = \frac{1}{t} \Rightarrow dx = -\frac{1}{t^2} dt$

$$I_1 = 2\int \frac{t}{\sqrt{t^2+1}\frac{(t^2-1)}{t^2}} \cdot \left(\frac{-1}{t^2}\right) dt$$

$$= 2\int \frac{-t}{(t^2-1)\sqrt{1+t^2}} dt$$

put $\sqrt{1+t^2} = u \Rightarrow 1 + t^2 = u^2$

$2tdt = 2udu$

$$= 2\int \frac{-u\, du}{(u^2-2)u} = -2\int \frac{du}{u^2 - (\sqrt{2})^2}$$

$$= -\frac{1}{\sqrt{2}} \ell n \left| \frac{-\sqrt{2}x + \sqrt{1+x^2}}{\sqrt{2}x + \sqrt{1+x^2}} \right| + c$$

and $I_2 = \ell n|x + \sqrt{1+x^2}| + c$

6. **(i)** $I = \int_0^{\pi/2} \frac{\cos x\, dx}{(1+\sin x)(2+\sin x)}$

$$I = \int_0^1 \frac{dt}{(1+t)(t+2)} = \int_0^1 \left(\frac{1}{t+1} - \frac{1}{t+2}\right) dt$$

$$= \left(\ell n(t+1) - \ell n(t+2)\right)_0^1$$

$$= (\ell n2 - \ell n3) - (\ell n1 - \ell n2) = \ell n\left(\frac{4}{3}\right)$$

(ii) $\int_1^2 \frac{x^3}{x^4(x^4+1)} dx = \int_1^{16} \frac{1}{t(t+1)}\left(\frac{dt}{4}\right)$

$$= \frac{1}{4}\int_1^{16}\left(\frac{1}{t} - \frac{1}{t+1}\right) dt$$

$$= \frac{1}{4}((\ell n16 - \ell n17) - (\ell n1 - \ell n2)) = \frac{1}{4}\ell n\left(\frac{32}{17}\right)$$

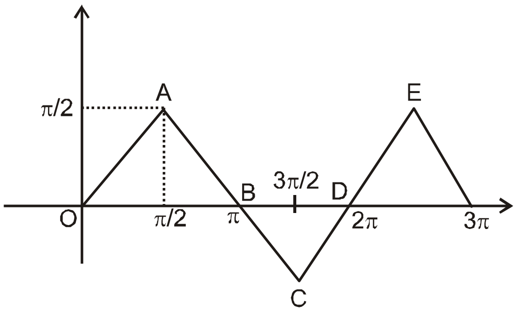
(iii) $\int_0^1 (4-2x)dx + \int_1^3 2dx + \int_3^5 (2x-4)dx$

$$= (4x - x^2)_0^1 + (2x)_1^3 + (x^2 - 4x)_3^5$$

$= 3 + 4 + 5-(-3) = 15$

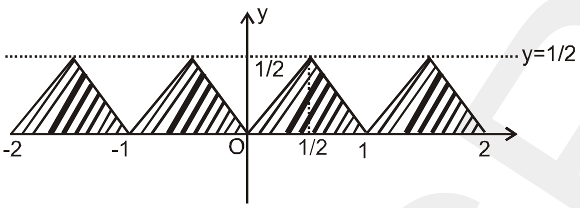
7. **(i)** $\int_0^{3\pi} \sin^{-1} \sin x\, dx$

= Area of (OAB) – Area of ΔBCD + Area of DEF



$$= \frac{1}{2}\pi\left(\frac{\pi}{2}\right) - \frac{1}{2}\pi\left(\frac{\pi}{2}\right) + \frac{1}{2}(\pi)\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4}$$

(ii) $\int_{-2}^{2} \min(\{x\}, \{-x\})dx$



Area $= 4\left(\frac{1}{2}\times 1\times\frac{1}{2}\right) = 1$

(iii) $\int_{0}^{\infty} \frac{x\log x}{(1+x^2)^2}dx$

Put x = tan θ

$$I = \int_{0}^{\pi/2} \frac{\tan\theta \log\tan\theta.\sec^2\theta\, d\theta}{\sec^4\theta\, d\theta}$$

$$= \int_{0}^{\pi/2} \frac{\sin 2\theta}{2}\log\tan\theta\, d\theta \quad ...(1)$$

By property

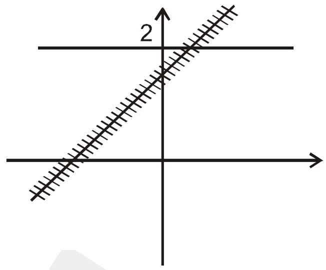
$$I = \frac{1}{2}\int_{0}^{\pi/2} \sin 2\left(\frac{\pi}{2}-\theta\right)\log\tan\left(\frac{\pi}{4}-\theta\right)d\theta$$

$$I = \frac{1}{2}\int_{0}^{\pi/2} \sin 2\theta \log\cot\theta\, d\theta \quad ...(2)$$

Adding (1) and (2)

$2I = 0 \Rightarrow I = 0$

8. **(A)** f(x) = min{x + 1, 2sgn(|x|)}



$$f(x) = \begin{cases} x+1, & x<1 \\ 2, & x \geq 1 \end{cases}$$

$$I = \int_{-5}^{1}(x+1)dx + \int_{1}^{4} 2\,dx = \left(\frac{x^2}{2}+x\right)_{-5}^{1} + (2x)_1^4$$

$$= \left(\frac{3}{2} - \left(\frac{25}{2}-5\right)\right) + 2(4-1) = -6 + 6 = 0$$

