MATHEMATICS DPP

DAILY PRACTICE PROBLEMS

DPP No. 69

Total Marks : 26

Max. Time : 28 min.

Topics : Permutation & Combination, Binomial Theorem

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.,1,2,3	(3 marks, 3 min.)	[9, 9]
Multiple choice objective (no negative marking) Q.4	(5 marks, 4 min.)	[5, 4]
Subjective Questions (no negative marking) Q. 5,6,7	(4 marks, 5 min.)	[12, 15]

1. The sum of all the four digit numbers that can be formed using the digits 1, 2, 3, 4 if repetition of digits is allowed, is

(A) 399996 (B) 388840 (C) 711040 (D) none of these

2. All possible three digit even numbers which can be formed with the condition that if 5 is one of the digit, then 7 is the next digit, is

(A) 5 (B) 325 (C) 345 (D) 365

3. Different words are formed by arranging the letters of the word "SUCCESS", find

(i) The number of words in which C are together but S's are seperated, is

(A) 120 (B) 96 (C) 24 (D) 420

(ii) The number of words in which no two C's and no two S's are together is

(A) 120 (B) 96 (C) 24 (D) 180

(iii) The number of words in which the consonants appear in alphabetic order is

(A) 42 (B) 40 (C) 420 (D) 280

4. If $(1 + 2x + 3x^2)^{10} = a_0 + a_1x + a_2x^2 + ... + a_{20}x^{20}$, then :

(A) $a_1 = 20$ (B) $a_2 = 210$

(C) $a_4 = 8085$ (D) $a_{20} = 2^2 . 3^7 . 7$

5. How many 10 digit numbers can be made with odd digits so that no two consecutive digits are same.

6. If repetitions are not permitted

(1) How many 3 digit numbers can be formed from the six digit 2, 3, 5, 6, 7 & 9 ?

(2) How many of these are less than 400 ?

(3) How many are even ?

(4) How many are odd ?

(5) How many are multiples of 5 ?

7. Consider the word W = "COMMISSIONER". Find

(i) Number of 5 lettered word containing two vowels and three consonants.

(ii) Number of ways in which all the letters of the word W can be arranged if alike letters are together but seperated from the other alike letters.

(iii) Number of ways in which letters of the word W can be arranged without changing order of alike letters.

MATHEMATICS

DPP
DAILY PRACTICE PROBLEMS

DPP No. 70

Total Marks : 28

Max. Time : 30 min.

Topics : Permutation & Combination, Binomial Theorem

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3,4	(3 marks, 3 min.)	[12, 12]
Subjective Questions (no negative marking) Q.5,6	(4 marks, 5 min.)	[8, 10]
Match the Following (no negative marking) Q.7	(8 marks, 8 min.)	[8, 8]

1. In the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n$, $n \in N$, if the sum of the coefficients of x^5 and x^{10} is 0, then n is :

(A) 25 (B) 20 (C) 15 (D) None of these

2. The sum of the coefficients of all the integral powers of x in the expansion of $(1+2\sqrt{x})^{40}$ is :

(A) $3^{40} + 1$ (B) $3^{40} - 1$ (C) $\frac{1}{2}(3^{40} - 1)$ (D) $\frac{1}{2}(3^{40} + 1)$

3. The coefficient of the term independent of x in the expansion of $\left(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}}\right)^{10}$ is :

(A) 70 (B) 112 (C) 105 (D) 210

4. A number of different seven digit numbers that can be written using only three digits 1, 2 & 3 under the condition that the digit 2 occurs exactly twice in each number, is

(A) 672 (B) 640 (C) 512 (D) None of these

5. There are 720 permutations of the digits 1, 2, 3, 4, 5, 6 suppose these permutations are arranged from smallest to largest numerical values beginning from 123456 and ending with 654321.

(a) What number falls on the 124^{th} position

(b) What is the position of the number 321546

6. How many different words can be formed out of the letters of the word 'ALLAHABAD'? In how many of them the vowels occupy the even positions?

7. Match the column

[Note : - Repetition is not allowed]

	Column- I		Column- II
(A)	Number of 3 - digit numbers which are even	(p)	7200
(B)	Number of 4 - digit numbers which are odd	(q)	328
(C)	Number of 4 - digit numbers which are multiples of 5	(r)	2240
(D)	Number of 7 - digit numbers where even digits occupies only even places	(s)	952

DPP 61 TO 70 (ANSWER KEY)

DPP NO. - 61

1. n = 12 **2.** 1 – f, if n is even and f, if n is odd

4. (A) **5.** (B) **6.** (B) **7.** (B)

13. (C) **14.** (A)(B)(C) **15.** (B)

DPP NO. - 62

1. (C) **2** (i) T_4 (ii) T_5, T_6 (iii) T_5 (iv) T_6

3. (C) **4.** (C) **5.** (B) **6.** (D)

7. (A)(C) **8.** (C)(D) **9.** (D)

DPP NO. - 63

1. $\frac{15015}{16}$ **3.** 20 **4.** (i) 280 (ii) 2^5 **5.** (A)

6. (C) **7.** (A) **8.** (A) **9.** 15e

DPP NO. - 64

1. (B) **2.** (B) **3.** (C) **4.** (B) **5.** (C) **6.** (C)

7. (B) **8.** $\frac{1}{4} + \log_e \frac{4}{5}$ **9.** (D) **10.** 60, 108

DPP NO. - 65

1. (C) **2.** (D) **3.** (D) **4.** (C)

5. (B) **6.** (C) **7.** (A) **8.** (D)

9. (C) **10.** (A) **11.** (C) **12.** (B)

13. (A, B, C, D) **14.** (A, C, D) **15.** (A, C)

DPP NO. - 66

1. (A) **2.** (D) **3.** (D) **4.** (B)

5. (B, C) **6.** 430 **7.** 25 **8.** 468000

DPP NO. - 67

1. (B) **2.** (3,4) **3.** (B, D) **4.** (B)

5. (C) **6.** (B) **7.** (A) **8.** (D)

DPP NO. - 68

1. (C) **2.** (D) **3.** (B) **4.** (C)

5. (D) **6.** (D) **7.** (B) **8.** (D)

DPP NO. - 69

1. (C) **2.** (D) **3.** **(i)** (C) **(ii)** (B) **(iii)** (A)

4. (A, B, C) **5.** (A, B) **6.** 5.4^9

7. (1) 120 (2) 40 (3) 40 (4) 80 (5) 20

8. (i) 6720 (ii) 2880 (iii) ${}^{12}P_4$

DPP NO. - 70

1. (C) **2.** (D) **3.** (D) **4.** (A)

5. (a) 213564 (b) 267 **6.** 7560, 60

7. (A) $\rightarrow$ (q), (B) $\rightarrow$ (r), (C) $\rightarrow$ (s), (D) $\rightarrow$ (p)

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CLASS : XI (MATHEMATICS)

