DPP 51 TO 60 (ANSWER KEY)

DPP NO. - 50

1. (D) **2.** (B) **3.** (C) **4.** (A) **5.** (B) **6.** (A)(B)(C)

DPP NO. - 51

1. (A) **2.** (A) **3.** (B) **4.** (A) **5.** (B) **6.** (B)

7. (A)→(r), (B)→(s), (C)→(q), (D)→ (p)

DPP NO. - 52

1. (D) **2.** (A) **3.** (B) **4.** (B) **5.** (A) **6.** 4

7. (A) → q, (B) → (s), (C) → p, (D) → r

DPP NO. - 53

1. (B) **2.** (C) **3.** (D) **4.** (C)(D) **5.** 10 **6.** 6

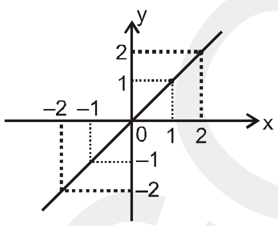
7. (A)→(r), (B)→(s), (C)→(p), (D)→(s)

DPP NO. - 54

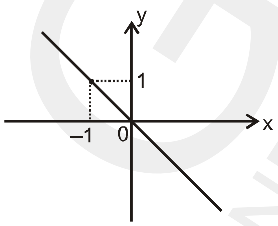
1. A **2.** A **3.** AC **4.** BD

5. (–2, –1), (–1, 2), (1, –2)

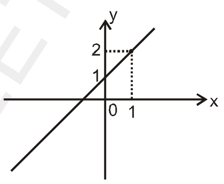
6. (i)



(ii)



(iii)



7. (A) → (s), (B) → (p), (C) → (s), (D) → (q)

DPP NO. - 55

1. B **2.** C **3.** A **4.** C **5.** D **6.** A

7. $x^2 + y^2 + 6x - 3y - 45 = 0$

DPP NO. - 56

1. A **2.** D **3.** C **4.** $m = \frac{12\sqrt{221}}{49}$ **6.** k = 1

7. 19

DPP NO. - 57

1. (B) **2.** (C) **3.** (A) **4.** (D) **5.** (D) **6.** (B)

7. (A)(B)(D)

DPP NO. - 58

1. (C) **2.** (D) **3.** (C) **4.** (D) **5.** (A) **6.** (B)

DPP NO. - 59

1. (i) $\left(\frac{2}{x}\right)^5 - 5\left(\frac{2}{x}\right)^3 + 10\left(\frac{2}{x}\right) - 10\left(\frac{x}{2}\right) + 5\left(\frac{x}{2}\right)^3 - \left(\frac{x}{2}\right)^5$

(ii) $y^8 + 8y^5 + 24y^2 + \frac{32}{y} + \frac{16}{y^4}$

2. ${}^{18}C_6$ **3.** n = 9 **4.** (i) ${}^{9}C_3$ (ii) $-2^7 \cdot {}^{12}C_7$

5. $\frac{17}{54}$ **6.** ${}^{11}C_5 \frac{a^6}{b^5}$, ${}^{11}C_6 \frac{a^5}{b^6}$, a b = 1 **7.** $\frac{1}{2\sqrt{3}}$

8. (C) **9.** (C) **10.** (A) **11.** (A)

12. (B) **13.** (A) **14.** (C) **15.** (A)

DPP NO. - 60

1. (B) **2.** (D) **3.** (B) **4.** (B)

5. (C) **6.** (D) **7.** (A) **8.** (A)

9. (A) **10.** (i) $-\frac{35x}{y}, \frac{35y}{x}$ (ii) $(-1)^n \frac{(2n)!}{n!\,n!} x^n$

13. (i) 4 (iii) 3, 03, 803 **14.** 101^{50}

15. $T_4 = -455 \times 3^{12}$ and $T_5 = 455 \times 3^{12}$

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अभ्यास ही सबसे बड़ा गुरू है।

CLASS : XI (MATHEMATICS)

DPP

DAILY PRACTICE PROBLEM

Solutions

DPP-51 to 60

DPP 51 : **Circle, Straight Lines, Pair of Straight Lines**
DPP 52 : **Circle, Straight Lines**
DPP 53 : **Straight Lines**
DPP 54 : **Circle, Straight Lines, Pair of Straight Lines**
DPP 55 : **Circle**
DPP 56 : **Circle, Straight Lines**
DPP 57 : **Sequence & Series, Circle, Straight Lines**
DPP 58 : **Sequence & Series, Circle, Fundamentals of Mathematics**
DPP 59 : **Binomial Theorem**
DPP 60 : **Binomial Theorem**