(B) $\int_{-3}^{5} f(|x|)dx = \int_{-3}^{-2} f(|x|)dx + \int_{-2}^{-1} f(|x|)dx$

$+ \int_{-1}^{0} f(|x|)dx + \int_{0}^{1} f(|x|)dx + \int_{1}^{2} f(|x|)dx$

$+ \int_{2}^{3} f(|x|)dx + \int_{3}^{4} f(|x|)dx + \int_{4}^{5} f(|x|)dx$

$$= \frac{1}{2}\left((-3)^2 + (-2)^2 + (-1)^2 + (0)^2 + (1)^2 + (2)^2 + (3)^2 + (4)^2\right)$$

$$= \frac{1}{2}(28 + 16) = \frac{44}{2} = 22$$

(C) $\int_{n}^{n+1}[x+[x+[x]]]dx = \int_{n}^{n+1} 3[x]dx = 3n$

∴ k = 3

∵ n < x < n + 1

⇒ [x] = n

(D) $f(x) = \int_{1}^{x} \frac{e^t}{t}dt$

DPP NO. - 69

1. (i) Number of words

$= {}^7C_4\ 4! = \frac{7\times 6\times 5}{3\times 2} \times 24 = 840$

(ii) Number of words

$= {}^5C_4\ 4! = 5 \times 24 = 120$

(iii) – – -- –

c 5(c + v) c

Number of words = $({}^5C_2\, 2!)\ ({}^5C_2\, 2!) = 400$

(iv) Number of words $= 2 \times {}^6C_3 \times 3! = 240$

(v) Number of words $= {}^6C_3 \times 4! = 480$

(vi) Number of words $= {}^5C_2 \times 2! \times 2 = 40$

(vii) Number of words $= 3 \times {}^5C_2 \times 2! = 60$

(viii) Number of words $= {}^5C_2 \times 4! = 240$

2. One digit number from 1 to 9 = 9

Two digit number □□

9 9 $= 9 \times 9 = 81$

Three digit number □□□

9 9 8 = 648

Total number of numbers = 738

3. A N R U V

All letter starting with A .... = 4! = 24

N .... = 4! = 24

R .... = 4! = 24

U .... = 4! = 24

VAN .... = 2! = 2

VARN ....= 1! = 1

VARUN .....= 1! = 1

100

4. $3^5 = 81 \times 3 = 243$

5. (i) Number of ways = selection of persons and then arrangement in chairs $= {}^4C_3 \times 3! = 24$

(ii) Number of ways = selection of chairs and then arrangement of persons in chairs $= {}^4C_3 \times 3! = 24$

6. $${}^7C_3 .4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)$$

$$= \frac{7.6.5}{3.2.1}.4!\left(\frac{1}{2}-\frac{1}{6}+\frac{1}{24}\right)$$

$= 35\,(12 - 4 + 1)$

$= 35\,(9) = 315$

7. Total no. of ways $= {}^4C_1 \times 4^4 = 1024$

8. Total no. of ways = 4! 5! = 2880

9. 2 alike and 1 different $= {}^8C_1\ {}^7C_1 = 56$

all different $= {}^8C_3 = 56$

total no. of ways = 112

10. Total no. ways in which all the guests can be invited $= {}^{11}C_5$

when 2 particular guest included $= {}^9C_3$

$\therefore$ required no. of ways $= {}^{11}C_5 - {}^9C_3 = 378$

11. The numbers having 1 at its unit's place $= 3 \times 2 = 6$

Similarly the numbers having 2,3,& 4 at its unit place will be 6 in each case.

$\therefore$ Sum of the digits at unit place $= 6(1 + 2 + 3 + 4) = 60$

Similarly the sum of the digits at tens place = 60

and the sum of the digits at hundredth place = 60

$\therefore$ Sum of the numbers $= 60 + 10 \times 60 + 100 \times 60$

$= 60 + 600 + 6000$

$= 6660$

12. Total number of ways of arranging 2 identical white balls. 3 identical red balls and 4 green balls of different shades

$$= \frac{9!}{2!3!} = 6.7!$$

Number of ways when balls of same colour are together $= 3! \times 4! = 6.4!$

$\therefore$ Number of ways of arranging the balls when atleast one ball is separated from the balls of the same colour $= 6.7! - 6.4! = 6(7! - 4!)$

13. Rectangle with side length

$1 \times 1,\ 1 \times 2,\ 1 \times 3,..........1 \times 8 \rightarrow 8$

$2 \times 2,\ 2 \times 3,\ 2 \times 4,..........2 \times 8 \rightarrow 7$

⋮

$8 \times 8 \rightarrow 1$

Total = 1 + 2 + 3 + ...+ 8

$$= \frac{8(9)}{2} = 36$$

14. Number of ways of filling 1 box with a green ball = 6

Number of ways of filling any two consective boxes with a green ball = 5

Number of ways of filling any three consective boxes with a green ball = 4

Number of ways of filling any four consective boxes with a green ball = 3

Number of ways of filling any five consective boxes with a green ball = 2

Number of ways of filling all boxes with green balls = 1

Total ways = 21

15. (i) Four different

${}^5C_4\ 4! = 5 \times 24 = 120$

(ii) 2 different 2 same

$${}^4C_2\ {}^1C_1 . \frac{4!}{2!} = 72$$

$\therefore$ Total permutations = 120 + 72 = 192

No change in BOMBAY & MUMBAI as both have same no. of letters with same no. of repetitions.

Following all the cases of combinations of 3 letters

(i) Three different

$^5C_3 = 10$

(ii) Two same 1 different

$^1C_1 \cdot {^4C_1} = 4$

Total combinations = 10 + 4 = 14

16. $10! - 9! = 9! \times 9$

The number of 10 digit numbers using the digits 0,1,2,.............9 = 9 × 9!

The number will be divisible by 4 if last two digits of the number is any of 04, 08, 12, 16, 20, 24, ...96 excluding 44 & 88

Number of numbers having 04,08, 20, 40, 60, 80 as their last two digits = 6 × 8!