DPP

DAILY PRACTICE PROBLEM

Solutions

DPP-61 to 70

DPP 61 : **Binomial Theorem**
DPP 62 : **Binomial Theorem**
DPP 63 : **Binomial Theorem**
DPP 64 : **Binomial Theorem, Permutation & Combination**
DPP 65 : **Binomial Theorem**
DPP 66 : **Circle, Permutation & Combination**
DPP 67 : **Circle, Permutation & Combination, Binomial Theorem**
DPP 68 : **Binomial Theorem**
DPP 69 : **Permutation & Combination, Binomial Theorem**
DPP 70 : **Permutation & Combination, Binomial Theorem**

DPP 61 TO 70 (HINTS AND SOLUTIONS)

DPP NO. - 61

1. For T_9 to be the numerically greatest term,

$$r = \left[\frac{n+1}{1+\left|\frac{x}{a}\right|}\right] = \left[\frac{n+1}{1+\left|\frac{1}{2}\right|}\right] = 8$$

$\Rightarrow 8 < \frac{2(n+1)}{3} < 9 \Rightarrow 11 < n < 12.5 \Rightarrow n = 12$

2. $p + f = (3\sqrt{3} + 5)^n = {}^nC_0(3\sqrt{3})^n 5^0 + {}^nC_1(3\sqrt{3})^{n-1} 5^1 + \ldots..$

$f' = (3\sqrt{3} - 5)^n = {}^nC_0 (3\sqrt{3})^n 5^0 - {}^nC_1(3\sqrt{3})^{n-1} 5^1 + \ldots...$

$p + f + f' = 2 [{}^nC_0(3\sqrt{3})^n + {}^nC_2 (3\sqrt{3})^{n-2}5^2 + \ldots...]$

$\Rightarrow$ p + f + f′ = even integer (if n is even)

$\Rightarrow f + f' = 1 \Rightarrow f' = 1 - f$

$p + f - f' = 2 [{}^nC_1(3\sqrt{3})^{n-1}(5) + {}^nC_3(3\sqrt{3})^{n-3}5^3 + \ldots.]$

(if n is odd)

$\Rightarrow f - f' = 0 \qquad \Rightarrow f' = f$

3. $I + f = (6\sqrt{6} + 14)^{2n+1}$

$= {}^{2n+1}C_0(6\sqrt{6})^{2n+1} + {}^{2n+1}C_1(6\sqrt{6})^{2n} . 14 + \ldots..$

$f' = (6\sqrt{6} - 14)^{2n+1} = {}^{2n+1}C_0(6\sqrt{6})^{2n+1} - {}^{2n+1}C_1(6\sqrt{6})^{2n} . 14 + \ldots..$

$I + f - f' = 2 [{}^{2n+1}C_1(6\sqrt{6})^{2n} . 14 + \ldots...]$

$\Rightarrow$ I + f – f′ = Even Integer

$\Rightarrow$ I = Even integer

$f - f' = 0$

4. For numerically greatest term

$$r = \left[\frac{n+1}{1+\left|\frac{x}{a}\right|}\right] = \left[\frac{9+1}{1+\left|\frac{4}{9}\right|}\right] \Rightarrow r = 6$$

Numerically greatest term $T_{r+1} = {}^9C_6(2)^3\left(\frac{9}{2}\right)^6$

5. For numerically greatest term

$$r = \left[\frac{n+1}{1+\left|\frac{x}{a}\right|}\right] = \left[\frac{34+1}{1+\left|\frac{6}{10}\right|}\right] \Rightarrow r = 21.$$

6. T_{22} is the numerically greatest term.

$(\sqrt{2} + 1)^6 = I + f$

$(\sqrt{2} - 1)^6 = f'$

$2[{}^6C_0 + {}^6C_2 . 2 + {}^6C_4 (2)^2 + \ldots\ldots..] = I + f + f'$

$f + f' = 1$ or $f' = 1 - f$

$I = 2 [{}^6C_0 + {}^6C_2.2 + {}^6C_4.4 + {}^6C_6.8] - 1$

$I = 2 [1 + 30 + 60 + 8] - 1 = 197$

7. $(5+2\sqrt{6})^n = p + f$

$(5-2\sqrt{6})^n = f' \Rightarrow 0 < f + f' < 2$

p + f + f ' = 2 [integer]

so f + f ' = integer = 1

$\because n \in N$

$(f - 1)(p + f) = -f' (p + f) = -(+1)^n = -1$

8. $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \ldots\ldots...$

$= 1 + 2 + 3 + \ldots.+ (n - 1) + n = \frac{n(n+1)}{2}$

9. $(C_0 + C_1)(C_1 + C_2)\ldots\ldots\ldots.(C_{n-1} + C_n)$

$= ({}^{n+1}C_1)({}^{n+1}C_2)\ldots\ldots..({}^{n+1}C_n)$

$= ((n + 1).C_0)\left(\frac{n+1}{2}.C_1\right)\ldots\ldots.\left(\frac{n+1}{n}.C_{n-1}\right)$

$= \frac{(n+1)^n}{n!} C_0C_1\ldots\ldots..C_{n-1}$

10. $(1 + x)^n = C_0 + C_1x + \ldots\ldots\ldots..+ C_nx^n$

$x(1 + x)^n = C_0x + C_1x^2 + \ldots\ldots\ldots..+ C_nx^{n+1}$

Differentiating w.r.t. x

$(1 + x)^n + n x (1 + x)^{n-1} = C_0 + 2C_1x + \ldots... + (n + 1) C_nx^n$

Putting x = – 1

$C_0 - 2C_1 + \ldots\ldots..+ (-1)^n (n + 1) C_n = 0$

11. $(1 + x)^n = {}^nC_0 + {}^nC_1 x + \ldots\ldots\ldots..+ {}^nC_nx^n$

$$\int_0^2 (1+x)^n dx = C_0x + \frac{C_1x^2}{2} + \ldots\ldots. + \left.\frac{C_nx^{n+1}}{n+1}\right|_0^2$$

$$\frac{3^{n+1}-1}{n+1} = 2.C_0 + \frac{2^2C_1}{2} + \ldots... + \frac{2^{n+1}C_n}{n+1}$$

12. ${}^nC_r + {}^{n-1}C_r + \ldots... + {}^rC_r$

= Co-efficient of x^r in $(1 + x)^n + (1 + x)^{n-1} + \ldots.. + (1 + x)^r$

= Co-efficient of x^r in $(1 + x)^r \left[\frac{(1+x)^{n-r+1}-1}{x}\right]$

= Co-efficient of x^{r+1} in $(1 + x)^{n+1} = {}^{n+1}C_{r+1}$

13. $\left(\sum_{r=0}^{10} {}^{10}C_r\right)\left(\sum_{k=0}^{10}(-1)^k \frac{{}^{10}C_k}{2^k}\right)$

$= ({}^{10}C_0 + ... + {}^{10}C_{10})\left({}^{10}C_0 - \frac{{}^{10}C_1}{2} + \frac{{}^{10}C_2}{2^2} ...... + \frac{{}^{10}C_{10}}{2^{10}}\right)$