DPP 51 TO 60 HINTS AND SOLUTIONS

DPP NO. - 51

1. one end A(3, 4)
centre of circle (2, 3)
other end = $(2(2)-3, 2(3)-4) \equiv (1, 2)$

2. $x^2 + y^2 - 2x + 4y - 20 = 0$
centre (1, –2) & Let radius r
Perimeter = $\pi r + 2r$

$36 = \frac{22}{7}r + 2r \Rightarrow r = 7$

$\Rightarrow$ Equaiton of circle is $(x-1)^2 + (y+2)^2 = 49$
$\Rightarrow x^2 + y^2 - 2x + 4y - 44 = 0$

3. $x^2 + y^2 - 6x - 2y + 5 = 0$ .... (i)
$x^2 + y^2 + 6x + 22y + 5 = 0$ .... (ii)
tangent at (2, –1) to the circle (i) is -
$2x - y - 3(x + 2) - (y - 1) + 5 = 0$
$\Rightarrow x + 2y = 0$ .... (iii)
on solving (ii) & (iii) $4y^2 + y^2 - 12y + 22y + 5 = 0$
$5(y^2 + 2y + 1) = 0$
$y = -1$ (both roots are same)
$\Rightarrow x + 2y = 0$ is tangent to the circle (ii)

4. Triangle formed by the lines
$3x + 4y = 24$, $x = 0$ & $y = 0$
Let equation of incircle is $(x - r)^2 + (y - r)^2 = r^2$
where r = length of $\perp$ from (r, r) on $3x + 4y = 24$

$\Rightarrow r = \left|\frac{3r + 4r - 24}{5}\right|$

$\Rightarrow \pm 5r = 7r - 24$
$\Rightarrow r = 12 , 2$
for incircle r = 2

5. Let centre is (h, 5) h > 0
this touch the line $3x - 4y = 0$

$\Rightarrow \left|\frac{3h - 20}{5}\right| = 5$

$\Rightarrow 3h - 20 = \pm 25$
$\Rightarrow 3h = 20 \pm 25$
h = 15 , – 5/3
but h > 0
$\Rightarrow$ h = 15
$\Rightarrow$ equation of circle $(x - 15)^2 + (y - 5)^2 = 25$
$\Rightarrow x^2 + y^2 - 30x - 10y + 225 = 0$

6. $\tan(90 - \alpha) = \frac{2-0}{8-x} = \frac{2}{8-x}$

$\cot\alpha = \frac{2}{8-x}$ .... (i)

$\tan(90 + \alpha) = \frac{y-0}{0-x} = \frac{-y}{x}$

$\cot\alpha = \frac{4-y}{3}$ .... (3)

$\frac{2}{8-x} = \frac{y}{x}$ $\quad$ $\frac{y}{x} = \frac{4-y}{3}$
$2x = 8y - xy$ $\quad$ $3y = 4x - xy$
$xy = 8y - 2x$ $\quad$ $xy = 4x - 3y$
$\Rightarrow 8y - 2x = 4x - 3y$ $\quad \Rightarrow 11y = 6x$

$y = \frac{6x}{11}$

$\Rightarrow x\left(\frac{6x}{11}\right) = 4x - 3\left(\frac{6x}{11}\right)$

$6x^2 = 44x - 18x$
$3x^2 = 22x - 9x$
$3x^2 = 13x$

$\Rightarrow x = 0$ or $x = \frac{13}{3} = 4\frac{1}{3}$

7. for point of intersection

$\frac{\partial f}{\partial x} = 0$ & $\frac{\partial f}{\partial y} = 0$

$\Rightarrow ax + hy + g = 0$
$by + hx + f = 0$
$\because (x_1, y_1)$ is point of intersection
$\Rightarrow ax_1 + hy_1 + g = 0$
& $by_1 + hx_1 + f = 0$
$\Rightarrow (ax_1 + hy_1)(by_1 + hx_1) = fg$

(B) $\because ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ represents a pair of straight line
so $abc + 2fgh - af^2 - bg^2 - bg^2 - ch^2 = 0$
$\Rightarrow af^2 + bg^2 + ch^2 = abc + 2fgh$