Number of numbers of having other two digits of the above group as the last two digits

$= 16 \times 7 \times 7! = 14 \times 8!$

∴ Total number of numbers

$= 6 \times 8! + 14 \times 8! = 20 \times 8!$

17. (A) No couple = Total ways - one couple - 2 couple

$= {^{12}C_4} - {^6C_1} . {^{10}C_1} . {^8C_1} - {^6C_2}$

= 495 – 240 – 15 = 240

(B) Exactly one couple = $^6C_1 \; {^{10}C_1} \; {^8C_1} = 240$

(C) At least one couple = Total – No. couple

= 495 – 240 = 255

(D) At most one couple = no couple + one couple

= 240 + 240 = 480

18. Groups of 3, 3, 2, 2, 2, 2

$$\frac{14!}{(3!)^2\, 2!(2!)^4\, 4!} = \frac{14!}{(3!)^2\,(2!)^5\, 4!}$$

19. Ways of distribution

	B_1	B_2	B_3
(i)	1	3	4
(ii)	2	2	4
(iii)	3	3	2

∴ Total no. of ways

$$= \left(\frac{8!}{1!3!4!} + \frac{8!}{2!2!4! \times 2!} + \frac{8!}{3!3!2!2!}\right) \times 3!$$

As given

$$\left(\frac{8!}{1!3!4!} + \frac{8!}{2!2!4!2!} + \frac{8!}{3!3!2! \times 2!}\right) \times 3! = K \cdot {^7P_3}$$

Solving K = 22

20. Number of seating arrangements of total 19 members = 18!

and number of ways when the members of same nationality sit together = 4! 5! 4! 4! 3! 3!

$= (3!)^2\,(4!)^3\,(5!)$

21. In the case of necklace, their is no distinction between clockwise and anticlockwise arrangements, then

$$\text{Total number of necklace} = \frac{1}{2}\left(\frac{^{23}P_{11}}{11}\right) = \frac{1}{22} \times \frac{23!}{12!}$$

22. A unique circle passes through three non-collinear points

Hence number of circles containg three of 11 points = $^{11}C_3$

But five of these points lie on the same circle and they form one cirlce

∴ Total number of circles

$= {^{11}C_3} - {^5C_3} + 1 = 165 - 10 + 1 = 156$

23. Number of seating arrangements according to given condition

= Number of seating arrangements when men & women are alternate – number of seating arragements when women are adjacent to her husband.

$= n!(n-1)! - 2(n-1)!$

24. Number of positive integral solution = $^{19-1}C_{4-1} = {^{18}C_3}$ = Coefficients of x^{19} in $(x + x^2 + x^3 + \ldots\ldots + x^{19})^4$

Number of ways in which 15 identical things can be distributed among 4 persons = $^{15+4-1}C_{15} = {^{18}C_{15}} = {^{18}C_3}$

25. Let the amount received by the sons be Rs. x , Rs. y and Rs. z respectively, then

$x \le y + z = 101 - x$

i.e., $2x \le 101$ ∴ $x \le 50,\ y \le 50,\ z \le 50$

$x + y + z = 101$

The corresponding multinomial is $(1 + x +, \ldots.\ x^{50})^3$

∴ coefficient of x^{101} in the expansion of $(1 + x +, \ldots.\ x^{50})^3$

$= 1 \times {^{103}C_{101}} - 3.\ {^{52}C_{50}}$

$= {^{103}C_2} - 3.\ {^{52}C_2}$

26. ∵ 8 things are different & 8 things are alike and we have to take 8 out of them

∴ required selection

= coefficient of x^8 in $(1 + x + x^2 + x^3 + \ldots x^8)(1 + x)^8$

⇒ coefficient of x^8 in $\left(\frac{1-x^9}{1-x}\right)(1+x)^8$

⇒ coefficient of x^8 in $(1 - x)^{-1}(1 + x)^8$

⇒ coefficient of x^8 in $(1 - x)^{-1}(2 - (1 - x))^8$

⇒ coefficient of x^8 in

$(1 - x)^{-1}[2^8 - {^8C_1} . 2^7(1 - x) + \ldots..]$

⇒ $2^8 = 256$

27. $N = 2^7 . 3^3 . 5^4 = 2^1 \times 3^1(2^6 . 3^2 . 5^4)$

Divisor but not 15 divisible by 6 in

$N = 2^1 \times 3^1\ (2^6 . 3^2 . 5^4)$

= (6 + 1)(2 + 1) = 7 × 3 = 21

28. $240 = 2^4 . 3^1 . 5^1$

We can choose divisors of the from 4n + 2 (n ≥ 0)

$= 2.(3^0 + 3^1)(5^0 + 5^1)$

Ways=4 since $(2^1.3^0.5^0,\ 2^1.3^0.5^1,\ 2^1.3^1.5^0,\ 2^1.3^1.5^1)$

29. $S = (1 + 2^p - 1)(2^0 + 2^1 + \ldots + 2^{p-1})$

$= 2^p \dfrac{2^p - 1}{2 - 1}$

$\Rightarrow\ 2^p(2^p - 1) = 2 \times 2^{p-1}(2^p - 1) = 2N$

30. Exponent of 2 in 100!

$$= \left[\frac{100}{2^1}\right] + \left[\frac{100}{2^2}\right] + \left[\frac{100}{2^3}\right] + \left[\frac{100}{2^4}\right] + \left[\frac{100}{2^5}\right] + \left[\frac{100}{2^6}\right]$$

$= 50 + 25 + 12 + 6 + 3 + 1 = 97$

Exponent of 3 in 100!

$$= \left[\frac{100}{3}\right] + \left[\frac{100}{3^2}\right] + \left[\frac{100}{3^3}\right] + \left[\frac{100}{3^4}\right] + \ldots\ldots$$