$= 2^{10} \times \left(1 - \frac{1}{2}\right)^{10} = 1$

14. For sum of co-efficient put x = 1
$a = 4^n$ & $b = 2^{2n}$ $\Rightarrow$ a = b

15. $\frac{{}^{11}C_0}{1} + \frac{{}^{11}C_1}{2} + \frac{{}^{11}C_2}{3} + ... + \frac{{}^{11}C_{10}}{11}$

$= \frac{1}{12}\left[\frac{12}{1}.{}^{11}C_0 + \frac{12}{2}.{}^{11}C_1 + \frac{12}{3}.{}^{11}C_2 + ... + \frac{12}{11}.{}^{11}C_{10}\right]$

$= \frac{1}{12}\left[{}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 + ... + {}^{12}C_{11}\right]$

$= \frac{1}{12}(2^{12} - 2) = \frac{2^{11} - 1}{6}$

DPP NO. - 62

1. $(x^{1/3} - x^{-1/2})^{15}$

$T_{r+1} = {}^{15}C_r\, x^{\left(\frac{15-r}{3}\right)} (-x^{-1/2})^r$

For constant term $\frac{15-r}{3} - \frac{r}{2} = 0$ $\Rightarrow$ r = 6

Co-efficient of $x^0 = {}^{15}C_6 = 5 \times 1001$ $\Rightarrow$ m = 1001

2. $(2x - 5)^6$

(i) Greatest binomial Co-efficient is of middle term = $T_{\frac{6}{2}+1} = T_4$

(ii) For greatest numerical term $r = \left[\frac{6+1}{1+\left|\frac{2}{5}\right|}\right]$

$= \left[\frac{35}{7}\right] = 5$

Since $\frac{n+1}{1+\left|\frac{x}{a}\right|}$ itself is an integer.

$\therefore$ T_5 and T_6 both terms have are greatest numerical value

(iii) The positive term of greatest numerical value is Algebraically greatest i.e. T_5.

(iv) The negative term of greatest numerical value is algebraically least i.e. T_6

3. $\int_0^1 (1-x)^n dx = \int_0^1 \left(C_0 - C_1x + C_2x^2 - C_3x^3 + ... + (-1)^n C_n x^n\right) dx$

$\Rightarrow \frac{1}{n+1} = \left[C_0 - \frac{C_1}{2} + \frac{C_2}{3} - .... + (-1)^n \frac{C_n}{n+1}\right]$

$\Rightarrow \frac{1}{3}\left(\frac{1}{n+1}\right)$

$= \frac{1}{3}\left[C_0 - \frac{C_1}{2} + \frac{C_2}{3} - .... + (-1)^n \frac{C_n}{n+1}\right]$

4. ${}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 = {}^{52}C_4$

5. ${}^{50}C_0 \times {}^{50}C_1 + {}^{50}C_1 \times {}^{50}C_2 + ... + {}^{50}C_{49} \times {}^{50}C_{50}$
$= {}^{50}C_0 \times {}^{50}C_{49} + {}^{50}C_1 \times {}^{50}C_{48} + ... + {}^{50}C_{49} \times {}^{50}C_0$
= co-eff. of x^{49} in $(1 + x)^{100}$ $= {}^{100}C_{49}$

6. Co-efficient of x^n in $(1 - x)^{-2} = {}^{2+n-1}C_1 = n + 1$

7. $(3x + 2)^{-1/2}$ has infinite expansion when $\left|\frac{3x}{2}\right| < 1$

$\Rightarrow x \in \left(-\frac{2}{3}, \frac{2}{3}\right)$

8. $(1 + x)^2 (1 - x)^{-2} = (1 + x^2 + 2x)(1 - x)^{-2}$
Co-efficient of $x^4 = {}^5C_4 + {}^3C_2 + 2\,{}^4C_3 = 16$

9. $(1 - x + 2x^2)^{12}$

General term = $\frac{12!}{r_1!\, r_2!\, r_3!} (1)^{r_1} (-x)^{r_2} (2x^2)^{r_3}$

$r_2 + 2r_3 = 4$ $\Rightarrow$ $r_3 = 0, r_2 = 4, r_1 = 8$
$r_3 = 1, r_2 = 2, r_1 = 9$
$r_3 = 2, r_2 = 0, r_1 = 10$

Co-efficient of $x^4 = \frac{12!}{4!\,8!} + \frac{12!}{2!\,10!}(2)^2 + \frac{12!}{2!\,9!} \times (2)$

$= {}^{12}C_8 + 4.\,{}^{12}C_{10} + 6.\,{}^{12}C_9$
$= {}^{12}C_3 + 3.\,{}^{13}C_3 + {}^{14}C_4$ (after solving)

10. (i) $(1 + x)^n = C_0 + C_1 x + ... + C_n x^n$
$(x + 1)^n = C_0x^n + C_1 x^{n-1} + ... + C_n$
$C_0C_3 + C_1C_4 + ...+ C_{n-3}C_n$
= Co-efficient of x^{n-3} in $(1 + x)^{2n} = {}^{2n}C_{n-3}$

(ii) $C_0C_r + C_1C_{r+1} + ...+ C_{n-r}C_n$
= co-efficient of x^{n-r} in $(1+x)^{2n}$
$= {}^{2n}C_{n-r}$

(iii) $(1 + x)^n = C_0 + C_1 x + ... + C_n x^n$
$(x - 1)^n = C_0x^n - C_1 x^{n-1} + ... + (-1)^n C_n$
$C_0^2 - C_1^2 + ... + (-1)^n C_n^2$
= co-efficient of x^n in $(x^2 - 1)^n = 0$ if n is odd
$= {}^nC_{n/2}(-1)^{n/2}$ if n is even

DPP NO. - 63

1. In the expansion of $(1 - 2x)^{-5/2}$

$T_{r+1} = \frac{\frac{5}{2}\left(\frac{5}{2}+1\right)\left(\frac{5}{2}+2\right)......\left(\frac{5}{2}+r-1\right)}{r!} \cdot (-1)^r (2)^r x^r$

$\therefore$ Coefficient of

$x^6 = \frac{\frac{5}{2}\left(\frac{5}{2}+1\right)\left(\frac{5}{2}+2\right)\left(\frac{5}{2}+3\right)\left(\frac{5}{2}+4\right)\left(\frac{5}{2}+5\right)}{6!}$

$= \dfrac{15015}{16}$

2. $\dfrac{\left(1+\frac{3}{4}x\right)^{-4}(16-3x)^{1/2}}{(8+x)^{2/3}} = \dfrac{(1-3x).4\left(1-\frac{3}{32}x\right)}{4\left(1+\frac{2x}{24}\right)}$