(C) If lines are parallel then $ax^2 + 2hxy + by^2 = 0$ is a perfect square
$\Rightarrow h^2 = ab$

(D) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c$
$= (a_1x+b_1y+c_1)(a_2x+b_2y+c_2)$
$\Rightarrow a = a_1a_2$, $2h = a_1b_2 + a_2b_1$, $b = b_1b_2$, $2g = a_1c_2 + a_2c_1$, $2f = b_1c_2 + b_2c_1$, $c = c_1c_2$

$$p_1p_2 = \frac{c_1c_2}{\sqrt{a_1^2 + b_1^2}\sqrt{a_2^2 + b_2^2}}$$

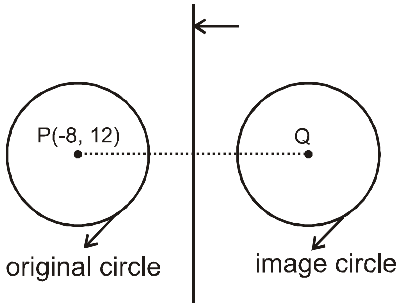
$$= \frac{c_1c_2}{\sqrt{(a_1b_2 - b_1b_2)^2 + (a_1b_2 + a_2b_1)^2}}$$

$$= \frac{c}{\sqrt{(a-b)^2 + 4h^2}}$$

DPP NO. - 52

1. Centre P (–8 , 12) & radius = $\sqrt{64 + 144 - 183} = 5$
given line mirror : $4x + 7y + 13 = 0$ .... (i)
clearly centre α of image circle is image of centre P in
line (i) and radius same as radius of original circle
$\therefore$ if Q is a $\equiv$ (x, y) then

$$\frac{x+8}{4} = \frac{y-12}{7} = \frac{-2(-32+84+13)}{16+49}$$



$\Rightarrow$ x = – 16 , y = – 2

$\therefore$ equation of image circle is $\rightarrow (x+16)^2 + (y+2)^2 = (5)^2$

$\Rightarrow x^2 + y^2 + 32x + 4y + 235 = 0$ **Ans.**

2. $x^2 + y^2 - 14x - 10y - 151 = 0$ .... (i) ;

centre = (7, 5) , r = 15

Let P $\equiv$ (2, –7)

$\therefore$ $S_1 = 4 + 49 - 28 + 70 - 151 = 56 < 0$

$\therefore$ D_{max} = PQ = PS + SQ = PS + r

$= \sqrt{(7-2)^2 + (5+7)^2} + 15 = 13 + 15 = 28$ **Ans.**

& D_{min} = PT = ST – PS = r – PS

= 15 – 13 = 2 **Ans.**

3. Equation of line (L) is

$7 - 5 = \frac{3}{2} 2(-5)$ $\Rightarrow$ 2y – 10 = 3x – 15

3x – 2y = 5

point E 2x + 3y – 12 = 0 $\Rightarrow$ 3x – 2y = 5 = 0

E(3, 2)

point C $\left[\frac{5}{3}, 0\right]$

Area of circle CEB

$$= \frac{1}{2}\begin{vmatrix} 0 & 5/3 & 3 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 \end{vmatrix} = \frac{1}{2}\left[\frac{10}{3} + 12\right] = \frac{46}{6} = \frac{23}{3}$$

4. Let line be ax+by+c = 0

$$\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}} + \frac{ax_2+by_2+c}{\sqrt{a^2+b^2}} + \frac{ax_3+by_3+c}{\sqrt{a^2+b^2}} = 0$$

$a(x_1+x_2+x_3) + b(x_1+x_2+x_3) + 3c = 0$

passes through $\left(\frac{x_1+x_2+x_3}{3} \; \frac{y_1+y_2+y_3}{3}\right)$ hence centroid

5. A (c cosα, c sinα) and B (c cosβ, c sinβ) are two points lies on a circle with centre at origin & radius c.
which represent that OAB is an isosceles Δ and perpendicular from oposite vertex to the base of an isosceles triangle bisects it.