$= 33 + 11 + 3 + 1 = 48$

coefficient of $x^{12} = (2^2)^{48} . (3)^{48} . 2$

$= (12)^{48}.2$

$= 48$

DPP NO. - 70

1. $$\int_{-1}^{3}\left(\tan^{-1}\frac{x}{(x^2+1)} + \tan^{-1}\left(\frac{x^2+1}{x}\right)\right)dx$$

$$= \int_{-1}^{0}\left(\tan^{-1}\frac{x}{(x^2+1)} - \pi + \cot^{-1}\left(\frac{x}{x^2+1}\right)\right)dx$$

$$+ \int_{0}^{3}\left(\tan^{-1}\frac{x}{(x^2+1)} + \cot^{-1}\left(\frac{x}{x^2+1}\right)\right)dx$$

$$\Rightarrow\ -\frac{\pi}{2}\int_{-1}^{0}dx + \int_{0}^{3}\frac{\pi}{2}dx$$

$\Rightarrow\ \pi$ **Ans.**

2. $$I = \int_{0}^{\pi/2}\frac{\sin x\,dx}{1+\sin x+\cos x}$$

$$I = \int_{0}^{\pi/2}\frac{\cos x\,dx}{1+\sin x+\cos x}$$

$$2I = \int_{0}^{\pi/2}\frac{(\sin x + \cos x)}{1+\sin x+\cos x}dx$$

$$2I = \int_{0}^{\pi/2}dx - \int_{0}^{\pi/2}\frac{dx}{1+\sin x+\cos x}$$

$$2I = \frac{\pi}{2} - \ell n2$$

$$I = \frac{\pi}{4} - \frac{1}{2}\ell n2$$

3. $$I_1 = \int_{-\pi/4}^{\pi/4}\ell n(\sin x + \cos x)\,dx$$

$$I_1 = \int_{-\pi/4}^{\pi/4}\ell n(-\sin x + \cos x)\,dx$$

$$2I_1 = \int_{-\pi/4}^{\pi/4}\ell n(\cos 2x)dx$$

$$= 2\int_{0}^{\pi/4}\ell n(\cos 2x)dx$$

$$= 2\int_{0}^{\pi/4}\ell n(\sin 2x)dx = I$$

$\therefore\ I_1 = I/2$

4. $$u = \int_{0}^{1}\frac{\tan(x+1)}{(x^2+1)}dx$$

put $x = \tan\theta$

$dx = \sec^2\theta d\theta$

$$= \int_{0}^{\pi/4}\frac{\ell n(1+\tan\theta)}{\sec^2\theta}(\sec^2\theta)d\theta$$

$$= \int_{0}^{\pi/4}\ell n\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta$$

$$= \int_{0}^{\pi/4}\ell n\left(\frac{2}{1+\tan\theta}\right)d\theta$$

$$u = \int_{0}^{\pi/4}\ell n2\,d\theta - u\ \Rightarrow\ u = \frac{\pi}{8}\ell n2$$

$$u = \frac{-v}{4}\ \Rightarrow\ 4u + v = 0$$

5. $A_{n+1} - A_n = \int_0^{\pi/2} \frac{\sin(2n+1)x - \sin(2n-1)x\,dx}{\sin x}$

$= \int_0^{\pi/2} \frac{2\cos 2nx \sin x}{\sin x} dx$

$= \left(\frac{2\sin 2nx}{2n}\right)_0^{\pi/2} = 0$

$\therefore \quad A_{n+1} = A_n$

Now $B_{n+1} - B_n = \int_0^{\pi/2} \left(\frac{\sin^2(n+1)x - \sin^2 nx}{(\sin^2 x)}\right) dx$

$= \int_0^{\pi/2} \frac{\sin(2n+1)x}{\sin x} dx = A_{n+1}$

6. $\int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right) dx$

$= \int_0^1 \tan^{-1}\left(\frac{x-(x-1)}{1+x(x-1)}\right) dx$

$= \int_0^1 \tan^{-1} x - \tan^{-1}(x-1)\,dx$

$= \int_0^1 \tan^{-1} x\,dx - \int_0^1 \tan^{-1}(x-1)\,dx$

$= \int_0^1 \tan^{-1} x\,dx - \int_{-1}^0 \tan^{-1} t\,dt$

$= 2\int_0^1 \tan^{-1} x\,dx$

$\because \int_0^1 \left(\tan^{-1}(1-x+x^2) + \tan^{-1}\left(\frac{1}{1-x+x^2}\right)\right) dx = \pi/2$

$\therefore \int_0^1 \tan^{-1}(1-x+x^2)\,dx$

$= \frac{\pi}{2} - 2\left\{\left(x\tan^{-1} x\right)_0^1 - \int_0^1 \frac{x\,dx}{(1+x^2)}\right\} = \ell n2$

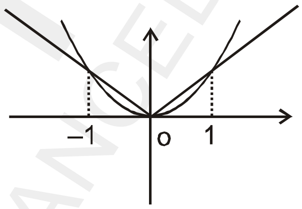
7. $x^2 F(x) = \int_4^x (4t^2 - 2F'(t))\,dt$

$\Rightarrow 2xF(x) + x^2 F'(x) = 4x^2 - 2F'(x)$

$\Rightarrow 8F(4) + 16 F'(4) = 64 - 2F'(4)$

$F'(4) = \frac{64}{18} = \frac{32}{9}$

8. (A) $\int_{-1}^1 \max\{|x|, x^2, x^4\}\,dx = 2\left(\frac{1}{2}\times 1 \times 1\right) = 1$



(B) $\int_{-a}^a f(x)\,dx = |k| \int_0^a (f(x) + f(-x))\,dx$

$\Rightarrow 2\int_0^a f(x)\,dx = |k| \left(\int_0^a (f(x)\,dx + \int_0^a f(-x)\,dx\right)$

(D) $\lim_{x\to 0} \frac{\sin^2 4x + x^2}{3x^2}$

$= \lim_{x\to 0} \frac{16\left(\frac{\sin 4x}{4x}\right)^2 x^2 + x^2}{x^3} = \frac{17}{3}$