$= \left(1-3x-\dfrac{3}{32}x\right)\left(1-\dfrac{x}{12}\right)$

$= 1 - \dfrac{x}{12} - 3x - \dfrac{3}{32}x = 1 - \dfrac{305}{96}x$

3. Co-efficient of x^7 in $(1-2x+x^3)^5$

$= \dfrac{n!}{r_1!r_2!r_3!}(1)^{r_1}(-2x)^{r_2}(x^3)^{r_3}$

$= r_2 + 3r_3 = 7$ & $r_1, r_2, r_3 \le 5$

(i) $r_2 = 4, r_3 = 1, r_1 = 0$ (ii) $r_2 = 1, r_3 = 2, r_1 = 2$

$= \dfrac{5!}{4!\,1!}(2)^4 + \dfrac{5!}{2!\,2!\,1!} \times (-2)^1 = 20$

4. (i) $(bc + ca + ab)^8$

$\dfrac{8!}{r_1!r_2!r_3!}(bc)^{r_1}(ca)^{r_2}(ab)^{r_3}$

$\left.\begin{array}{l} r_2 + r_3 = 5 \\ r_1 + r_3 = 4 \\ r_2 + r_1 = 7 \end{array}\right\} \Rightarrow r_2 = 4,\ r_1 = 3,\ r_3 = 1$

or $\dfrac{8!}{4!3!1!} = 280$

(ii) $(9x^2 + x - 8)^6 = a_0 + a_1x + a_2x^2 + ... a_{12}x^{12}$

$2^6 = a_0 + a_1 + ... + a_{12}$ $(x = 1)$

$0 = a_0 - a_1 + ...+ a_{12}$ $(x = -1)$

$\Rightarrow a_1 + a_3 + ... + a_{11} = 2^5$

5. $(1 + x)^{10} = a_0 + a_1 + a_2x^2 + ... + a_{10}x^{10}$

Put $x = i$,

$(1 + i)^{10} = a_0 - a_2 + a_4 + ... + a_{10} + i\,(a_1 - a_3 + ... + a_9)$

$a_0 - a_2 + a_4 + ... + a_{10}$ = real part of $(1 + i)^{10}$

$= 2^5\cos 10\pi/4$

$a_1 - a_3 + ...$ = imaginary part of $(1 + i)^{10}$

$= 2^5 \sin 10\pi/4$ ....(2)

$(1)^2 + (2)^2 = 2^{10}$

6. n^{th} term of the given series is given by

$t_n = \dfrac{1+2+3+...n}{(n+1)!} = \dfrac{\frac{n(n+1)}{2}}{(n+1)!}$

$= \dfrac{1}{2}\dfrac{n}{n!} = \dfrac{1}{2} \cdot \dfrac{1}{(n-1)!}$

Sum $= \displaystyle\sum_{n=1}^{\infty} t_n = \sum_{n=1}^{\infty}\frac{1}{2}\frac{1}{(n-1)!}$

$= \dfrac{1}{2}\left[\dfrac{1}{0!}+\dfrac{1}{1!}+\dfrac{1}{2!}+....\text{to}\,\infty\right] = \dfrac{e}{2}$

7. As we know

$e^x = 1 + \dfrac{x}{1!}+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+\dfrac{x^4}{4!}+\dfrac{x^5}{5!}+\dfrac{x^6}{6!}+....$

$\therefore$ $e^{2x} =$

$\dfrac{(2x)}{1!}+\dfrac{(2x)^2}{2!}+\dfrac{(2x)^3}{3!}+\dfrac{(2x)^4}{4!}+\dfrac{(2x)^5}{5!}+\dfrac{(2x)^6}{6!}+....$

Hence coefficient of x^6 is $= \dfrac{2^6}{6!} = \dfrac{64}{720} = \dfrac{4}{45}$

8. Let $x = \dfrac{1}{2}$, then the sum of the given series

$= x - \dfrac{x^2}{2}+\dfrac{x^3}{3}-\dfrac{x^4}{4} + ...$ to ∞

$= \log_e(1 + x)$

$= \log_e\left(1+\dfrac{1}{2}\right) = \log_e\left(\dfrac{3}{2}\right)$

9. Given series is $\displaystyle\sum_{n=1}^{\infty}\frac{n^3}{(n-1)!} = \sum_{n=1}^{\infty}\frac{n^4}{n!}$

Now $n^4 = P_4(n) = a_0 + a_1n + a_2n(n-1)$
$+ a_3n(n-1)(n-2) + a_4n(n-1)(n-2)(n-3)$

Equating coefficients of n^4, we get

$a_4 = 1,\ a_0 = 0,\ 1 = a_1,\ 16 = a_0 + 2a_1 + 2a_2$

$81 = a_0 + 3a_1 + 6a_2 + 6a_3$

Solving these equations, we get

$a_0 = 0,\ a_1 = 1,\ a_2 = 7,\ a_3 = 6,\ a_4 = 1$

So $\displaystyle\sum_{n=1}^{\infty}\frac{n^4}{n!} = (1 + 7 + 6 + 1)e = 15e$

10. $y = \left(x-\dfrac{x^2}{2}+\dfrac{x^3}{2}-\dfrac{x^4}{2}+....\text{to}\,\infty\right) = \log_e(1 + x)$

$\Rightarrow e^y = (1 + x)$

$\Rightarrow x = (e^y - 1) = \left[\left(1+y+\dfrac{y^2}{2!}+\dfrac{y^3}{3!}....\right)-1\right]$

$\Rightarrow x = \left(y+\dfrac{y^2}{2!}+\dfrac{y^3}{3!}....\text{to}\,\infty\right)$

DPP NO. - 64

1. $(1 + x)^n = \displaystyle\sum_{r=0}^{n} a_r x^r = a_0 + a_1x + ...+ a_nx^n$

$b_r = 1 + \dfrac{a_r}{a_{r-1}} = 1+\dfrac{n-r+1}{r} = \dfrac{n+1}{r}$

$\displaystyle\prod_{n=1}^{n} b_r = b_1b_2........b_n = \frac{(n+1)^n}{1.2.3......n} = \frac{(101)^{100}}{100!}$

$\Rightarrow$ $n = 100$

2. ${}^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$

$r^2 = 3r$ or $r = 0, 3$

3. ${}^{18}C_{r-2} + 2.{}^{18}C_{r-1} + {}^{18}C_r \geq {}^{20}C_{13}$

or ${}^{19}C_{r-1} + {}^{19}C_r \geq {}^{20}C_{13}$

or ${}^{20}C_r \geq {}^{20}C_{13}$

$r = 7,8,9,10,11,12,13$

4. $S = \sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} 2^m$

$S = {}^{100}C_0(x-3)^{100} + {}^{100}C_1(x-3)^{99}.2 + ... + {}^{100}C_{100} . 2^{100}$

$S = (2 + (x-3))^{100} = (x-1)^{100}$

Co-efficient of $x^{52} = {}^{100}C_{52} = {}^{100}C_{48}$

5. $\frac{1}{1!(n-1)!} + \frac{1}{2!(n-2)!} + ... + \frac{1}{1!(n-1)!}$

$= \frac{1}{n!} [{}^nC_1 + {}^nC_2 + ... + {}^nC_{n-1}]$(multiply and divde by n!)