6. passes (0, β)

$\Rightarrow$ β = A

2B = β

B = 2β

hence two sides are

y – 2x – β = 0 $\Rightarrow$ 2y – x – 2β = 0

Angle bisector

y – 2x – β = ± (2y – x – 2β)

+ve y – 2x – β = 2y – x – 2β

2y + x – β = 0

passes (1, 2) β = 3

–ve 3y – 3x – 3β = 0

y – x – β = 0

passes (1, 2)

2 – 1 – β = 0

β = 1

a = 3 b = 1

a + b = 4

7. **(A)**

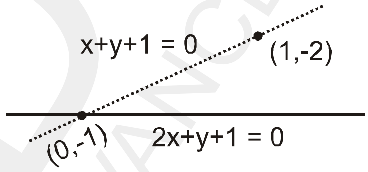


Image of (1, –2)

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{-2[2-2+1]}{5} = -\frac{2}{5}$$

x = –4/5 + 1 = 1/5

y = –2/5 – 2 = – 12/5

image $\equiv$ (1/5 , – 12/5)

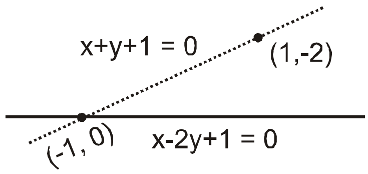
Equation of image line is -

$$y + 1 = \frac{-12/5+1}{1/5-0}(x-0)$$

y + 1 = – 7x

7x + y + 1 = 0

(B)



$$\frac{x-1}{1} = \frac{y+2}{-2} = \frac{-2(1+4+1)}{5} = +2/5$$

x = – 7/5 y = 14/5

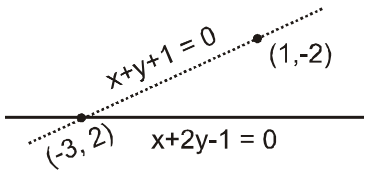
$$y - 0 = \frac{\frac{14}{5} - 0}{-\frac{7}{5}+1}(x+1)$$

$$y = \frac{-14}{2}(x+1)$$

y = –7x – 7

7x + y + 7 = 0

(C)



$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{-2(1-4-1)}{5}$$

$x = \frac{13}{5}$ $y = \frac{6}{5}$

$y - 2 = \frac{-1}{7}(x + 3)$

$7y - 14 = -x - 3$

$x + 7y - 11 = 0$

(D)

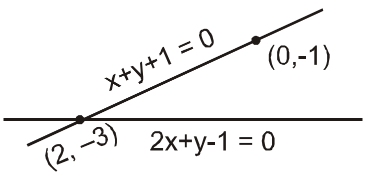


image of (0, –1)

$$\frac{x-0}{2} = \frac{y+1}{1} = \frac{-2(-1-1)}{5} = \frac{4}{5}$$

$x = \frac{8}{5}$ $y = -\frac{1}{5}$

Equation of line $y + 3 = \dfrac{-\frac{1}{5}+3}{\frac{8}{5}-2}(x-2)$

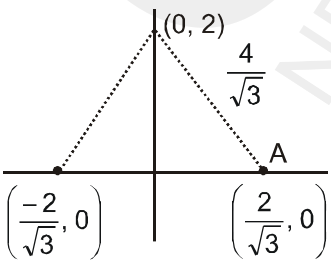
$y + 3 = \frac{-14}{2}$ (x - 2)

$y + 3 = -7x + 14$

$7x + y - 11 = 0$

DPP NO. - 53

1.



Point of intersection is (0, 0)

2. $(px + 2y - 1) + \lambda (2x + py - 1) = 0$
passed (p, q)
$(p^2 + q^2 - 1) + \lambda (2pq - 1) = 0$

$$\lambda = \frac{1-2p^2}{p^2+q^2-1}$$

3. Let $A(r_1, r_1)$ $B(r_2, r_2)$, $C(r_3, r_3)$
$\Rightarrow$ OA .OB.OC = $2\sqrt{2}\, r_1r_2r_3$
on putting point (r, r) in the curve
$r^3 + 3r^3 - 30r^2 + 72r - 55 = 0$
$4r^3 - 302 + 72r - 55 = 0$
it has roots r_1, r_2, r_3

$\Rightarrow r_1r_2r_3 = \frac{55}{4}$ $\Rightarrow$ OA.OB.OC = $\frac{55}{\sqrt{2}}$