DPP NO. - 71

1. $I = \int_2^4 \left(\log_x 2 - \frac{(\log_x 2)^2}{\ell n2}\right) dx$

$= \int_2^4 \left(\frac{\ell n2}{\ell nx} - \frac{1}{\ell n2}\frac{(\ell n2)^2}{(\ell nx)^2}\right) dx$

put $\ell n\ x = t$

$\therefore x = e^t \quad \therefore dx = e^t\,dt$

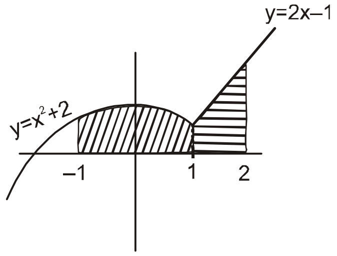
$= \ell n2 \int_{\ell n2}^{\ell n4} \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt = \ell n2 \left\{\frac{e^t}{t}\right\}_{\ell n2}^{\ell n4}$

$= \ell n2 \left\{\frac{4}{\ell n4} - \frac{2}{\ell n2}\right\} = 0$

2. $I = \int_1^{\sqrt2}(1-1)\,dx + \int_{\sqrt2}^{\sqrt2}(2-1)\,dx + \int_{\sqrt3}^{2}(3-1)\,dx$

$= 4 - \sqrt3 - \sqrt2$

3. Area $= \int_{-1}^{1}(2-x^2)dx + \int_1^2(2x-1)dx = \frac{16}{3}$

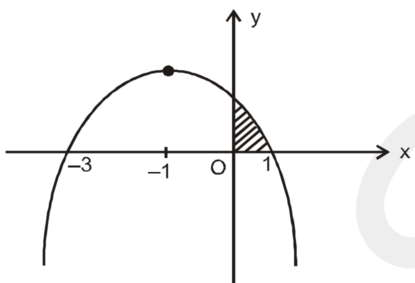


Figure

4. $0 < y < 3 - 2x - x^2$, $x > 0$

$y = 3 - 2x - x^2 \Rightarrow (x + 1)^2 = -(y - 2)$

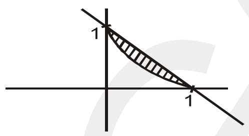
vertex $(-1, 2)$ at $y = 0$, $x = -3$, $x = 1$



Figure

Area $= \int_0^1 (3-2x-x^2)\,dx$ **Ans.**

5.



Figure

Area $= \int_0^1 \left((1-x)-(1-\sqrt{x})^2\right)dx = \frac{4}{3} - 1 = \frac{1}{3}$

6. put $x = \tan\theta$ $dx = \sec^2\theta d\theta$

$\therefore I = \int_{-\pi/6}^{\pi/6}\left(\frac{\cos^{-1}(\sin 2\theta)+\tan^{-1}(\tan 2\theta)}{e^{\tan\theta}+1}\right)\sec^2\theta d\theta$

$I = \int_{-\pi/6}^{\pi/6}\left(\frac{\left(\frac{\pi}{2}-2\theta\right)+2\theta}{e^{\tan\theta}+1}\right)\sec^2\theta d\theta$

$= \frac{\pi}{2}\int_{-\pi/6}^{\pi/6}\left(\frac{\sec^2\theta\, d\theta}{e^{\tan\theta}+1}\right)$

$I = \frac{\pi}{2}\int_{-\pi/6}^{\pi/6}\left(\frac{\sec^2\theta\, d\theta}{e^{-\tan\theta}+1}\right)$

$\therefore 2I = \frac{\pi}{2}\int_{-\pi/6}^{\pi/6}\frac{\sec^2\theta(e^{\tan\theta}+1)\, d\theta}{(e^{\tan\theta}+1)}$

$= \frac{\pi}{2}.2\int_0^{\pi/6}\sec^2\theta\, d\theta$

$2I = \pi\frac{1}{\sqrt3}$ $\therefore$ $I = \frac{\pi}{2\sqrt3}$

7. Let $I_1 = \int_0^\infty \frac{dx}{x^2+2x\cos\theta+\sin^2\theta+\cos^2\theta}$

$= \int_0^\infty \frac{dx}{(x+\cos\theta)^2+\sin^2\theta} = \frac{1}{\sin\theta}\left[\tan^{-1}\frac{x+\cos\theta}{\sin\theta}\right]_0^\infty$

$= \frac{1}{\sin\theta}\left[\frac{\pi}{2}-\left(\frac{\pi}{2}-\theta\right)\right] = \frac{\theta}{\sin\theta}$

Again Let $I_2 = 2\int_0^1 \frac{dx}{(x+\cos\theta)^2+\sin^2\theta}$

$= 2.\frac{1}{\sin\theta}\left[\tan^{-1}\left(\frac{x+\cos\theta}{\sin\theta}\right)\right]_0^1 = \frac{2}{\sin\theta}\cdot\frac{\theta}{2} = \frac{\theta}{\sin\theta}$

$\therefore$ $I_1 = I_2$ Hence Proved.

8. **(A)** $\lim_{x\to1}\frac{(f(x^2)-x^2)2x}{2(x-1)}$

$= \lim_{x\to1}\frac{(f'(x^2)2x-2x)2x+(f(x^2)-x^2)2}{2}$

$= \frac{(2f'(1)-2)2+2(f(1)-1)}{2}$

$= (2(3) - 2) + (1 - 1)$

$= 4$

(B) $\int_0^1(3ax^2+2bx+c)dx + \int_1^3(3ax^2+2bx+c)dx$

$= \int_1^3(3ax^2+2bx+c)dx$

$\Rightarrow \int_0^a(3ax^2+2bx+c)dx = 0$

$\Rightarrow \left(ax^3+bx^2+cx\right)_0^1 = 0$

$\Rightarrow a + b + c = 0$

(D) $f'(x) = 3x^2 + 2(a + 2)x + 3a \geq 0$ (for monotonic)

$\Rightarrow$ $D \leq 0$

$\Rightarrow$ $4(a + 2)^2 - 4(3)(3a) \leq 0$

$\Rightarrow$ $(a + 2)^2 - 9a \leq 0$

$\Rightarrow$ $a^2 - 5a + 4 \leq 0$

$(a-1)(a-4) \le 0$

$a \in [1, 4]$

integral value = 1, 2, 3, 4 = four values.