$= \frac{1}{n!} [2^n - 2] = \frac{2}{n!} (2^{n-1} - 1)$

6. $(1 + x)^{21} [1 + (1 + x) + ... (1 + x)^9] = (1 + x)^{21}$

$\left[\frac{(1+x)^{10}-1}{x}\right] = \frac{(1+x)^{31}-(1+x)^{21}}{x}$

Coefficient of $x^5 = {}^{31}C_6 - {}^{21}C_6$

7. $(x^4 - 1)^5 (x-1)^{-5}$

$= {}^5C_0 (x-1)^{-5} - {}^5C_1 x^4 (x-1)^{-5} + {}^5C_2 x^8 (x-1)^{-5}$

$= {}^5C_0 \times {}^{14}C_4 - {}^5C_1 \times {}^{10}C_6 + {}^5C_2 \times {}^6C_2 = 101$

8. $\frac{1}{2}\left(\frac{1}{5}\right)^2 + \frac{2}{3}\left(\frac{1}{5}\right)^3 + \frac{3}{4}\left(\frac{1}{5}\right)^4 + ...$

$= \left(1-\frac{1}{2}\right)\left(\frac{1}{5}\right)^2 + \left(1-\frac{1}{3}\right)\left(\frac{1}{5}\right)^3 + \left(1-\frac{1}{4}\right)\left(\frac{1}{5}\right)^4 + ...$

$= \left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^3 + \left(\frac{1}{5}\right)^4 + ... - \left[\frac{1}{2}\left(\frac{1}{5}\right)^2 + \frac{1}{3}\left(\frac{1}{5}\right)^3 + \frac{1}{4}\left(\frac{1}{5}\right)^4 + ...\right]$

$= \left[\frac{1}{5} + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^3 + \left(\frac{1}{5}\right)^4 + ...\right]$

$- \left[\frac{1}{5} + \frac{1}{2}\left(\frac{1}{5}\right)^2 + \frac{1}{3}\left(\frac{1}{5}\right)^3 + \frac{1}{4}\left(\frac{1}{5}\right)^4 + ...\right]$

adding and subtracting $\frac{1}{5}$

$= \frac{\frac{1}{5}}{1-\frac{1}{5}} - \left(-\log_e\left(1-\frac{1}{5}\right)\right) = \frac{1}{4} + \log_e \frac{4}{5}$

9. Four digit numbers

_ _ _ _20 = 3 × 2 numbers

_ _ _ _12 = 2 × 2 numbers

_ _ _ _32 = 2 × 2 numbers

_ _ _ _52 = 2 × 2 numbers

Three digit numbers = 18

_ _ _ _20 = 3

_ _ _ _12 = 2

_ _ _ _32 = 2

_ _ _ _52 = 2

Total = 9

Two digit numbers = 4

Total = 31

10. (1), 2, (3), 4, (5), 6

(i) 4 × 5 × 3 = 60 (ii) 6 × 6 × 3 = 108

DPP NO. - 65

1. $\left(x^3 - \frac{1}{x^2}\right)^n$

General term $= \frac{n!}{r!(n-r)!} (-1)^{n-r} x^{5r-2n}$

If $5r - 2n = 5$, then $5r = 2n + 5 \Rightarrow r = \frac{2n}{5} + 1$

If $5r - 2n = 10$, then $5r = 2n + 10 \Rightarrow r = \frac{2n}{5} + 2$

Let $n = 5k$

Now $\frac{5k!}{(2k+1)!(3k-1)!} - \frac{5k!}{(2k+2)!(3k-2)!} = 0$

$\Rightarrow \frac{1}{3k-1} - \frac{1}{2k+2} = 0 \Rightarrow k = 3 \Rightarrow n = 15$

2. $(1+2\sqrt{x})^{40} = {}^{40}C_0 + {}^{40}C_1\ 2\sqrt{x} + ...+ {}^{40}C_{40}\ (2\sqrt{x})^{40}$

$(1-2\sqrt{x})^{40} = {}^{40}C_0 - {}^{40}C_1\ 2\sqrt{x} + ...+ {}^{40}C_{40}\ (2\sqrt{x})^{40}$

$(1+2\sqrt{x})^{40} + (1-2\sqrt{x})^{40}$

$= 2\ [{}^{40}C_0 + {}^{40}C_2\ (2\sqrt{x})^2 + ...+ {}^{40}C_{40}\ (2\sqrt{x})^{40}]$

Putting x = 1

${}^{40}C_0 + {}^{40}C_2(2)^2 + ...+ {}^{40}C_{40}\ (2)^{40} = \frac{3^{40}+1}{2}$

3. $\left(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}}\right)^{10} = \left(x^{1/3} + 1 - 1 - \frac{1}{\sqrt{x}}\right)^{10}$

$T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r} \left(-\frac{1}{\sqrt{x}}\right)^r$

For independent term $\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow r = 4$

Coefficient of the term independent of x $= {}^{10}C_4$

5. $f(n) = 10^n + 3.4^{n+2} + 5$

put n = 1

f(1) = 10 + 192 + 5 = 207 this is divisible by 3 and 9

6. $\left\{\frac{3^{1001}}{82}\right\} = \left\{\frac{3.(82-1)^{250}}{82}\right\}$

$= \left\{\frac{3.[{}^{250}C_0(82)^{250} + {}^{250}C_1(82)^{249}(-1) + ... + {}^{250}C_{250}]}{82}\right\} =$

$\frac{3}{82}$

7. $\because$ $(1+x)^n = C_0 + C_1x + ... + C_nx^n$
Multiply by x & then differentiate
$(1+x)^n + x.\ n(1+x)^{n-1} = C_0 + 2C_1x + ... + (n+1)C_nx^n$ ....(i)
and $(x+1)^n = C_0x^n + C_1x^{n-1} + ... + C_n$ ....(ii)
Multiply (i) & (ii) & equate the coefficient of x^n on both side
$C_0^2 + 2C_1^2 + ... + (n+1)\ C_n^2 = {}^{2n}C_n + n.\ {}^{2n-1}C_{n-1}$

$= \frac{(2n)!}{(n!)^2} + n\frac{(2n-1)!}{n!(n-1)!} = (n+2)\frac{(2n-1)!}{n!(n-1)!}$

8. $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$

$\Rightarrow S = \sum_{r=0}^{n} \frac{n-2r}{{}^nC_r}$

$S = \sum_{r=0}^{n} \frac{n-2(n-r)}{{}^nC_r}$

$2S = 0 \quad \Rightarrow \quad S = 0$

9. $= a\sum_{r=1}^{n}(-1)^{r-1}.\ {}^nC_r - \sum_{r=1}^{n} r.\ {}^nC_r(-1)^{r-1} = a[{}^nC_1 - {}^nC_2 + {}^nC_3$