$\Rightarrow \frac{4\sqrt{2}\,\text{OA.OB.OC}}{55} = 4$

4. points of intersection (7,1)
so line is $y - 1 = m(x - 7)$
$y = mx + 1 - 7m$

$$5 = \left|\frac{1-7m}{\sqrt{1+m^2}}\right|$$

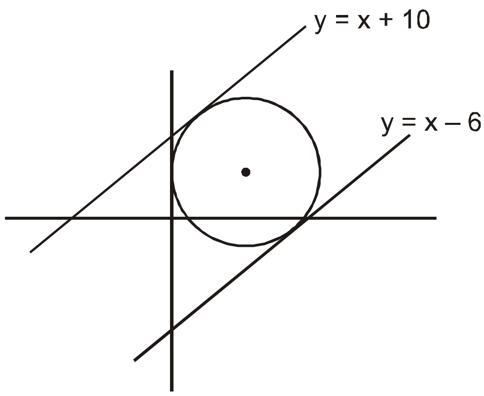
$25(1+m^2) = 49m2 - 14m + 1$
$24m2 - 14m - 24 = 0$

$m = -\frac{3}{4}, \frac{4}{3}$

hence line is $4x - 3y = 25$ & $3x + 4y = 25$

5. centre is (h, k) & touch the line x = 0
$\Rightarrow$ radius = h
dist between both lines

$2r = \frac{16}{\sqrt{2}}$



$\Rightarrow r = 4\sqrt{2} = h$ ...... (i)
length of $\perp$ from center n(h, k) = radius

$\Rightarrow \left|\frac{h-k+10}{\sqrt{2}}\right| = 4\sqrt{2}$

$\Rightarrow 4\sqrt{2} - k + 10 = 8$ $\Rightarrow 4\sqrt{2} + 2 = k$

$h + k = 4\sqrt{2} + 4\sqrt{2} + 2$

$= 8\sqrt{2} + 2 = a + b\sqrt{a}$

$\Rightarrow a = 2 \;\&\; b = 8$

$a + b = 10$

6. $x^2 + y^2 - 2x - 2y + 1 = 0$ ...... (i)

$x^2 + y^2 - 16x - 2y + 61 = 0$ .... (ii)

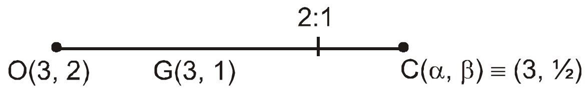
$3x - 4y + k = 0$ .... (iii)

solving (i) & (iii)

$$x^2 + \left(\frac{3x-k}{4}\right)^2 - 2x - 2\left(\frac{3x-k}{4}\right) + 1 = 0$$

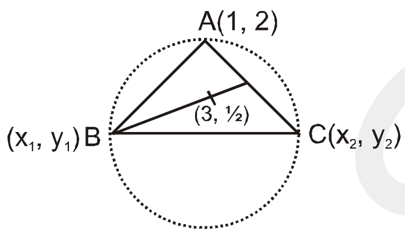
$\Rightarrow \; 16x^2 + 9x$

7. **(B)**



$$\frac{2\alpha+3}{3} = 3 \quad \alpha = 3$$

$$\frac{2\beta+2}{3} = 1 \quad \beta = \frac{1}{2}$$

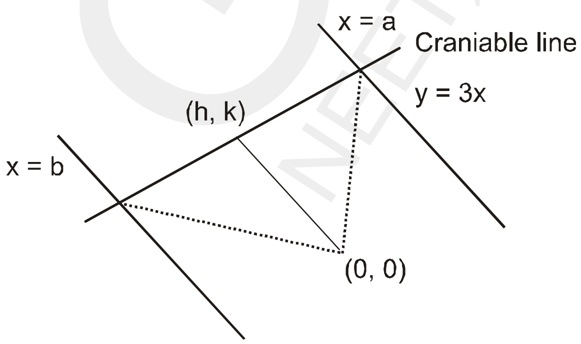


$x_1 + x_2 + 1 = 9$

DPP NO. - 54

1. Equation of line

$$y - k = \frac{-h}{k}(x - h)$$



$ky - k^2 = -hx + h^2$

$hx + ky = h^2 + k^2$ .... (i)

homogenisnry pair of st. line with the help of (i)

$$x^2 - (a+b)x\left(\frac{hx+ky}{h^2+k^2}\right) + ab\left(\frac{hx+ky}{h^2+k^2}\right)^2 = 0$$