DPP NO. - 72

2. $\displaystyle\int_0^{\pi/4} \tan^{-1}\left(\frac{2}{2\sec^2\theta - 2\tan\theta}\right)\sec^2\theta\, d\theta$

put $\tan\theta = x$

$\therefore \displaystyle\int_0^{1} \tan^{-1}\left(\frac{1}{1+x^2-x}\right)dx$

$= \displaystyle\int_0^{1} \tan^{1}\left(\frac{x-(x-1)}{1+x(x-1)}\right)dx$

$= \displaystyle\int_0^{1} \tan^{-1}x - \tan^{-1}(x-1)\,dx$

$= \displaystyle\int_0^{1} \tan^{-1}x\, dx - \int_0^{1} \tan^{-1}(x-1)\,dx$

put $x - 1 = -k$ in I_2

$= \displaystyle\int_0^{1} \tan^{-1}x\,dx + \int_1^{0} \tan^{-1}k(-dk)$

$= 2\displaystyle\int_0^{1} \tan^{-1}x\,dx = 2\alpha$

3. $y = 1 + 4x - x^2 \Rightarrow (x-2)^2 = -(y-5)$

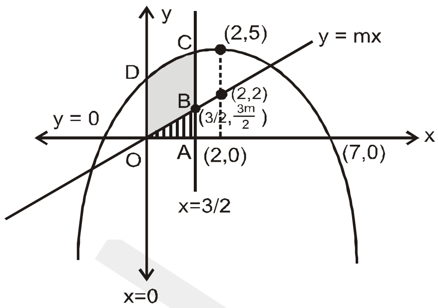
vertex (2, 5)

Area OACD

$= \displaystyle\int_0^{3/2} (1+4x-x^2)\,dx = \left(x + \frac{4x^2}{2} - \frac{x^3}{3}\right)_0^{3/2}$

$= \dfrac{39}{8}$ ......(1)

$\Delta OAB = \dfrac{1}{2}\times\dfrac{3}{2}\times\dfrac{3}{2}\times m = \dfrac{9}{8}m$ ......(2)



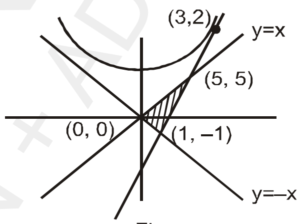
Figure

From (1) and (2)

$\dfrac{9}{8}m = \dfrac{1}{2}\left(\dfrac{39}{8}\right) \Rightarrow m = \dfrac{39}{2\times 9} = \dfrac{13}{6}$

4. $\left.\dfrac{dy}{dx}\right|_{(3,2)} = \dfrac{3}{2}$. Tangent $y = \dfrac{3x}{2} - \dfrac{5}{2}$

$y = x,\quad y = \dfrac{3x}{2} - \dfrac{5}{2} \Rightarrow (5, 5)$

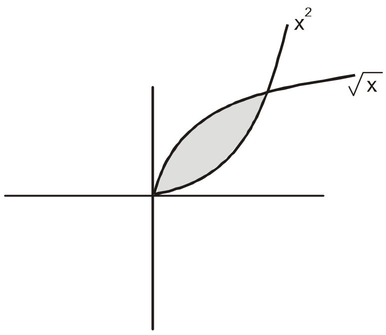


Figure

$y = x,\quad y = \dfrac{3x}{2} - \dfrac{5}{2} \Rightarrow (1, -1)$

closed figure formed is right angled triangle. Its area is $\dfrac{1}{2}(\sqrt{2})(5\sqrt{2}) = 5$

5. $\left\{(x,y); x^2 \le y \le \sqrt{x}\right\} = \displaystyle\int_0^{1}\left(\sqrt{x} - x^2\right)dx$



$= \left.\dfrac{x^{3/2}}{3/2} - \dfrac{x^3}{3}\right|_0^1 = \dfrac{2}{3} - \dfrac{1}{3} = \dfrac{1}{3}$

9. using f (a+b–x) on for f(x) we get

$$I = \int_{-(\pi/4)^{1/3}}^{(\pi/4)^{1/3}} \frac{x^2 . e^{x^7}}{(1+\sin^2 x^3)(1+e^{x^7})} dx$$

$$\therefore \quad 2I = \int_{-(\pi/4)^{1/3}}^{(\pi/4)^{1/3}} \frac{x^2 . dx}{(1+\sin^2 x^3)}$$

put $x^3 = t \quad \therefore \ x^2 dx = \frac{dt}{3}$

$$2I = \frac{1}{3}\int_{-\pi/4}^{\pi/4} \frac{dt}{(1+\sin^2 t)}$$

$$I = \frac{1}{6} 2 \int_0^{\pi/4} \frac{dt}{(1+\sin^2 t)}$$

$$= \frac{1}{3}\int_0^{\pi/4} \frac{\sec^2 t\, dt}{(\sec^2 t + \tan^2 t)}$$

$$= \frac{1}{3}\int_0^{\pi/4} \frac{\sec^2 t\, dt}{1+2\tan^2 t}$$

put $\tan t = k$ we get $\frac{1}{3}\int_0^1 \frac{dk}{1+2k^2}$

$$= \frac{1}{6}\int_0^1 \frac{dk}{\left(\frac{1}{\sqrt{2}}\right)^2 + k^2}$$

$$= \frac{\sqrt{2}}{6} \tan^{-1}\left(k\sqrt{2}\right)_0^1 = \frac{1}{3\sqrt{2}} \tan^{-1}\sqrt{2} \quad \textbf{Ans.}$$

DPP NO. - 73

1. $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$

$(\sin y + y\cos y)dy = x(2\log x + 1)dx$

$\Rightarrow \ -\cos y + \{(y\sin y) + \cos y\}$

$$= 2\left\{\frac{x^2}{2}\ln x - \frac{1}{2}\int \frac{x}{1} dx\right\} + \frac{x^2}{2}$$