$.....+(-1)^{n-1}.{}^nC_n] - n\sum_{r=1}^{n}(-1)^{r-1}\ {}^{n-1}C_{r-1}$

$= a(1) - n[{}^{n-1}C_0 - {}^{n-1}C_1 + ...+(-1)^{(n-1)}\ {}^{n-1}C_{n-1}]$
$= a - n(0) = a$

10. $3\ .\ {}^nC_0 - 8\ .\ {}^nC_1 + 13\ .\ {}^nC_2 - 18\ {}^nC_3 + ...$ up to $(n+1)$ terms
$(1 + x^5)^n = C_0 + C_1x^5 + C_2x^{10} + ... + C_nx^{5n}$
Multiplying by x^3 and differentiating w.r.t. x
$x^3\ .\ n(1 + x^5)^{n-1}\ .\ 5x^4 + 3x^2\ (1 + x^5)^n = 3C_0\ x^2 + 8C_1\ x^7 + 13C_2\ x^{12} + ... + (5n + 3)\ C_n\ x^{5n+2}$
Now put $x = -1$
$3C_0 - 8C_1 + 13\ C_2 + ...+ (n + 1)$ terms $= 0$

11. $\left(\left(x+\frac{1}{x}\right)^2 - 1\right)^n = {}^nC_0\left(x+\frac{1}{x}\right)^{2n} - {}^nC_1\left(x+\frac{1}{x}\right)^{2n-2} + ...$

$+ {}^nC_n(-1)^n$
Total number of terms $= 2n + 1$

12. $(1 + x + 2x^2)^{20} = a_0 + a_1x + ... + a_{40}x^{40}$
$x = 1$, then $a_0 + a_1 + ... + a_{40} = 4^{20}$
$x = -1$, then $a_0 - a_1 + a_2 - .... + a_{40} = 2^{20}$
$2^{20} + 2^{40} = 2[a_0 + a_2 + ... + a_{38} + a_{40}]$
$\Rightarrow a_0 + a_2 + ... + a_{38} = 2^{19} + 2^{39} - 2^{20}$
$= 2^{19}\ (2^{20} - 1)\ \because\ a_{40} = a^{20}$

13. $\left(4^{1/3} + \frac{1}{6^{1/4}}\right)^{20}$

$T_{r+1} = {}^{20}C_r(4^{1/3})^{20-r}(6^{-1/4})^r$
For rational terms
$20 - r = 3k$ & $r = 4p$, where $k, p \in I$
$\Rightarrow r = 20$ & $r = 8$
$\therefore$ no. of rational terms = 2
$\therefore$ no. of irrational terms = 19

14. $(9+\sqrt{80})^n = I + f$

$\Rightarrow (9-\sqrt{80})^n = f'$

$2[{}^nC_0\ (9)^n + {}^nC_2\ (9)^{n-2}\ (\sqrt{80})^2 + ...] = I + f + f'$

$\therefore$ $I = 2(\text{integer}) - 1$

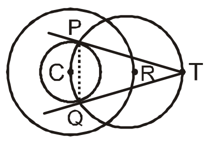
$(\because f + f' = 1)$ $\quad\therefore\ (I + f)\ (1 - f) = 1$

15. $7^9 + 9^7 = (8-1)^9 + (8+1)^7 = {}^9C_0(8)^9 - {}^9C_1(8)^8 + {}^9C_2(8)^7$ $.....+ {}^9C_8(8) - {}^9C_9 + {}^7C_0(8)^7 + ...+ {}^7C_6(8) + {}^7C_7$
This is divisible by 64 & 16

DPP NO. - 66

1. $x^2 + y^2 - 6x + 8y = 0$
Required chord whose middle point is $(5, -3)$, is
$T = S_1$
$\Rightarrow 5x + (-3)y + (-3)(x+5) + 4(y-3)$
$= 25 + 9 - 30 - 24$
$\Rightarrow 2x + y - 27 = -20$
$\Rightarrow 2x + y - 7 = 0$

2. Required distance = TR
= CT – CR
= 2 × 6 – radius of director circle
$= 12 - 4\sqrt{2}$

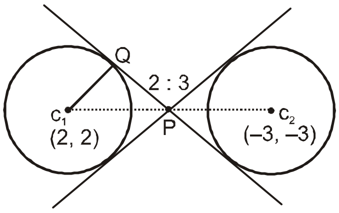


3. $x^2 + y^2 - 4x - 4y + 4 = 0$ .... (i) ; $c_1 \equiv (2, 2)$, $r_1 = 2$
$x^2 + y^2 + 6x + 6y + 9 = 0$ .... (ii) ; $c_2 \equiv (-3, -3)$, $r_2 = 3$
$\therefore$ distance between centres

$= c_1c_2 = \sqrt{(2+3)^2 + (2+3)^2} = 5\sqrt{2}$
$r_1 + r_2 = 5$
$|r_1 - r_2| = 1$
$\therefore$ $c_1c_2 > r_1 + r_2$

'P' is a point of intersection of common internal tangent. which lies on the line segment c_1c_2 joining centres of two circle and divides this segment c_1c_2 internally in the ratio of radius of given circles.
i.e. in the ratio 2 : 3

$$\therefore P \equiv \left(\frac{2\times(-3)+3(2)}{2+3}, \frac{2\times(-3)+3(2)}{2+3}\right)$$



$\equiv (0, 0)$

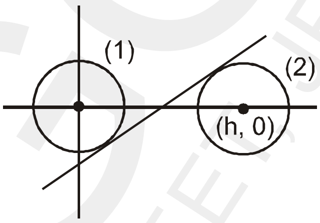
Let slope of internal tangent be m
$\therefore$ equation of this tangent is -
$y - 0 = m(x - 0) \Rightarrow y = mx$
$\Rightarrow mx - y = 0$
$\therefore c_1Q = r_1$

$$\Rightarrow \left|\frac{2\times m - 2\times 1}{\sqrt{m^2+1}}\right| = 2$$

$\Rightarrow 4(m-1)^2 = 4(m^2+1)$
$\Rightarrow m^2 + 1 - 2m = m^2 + 1$
$\Rightarrow m = 0$
$\therefore$ common tangents are -
$x = 0$ & $y = 0$

4. $x^2 + y^2 = 1$ ......(i) ; $c_1 \equiv (0, 0)$, $r_1 = 1$
$(x - h)^2 + (y^2) = 1$ ......(ii) ; $c_2 \equiv (h, 0)$, $r_2 = 1$
length of transverse common tangent

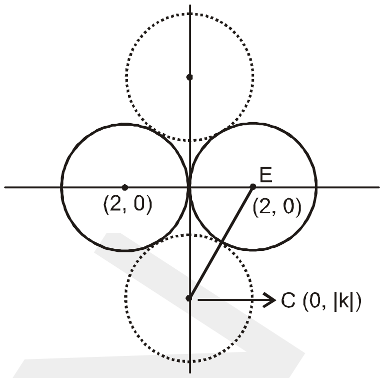
$$= \sqrt{(c_1c_2)^2 - (r_1+r_2)^2} = 2\sqrt{3}$$



$= h^2 - (2)^2 = 12$
$= h^2 = 16 \Rightarrow h = \pm 4$ **Ans. (B)**

5. $x^2 + y^2 - 4|x| = 0$
clearly centre of the required circle is c (0, | k |)
& $CE = r_1 + r_2 = 2 + 2$

$$\Rightarrow \sqrt{(2-0)^2 + (0-|k|^2)} = 4$$



$\Rightarrow k^2 = 12 \Rightarrow k = \pm 2\sqrt{3}$
$\therefore$ Equation of required circles (shown in figure as dotted circles) is
$(x - 0)^2 + (y - |k|^2) = 2^2$
$\Rightarrow x^2 + y^2 \pm 4\sqrt{3}y + 8 = 0$

6. Word $\Rightarrow$ SHWETA
Dictionary order AEHSTW
Word begin with A = 5!
E = 5!
H = 5!
SA = 4!
SE = 4!
SHA = 3!
SHE = 3!
SHT = 3!
Words begin with SHWA = 2!
SHWEA = 1!
SHWETA = 1!
Rank = 3 5! + 2 4! + 3 3! + 2! + 1 + 1 = 360 + 48 + 18 + 2 + 2 = 430

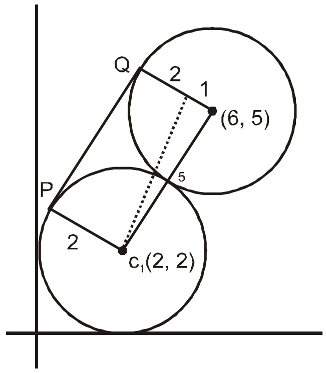
7. 4 .... 5 Required number is begin with 4 and end with 5 then total numbers = 5 × 5 = 25

8. ${}^{26}P_2 \cdot {}^{10}P_3 \Rightarrow 26 \times 25 \times 10 \times 9 \times 8$
$720 \times 650 = 468000$

DPP NO. - 67

1. $(x - 2)^2 + (y - 2)^2 = 4$ .... (i)
$(x - 6)^2 + (y - 5)^2 = 9$ ...... (ii)

$c_1c_2 = \sqrt{16+9} = 5$
$PQ = c_1M$ = length of external common tangent
$c_2M = 1$
$c_1M^2 = c_1c_2^2 - c_2M^2 = 25 - 1$
$c_1M^2 = 24$
$c_1M = \sqrt{24}$



2. Let mid point of chord is p(h, k)
equation of chord x.h + yk = $h^2 + k^2$ ....(i)
$xh + yk - (x^2 + k^2) = 0$
$3x + 4y - 5 = 0$ ....(ii)

(i) and (ii) are same chord $\frac{h}{3} = \frac{k}{4} = \frac{h^2+k^2}{25}$

$4h = 3k$ .... (i)
$\Rightarrow 25k = 4h^2 + 4k^2$ .... (ii)

$25k = 4.\frac{9k^2}{16} + 4k^2$

$25k = \frac{25k^2}{4}$

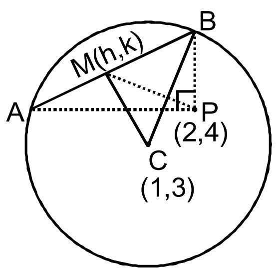
k = 0 or 4
h = 0 or 3   h = 0 , k = 0 not possible
P ≡ (3, 4)

3. Let $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$
∵ it cuts $x^2 + y^2 = 4$ orthogonally ⇒ c = 4
Moreover – 2g + 2f + 9 = 0
(∵ (– g, – f) satisfy the given equation)
∴ $S \equiv x^2 + y^2 + 2gx + 2fy + 4 = 0$
$\Rightarrow x^2 + y^2 + (2f + 9)x + 2fy + 4 = 0$
$\Rightarrow (x^2 + y^2 + 9x + 4) + 2f(x + y) = 0$
It is of the form S + λP = 0 and hence passes through the intersection of S = 0 and P = 0 which when solved give (– 1/2, 1/2), (– 4, 4).

4. Let M be (h, k) M(h, k) $r = \sqrt{1+9+6}$
AM = BM = PM = 4

$= \sqrt{(h-2)^2 + (k-4)^2}$

$CB^2 = CM^2 + MB^2$
$16 = (h-1)^2 + (k-3)^2 + (h-2)^2 + (k-4)^2$



~~16~~ = $2h^2 + 2k^2 - 6h - 14k + 1 + 9 + 4 +$ ~~16~~
$2h^2 + 2k^2 - 6h - 14k + 14 = 0$
$x^2 + y^2 - 3x - 7y + 7 = 0$

5. $x^2 + y^2 = 10$ .... (1)
and $x^2 + y^2 + 4x - 3y + 2 = 0$ .... (2)

Let P' (h, k) is required point of intersection of tangent at (1) at A & B
clearly AB is common chord of (1) & (2)
$S_1 - S_2 = a$
⇒ Equation of AB ⇒ $4x - 3y + 12 = 0$ .... (3)
But AB is also chord of contact of point p (h,k) w.r.t. (1) ∴ again equaiton of AB is -
T = 0
⇒ $xh + yk - 10 = 0$ .... (4)
(3) & (4) are equation of same line

$\therefore \frac{h}{4} = \frac{k}{-3} = \frac{-10}{12}$

$\Rightarrow h = -\frac{10}{3}, k = \frac{5}{2}$ **Ans. (C)**

7. 52 × 51 = 2652

8. ${}^{59}C_{30} + {}^{59}C_{31} + ... + {}^{59}C_{59}$
${}^{59}C_0 + {}^{59}C_2 + ... + {}^{59}C_{58} = 2^{59-1}$

DPP NO. - 68

1. $(1 + x)^m (1 - x)^n$
$3 = {}^mC_0\, {}^nC_1(-1) + {}^mC_1 . {}^nC_0$
or 3 = m – n ...(i)
$-6 = {}^mC_0 . {}^nC_2 + {}^mC_2 . {}^nC_0 - {}^mC_1\, {}^nC_1$

or $-6 = \frac{n(n-1)}{2} + \frac{m(m-1)}{2} - mn$ ...(ii)

from (i) & (ii) n = 9 & m = 12

2. ${}^nC_r + 2.{}^nC_{r-1} + {}^nC_{r-2} = {}^nC_r + {}^nC_{r-1} + {}^nC_{r-1} + {}^nC_{r-2}$
$= {}^{n+1}C_r + {}^{n+1}C_{r-1} = {}^{n+2}C_r$

3. $(a - b)^n$
$T_5 + T_6 = {}^nC_4(a)^{n-4}(-b)^4 + {}^nC_5(a)^{n-5}(-b)^5 = 0$

$\Rightarrow a + \frac{n-4}{5}(-b) = 0 \Rightarrow a/b = \frac{n-4}{5}$

4. $S = \sum_{i=0}^{m} \binom{10}{i}\binom{20}{m-i}$

$= {}^{10}C_0 . {}^{20}C_m + {}^{10}C_1 . {}^{20}C_{m-1} + ...$
⇒ S = coefficient of x^m in $(1+x)^{10}(1+x)^{20} = {}^{30}C_m$
S is maximum when m = 15