$\Rightarrow \ y \sin y = x^2 \log x$

2. $(1 + y^2)\, dx + x\, dy - 2e^{\tan^{-1} y} dy = 0$

$$e^{\tan^{-1} y} dx + x \frac{e^{\tan^{-1} y}}{1+y^2} dy - \frac{2e^{2\tan^{-1} y}}{1+y^2} dy = 0$$

$d(xe^{\tan^{-1} y}) - e^{2\tan^{-1} y}\, d(2\tan^{-1} y) = 0$

$\therefore \ xe^{\tan^{-1} y} - e^{2\tan^{-1} y} = k$

i.e. $xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$

3. $\because \ \frac{dy}{dx} - \frac{\tan(1/x)}{x^2} y = -\frac{\sec(1/x)}{x^2}$ ......(1)

$\because \ I.F. = e^{-\int \frac{\tan(1/x)}{x^2} dx}$

$= e^{\ln \sec(1/x)} = \sec(1/x)$

$\therefore \ y \sec(1/x) = -\int \frac{\sec^2(1/x)}{x^2} dx$

$\Rightarrow \ y \sec(1/x) = \tan(1/x) + c$ ......(2)

If $x \to \infty$; $y \to -1$

$\Rightarrow \ (-1)(1) = 0 + c \Rightarrow c = -1$ put in (2)

$y = \sin(1/x) - \cos(1/x)$

4. $(x^2 + y^3)(2x^2\, dx + 3y\, dy) = 6\,(2x\, dx + 3y^2\, dy)$

$\Rightarrow \ 2x^2\, dx + 3y\, dy = 6 \frac{(2x dx + 3y^2 dy)}{(x^2 + y^3)}$

$\Rightarrow \ \int 2x^2 dx + \int 3y\, dy = 6 \frac{d(x^2 + y^2)}{(x^2 + y^3)}$

$\Rightarrow \ \frac{2}{3}x^3 + \frac{3}{2}y^2 = 6 \ln(x^2 + y^3) + c$

5. $(2x\, \ell n\, y)\, dx + \left(\frac{x^2}{y} + 3y^2\right) dy = 0$

$\Rightarrow \ d((\ell n\, y)\, x^2) + 3y^2\, dy = 0$

$\Rightarrow \ (\ell n\, y)\, x^2 + y^3 = c$

6. $\frac{dy}{dx} = \frac{1}{x\cos y + 2\sin y \cos y}$

$\therefore \ \frac{dx}{dy} = x \cos y + 2 \sin y \cos y$

$$\frac{dx}{dy} + (-\cos y)\, x = 2 \sin y \cos y$$

$\therefore$ I.F. $= e^{-\int \cos y\, dy} = e^{-\sin y}$

$\therefore$ The solution is

$x.\, e^{-\sin y} = 2\int e^{-\sin y}\,.\, \sin y \cos y\, dy$

$= -2 \sin y\, e^{-\sin y} - 2 \int(-e^{-\sin y}) \cos y\, dx$

$= -2 \sin y\, e^{-\sin y} + 2 \int e^{-\sin y} \cos y\, dy$

$= -2 \sin y\, e^{-\sin y} - 2\, e^{-\sin y} + c$

i.e. $x = -2 \sin y - 2 + c\, e^{\sin y} = ce^{\sin y} - 2(1 + \sin y)$

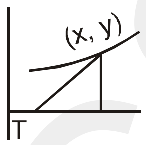
$\therefore$ $k = 2$

7. $Y - y = \frac{dy}{dx}(X - x)$

$\therefore$ coordinates of T are $\left(x - y\frac{dx}{dy}, 0\right)$

$\therefore$ sum of subtangent and abscissa $\equiv y\frac{dx}{dy} + x = a$

$\therefore$ $y\frac{dx}{dy} = a - x$



i.e. $\frac{dx}{a-x} = \frac{dy}{y}$

$\therefore$ $cy = \frac{1}{a-x}$

$\therefore$ $c = -\frac{1}{a^2}$

$\therefore$ equation of the curve is $(x - a)\, y = a^2$

8. $I = \int_4^{10} \frac{[(x-14)^2]}{[x^2]+[(x-14)^2]}\, dx$

$\Rightarrow 2I = \int_4^{10} dx = 6 \quad \Rightarrow \quad I = 3$

DPP NO. - 74

1. Total – (No 6 present)
$= 9 \times 10 \times 10 - 8 \times 9 \times 9$
$= 900 - 648 = 252$

2. Prob. $= \frac{{}^{8}C_6}{{}^{13}C_6}$

3. Let $a = 2x + 1$, $b = 2y + 1$, $c = 2z + 1$
$a + b + c = 13, \quad x, y, z \in w$
$2x + 2y + 2z + 3 = 13$
$x + y + z = 5$
so required solution $= {}^{5+3-1}C_{3-1} = {}^{7}C_2$

4. Number of triangles $= {}^{18}C_3 - 3 \cdot ({}^{7}C_3)$

$= \frac{18.17.16}{3.2.1} - 3 \cdot \frac{76.5}{3.2.1} = 816 - 105 = 711$

Total way $= {}^{8}C_3$, Probability $= \frac{711}{{}^{18}C_3}$

5. $(x - 15)(x + 2) \le 0$
$-2 \le x \le 15$
x is natural number
$x = 1, 2, 3, \ldots.. 15$

probability $= \frac{15}{100} = \frac{3}{20}$

6. Probability $= \frac{360}{9 \times 10 \times 10}$

7. Total case = 90
Now divisible by 6 = 15
divisible by 8 = 11
common number = 3
Favarable case = 15 + 11 – 3 = 23

Probability $= \frac{23}{90}$

9. (i) $\left(\frac{1}{2}\right)^{20}\left({}^{20}C_{11} + {}^{20}C_{12} + \ldots\ldots + {}^{20}C_{20}\right)$