5. $(1 + t^2)^{12}(1 + t^{12} + t^{24} + t^{36})$
$= (1 + t^{12} + t^{24})(1 + t^2)^{12}$
coefficient of $t^{24} = {}^{12}C_{12} + {}^{12}C_6 + {}^{12}C_0 = {}^{12}C_6 + 2$

6. ${}^{(n-1)}C_r = (k^2 - 3)\, {}^nC_{r+1}$
or ${}^{(n-1)}C_{n-(r+1)} = (k^2 - 3)\, {}^nC_{n-(r+1)}$

$1 \ge k^2 - 3 > 0 \Rightarrow k \in [-2, -\sqrt{3}) \cup (\sqrt{3}, 2]$

7. $S = {}^{30}C_0\, {}^{30}C_{20} - {}^{30}C_1\, {}^{30}C_{19} + {}^{30}C_2\, {}^{30}C_{18}$.................
S = Co-efficient of x^{20} in $(1-x)^{30}(1 + x)^{30}$
S = Co-efficient of x^{20} in $(1-x^2)^{30} = {}^{30}C_{10}$

8. $B_{10}\sum_{r=1}^{10} A_rB_r - C_{10}\sum_{r=1}^{10}(A_r)^2 = {}^{20}B_{10}\,({}^{30}C_{20} - 1) - {}^{30}C_{10}$

$({}^{20}C_{10} - 1) = {}^{30}C_{10} - {}^{20}C_{10} = C_{10} - B_{10}$

9. $S = 2^k\,{}^nC_0.\,{}^nC_k - 2^{k-1}\,{}^nC_1\,{}^{n-1}C_{k-1} + 2^{k-2}\,{}^nC_2\,{}^{n-2}C_{k-2}$ +.............

$\Rightarrow S = \sum_{r=0}^{k}(-1)^r\,{}^nC_r\,{}^{n-r}C_{k-r}\cdot 2^{k-r}$

$\Rightarrow S = \sum_{r=0}^{k}(-1)^r\,\frac{n!}{r!(n-r)!}\times\frac{(n-r)!.2^{k-r}}{(n-k)!(k-r)!}$

$= \sum_{r=0}^{k}(-1)^r\,.\,2^{k-r}\,\frac{n!}{k!(n-k)!}\times\frac{k!}{r!(k-r)!}$

$= 2^k\,{}^nC_k\left(1-\frac{1}{2}\right)^k = {}^nC_k$

10. ${}^nC_m + {}^{n-1}C_m + {}^{n-2}C_m$ + ...+ ${}^mC_m$

= Co-efficient of x^m in $(1+x)^n + (1+x)^{n-1}$ + ...+ $(1+x)^m$

= Co-efficient of x^m in $(1+x)^m\left[\frac{(1+x)^{n-m+1}-1}{x}\right]$

$= {}^{n+1}C_{m+1}$

$S = {}^nC_m + 2.\,{}^{n-1}C_m + 3.\,{}^{n-2}C_m$ + ...

$\Rightarrow$ S = Co-efficient of x^m in $(1+x)^n + 2.(1+x)^{n-1} + 3(1+x)^{n-2}$ +.......

Let $S' = (1+x)^n + 2.(1+x)^{n-1} + 3(1+x)^{n-2}$ + ...+ $(n-m+1)(1+x)^m$ ......(i)

$\Rightarrow \frac{S'}{(1+x)} = (1+x)^{n-1} + 2.(1+x)^{n-2}$ +.... + $(n-m+1)(1+x)^{m-1}$ ......(ii)

from (i) – (ii)

$\Rightarrow \frac{xS'}{1+x} = (1+x)^n + (1+x)^{n-1} +...+(1+x)^m - (n-m+1)(1+x)^{m-1}$

$\Rightarrow \frac{xS'}{1+x} = (1+x)^m\left[\frac{(1+x)^{n-m+1}-1}{x}\right] - (n-m+1)(1+x)^{m-1}$

$\Rightarrow S' = \frac{(1+x)^{n+2}-(1+x)^{m+1}}{x^2} - \frac{(n-m+1)(1+x)^m}{x}$

$\Rightarrow$ S = Co-efficient of x^m in S' = ${}^{n+2}C_{m+2}$

DPP NO. - 69

1. n^{n-1} (Sum of digit) . $\frac{(10^n-1)}{10-1}$

$= 4^3(10)\left(\frac{10^4-1}{9}\right) = 711040$

2. Numbers in which 5 is included = 5

Numbers in which 5 is always excluded 8 × 9 × 5 = 360 + 5 = 365

3. **(i)** $3!\,{}^4C_3 = 24$

(ii) Required permutation = All S's are seperated – All S's are seperated but C's together

$= {}^5C_3 \times \frac{4!}{2!} - {}^4C_3 \times 3! = 120 - 24 = 96$

(iii) Required words = ${}^7C_5 \times 2! = 21 \times 2 = 42$

4. General term = $\frac{10!}{r_1!r_2!r_3!}(1)^{r_1}(2x)^{r_2}(3x^2)^{r_3}$

a_1 = Coeff. of x

$r_2 + 2r_3 = 1 \Rightarrow r_2 = 1, r_1 = 9, r_3 = 0$

$\therefore a_1 = \frac{10!}{1!9!}(2)^1 = 20$

a_2 = Coeff. of x^2

$r_2 + 2r_3 = 2 \Rightarrow r_2 = 2, r_1 = 8, r_3 = 0$

$r_2 = 0, r_1 = 9, r_3 = 1$

$a_2 = \frac{10!}{2!8!}(2)^2 + \frac{10!}{9!1!}(3) = 210$

a_4 = coeff. of x^4

$r_2 + 2r_3 = 4 \Rightarrow r_2 = 4, r_1 = 6, r_3 = 0$

$r_2 = 2, r_1 = 7, r_3 = 1$

$r_2 = 0, r_1 = 8, r_3 = 2$

$a_4 = \frac{10!}{4!6!}(2)^4 + \frac{10!}{2!7!1!}(2)^2(3) + \frac{10!}{8!2!}(3)^2$

= 8085

$a_{20} = 3^{10}$

5. 5×4^9 {First place can be fill by 5 ways after that each place can be filled by 4 ways}

6. (1) ${}^6P_3 = 120$

(2) 2 × 5 × 4 = 40

(3) 4 × 5 × 2 = 40

(4) 120 – 40 = 80

(5) 4 × 5 × 1 = 20

7. (i) Both vowels are same

[consonan to are diff. = ${}^2C_1\,{}^5C_3\,\frac{5!}{2!}$ two consonants are same.

${}^2C_1\,{}^2C_1\,{}^4C_1\,\frac{5!}{2!\,2!}$