(ii) $\frac{26}{52}.\frac{26}{51} + \frac{26}{52}.\frac{26}{51}$

10. (i) $\frac{{}^{6}C_1.\,{}^{10}C_1.\,{}^{4}C_1}{{}^{12}C_4}$ (ii) $\frac{{}^{6}C_2}{{}^{12}C_4}$

DPP NO. - 75

1. 4 cases (4, 5), (6, 3), (3, 6), (5, 4)

32 unfavarable cases

$$P = \frac{4\times4\times32}{36\times36\times36} + \frac{4\times32\times4}{36\times36\times36} + \frac{32\times4\times4}{36\times36\times36}$$

$$= \frac{8}{243}$$

2. $(A'\cap B') = 1 - P(A\cup B) = 1 - (P(A) + P(B) - P(A\cap B))$

$= 1 - (0.59 + 0.30 - 0.21) = 1 - (0.59 + 0.09)$

$= 1 - 0.68 = 0.32$

3. Let two non-negative number x and y

$x = 5a + \alpha$, $y = 5b + \beta$

where $0 \le \alpha \le 4$ and $0 \le \beta \le 4$

Now $x^2 + y^2 = (5a + \alpha)^2 + (5b + \beta)^2$

$= 25(a^2 + b^2) + 10(a\alpha + b\beta) + \alpha^2 + \beta^2$

$\Rightarrow$ $x^2 + y^2$ is divisible by 5 if and only if $\alpha^2 + \beta^2$ is divisible by 5

total way choosing α and $\beta = 5 \times 5 = 25$

and $\alpha^2 + \beta^2$ will be divisible by 5 if

$(\alpha, \beta) \in \{(0, 0), (1, 2), (1, 3), (2, 1), (2, 4), (3, 1), (3, 4), (4, 2), (4, 3)\}$

Favarable cases = 9

Probability = $\frac{9}{25}$

4. Probability $= P(A)\left[P(\bar{B})P(B) + P(\bar{B})P(\bar{B})P(B) + ...\right] +$

$P(\bar{A}).P(A).\left(P(\bar{B})P(\bar{B})P(B) + P(\bar{B})^3 P(B) + ...\right) + ...$

$= P(A)P(\bar{B}) + P(A)P(\bar{B})^2 P(\bar{A}) + ...$

$$= \frac{P(A)P(\bar{B})}{1 - P(\bar{A})P(\bar{B})}$$

$$= \frac{\frac{3}{5}\times\frac{2}{7}}{1 - \frac{2}{5}\times\frac{2}{7}} = \frac{6}{31}$$

5.

R_1	R_2	R_3	R_4	R_5	Total ways
2	1	1	1	1	${}^3C_2 \times {}^3C_1 \times {}^3C_1 = 27$
1	1	2	1	1	${}^3C_1 \times {}^3C_2 \times {}^3C_1 = 27$
1	1	1	1	2	${}^3C_1 \times {}^3C_1 \times {}^3C_2 = 27$

ways $= 81\left(\frac{6!}{2!\,3!}\right) = 4860$

6. Total number of combinations of numbers on the cube and the tetrahedon = 6 × 4 = 24

Favourable number of ways of getting a sum not less than 5

= sum of coefficients of x^6 , x^7,.........x^{10} inthe product

$= (x + x^2 + x^3 + x^4 + x^5 + x^6)(x + x^2 + x^3 + x^4)$

$= (x^2 + 2x^3 + 3x^4 + 4x^5 + 4x^6 + 4x^7 + 3x^8 + 2x^9 + x^{10})$

$= 4 + 4 + 4 + 3 + 2 + 1 = 18$

$\therefore$ Required probability $= \frac{18}{6\times4} = \frac{3}{4}$

7. $$P(B/A\cup B^c) = \frac{P[B\cap(A\cup B^c)]}{P(A\cup B^c)}$$

$P = P(A\cap B) = P(A) - P(A\cap B^c) = 0.7 - 0.5 = 0.2$

Again $P(A\cup B^c) = P(A) + P(B^c) - P(A\cap B^c)$

$= 0.7 + (1 - 0.4) - 0.5 = 0.8$

$\therefore$ $P(B/A\cup B^c) = \frac{0.2}{0.8} = 0.25$

8. $$P(E_1) = P(E_2) = \frac{\frac{10!}{2!}}{\frac{11!}{2!\,2!}} = \frac{2}{11}$$

$$P(E_1\cap E_2) = \frac{9!}{\frac{11!}{2!\,2!}} = \frac{2}{55}$$

$P(E_1\cup E_2) = P(E_1) + P(E_2) - P(E_1\cap E_2)$

$$= \frac{2}{11} + \frac{2}{11} - \frac{2}{55} = \frac{18}{55}$$

$$P(E_1/E_2) = \frac{P(E_1\cap E_2)}{P(E_2)} = \frac{\frac{2}{55}}{\frac{2}{11}} = \frac{1}{5}$$

9. (i) ${}^6C_4 \times 4! = 15 \times 2\,4 = 360$

and ${}^2C_1.{}^5C_2.\frac{4!}{2!} = 20 \times 12 = 240$

and ${}^2C_2.\frac{4!}{2!2!} = 6$

Total = 606

(ii) when both S appear in the 4 letter word ways

$= {}^5C_2 . \frac{4!}{2!} + {}^2C_2 . \frac{4!}{2!2!}$

$= 120 + 6 = 126$

$\text{Probability} = \frac{126}{606} = \frac{21}{101}$

$\text{(iii) Probability} = \frac{{}^6C_4 . 4!}{606}$

10. $(73)^{75^{64^{76}}} = (73)^{(75)^{\text{multiple of 4}}} = (73)$

DPP NO. - 76

1. $P(A) = \frac{11}{36}$, cases (4, 6)(6, 4)

$P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

$\therefore \; P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1/18}{11/36} = \frac{2}{11}$

5. Total function = $2^4 = 16$

Onto function = 16 – 2 = 14

One one function = 0

(i) Probability = $\frac{14}{16} = \frac{7}{8}$

(ii) Probability = 0