Coeff. of x^2 + coeff of $y^2 = 0$

$$1 - \frac{h(a+b)}{h^2+k^2} + \frac{abh^2}{(qh^2+k^2)^2} + \frac{abk^2}{(h^2+k^2)^2} = 0$$

$$1 - \frac{h(a+b)}{h^2+k^2} + \frac{ab}{(h^2+k^2)} = 0$$

$h^2 + k^2 - h(a+b) + ab = 0$

Locus is $x^2 + y^2 - x(a+b) + ab = 0$

2. $2x^2 + 3xy + by^2 - 11x + 13y + c = 0$

$\because \; \perp^r \;\Rightarrow\; b = -2$

$\because$ pair of st. lines $\Rightarrow \Delta = 0$

$$\begin{vmatrix} 2 & 3/2 & -11/2 \\ 3/2 & -2 & 13/2 \\ -11/2 & 13/2 & c \end{vmatrix} = 0$$

$$2\left(-2c - \frac{169}{4}\right) - \frac{3}{2}\left(\frac{3c}{2} + \frac{143}{4}\right) - \frac{11}{2}\left(\frac{39}{4} - 11\right) = 0$$

$$\frac{-21(8c+169)}{4} - \frac{3}{2}\frac{(6c+143)}{4} - \frac{11}{2}\frac{(39-44)}{4} = 0$$

$c = -21$

3. $x^2 + y^2 = 8$ .....(i)

Equaiton of director circle of (i) is -

$x^2 + y^2 = \left(\sqrt{2} \times 2\sqrt{2}\right)^2$

$x^2 + y^2 = 16$ .....(ii)

We know that all points from which tangent to the circle are mutually perpendicular, lie on its director circle.

But here point also lies on the line $x = 3$ .....(iii)

$\therefore$ solving (iii) & (ii)

$y^2 = 16 - 9 = 7 \quad \Rightarrow \quad y = \pm\sqrt{7}$

$\therefore$ required points are $\left(3, \sqrt{7}\right)$ & $\left(3, -\sqrt{7}\right)$

Ans. A,C

4. $x^2 + y^2 - 6x - 8y + 21 = 0$ .... (i) ;

$c_1 \equiv (3, 4)$, $r_1 = \sqrt{9+16-21} = 2$

Let required radius of circle is r

centre is (0, 0) (given). If this circle touches the circle (i) then

$r_1 + r_2 = c_1c_2$ or $|r_1 - r_2| = c_1c_2$

$\Rightarrow 2 + r = \sqrt{(3-0)^2 + (4-0)^2}$

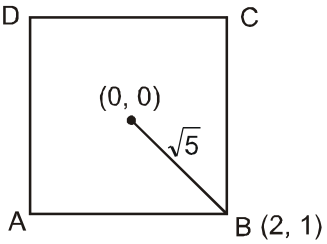
or $|2 - r| = \sqrt{(3-0)^2 + (4-0)^2}$

$\Rightarrow r = 3 \qquad \Rightarrow 2 - r = \pm 5$

$r = -3, 7$

$\therefore$ r = 3, 7 are valid **Ans. B, D**

5.



Vertex D will be (–2, –1)

for vertex A & C , $m_{OB} = \frac{1}{2}$

$\Rightarrow m_{AC} = -2$

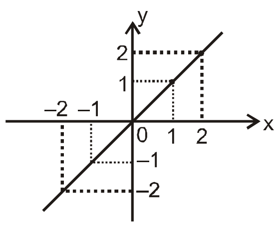
$\Rightarrow$ other vertices are $\left(0 \pm \sqrt{5}\left(-\frac{1}{\sqrt{5}}\right), 0 \pm \sqrt{5}\left(\frac{-2}{\sqrt{5}}\right)\right)$

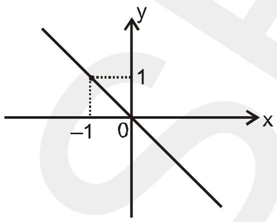
(1 , –2) & (–1 , 2)

6. **To plot a straight line we required co-ordinate of only two points**

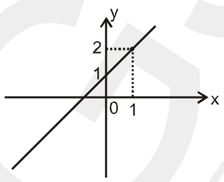
(i) $y = x$
$x = 0, y = 0$
$x = 1, y = 1$

(ii) $y = -x$
$x = 0, y = 0$
$x = -1, y = +1$





(iii) $y = x + 1$
$x = 0, y = 1$
$x = 1, y = 2$



7. **(A)** $t^2 + \sqrt{2}t - 42 = 0$

$t = \frac{-\sqrt{2} \pm \sqrt{2+168}}{2}$

$x + 7y = \frac{-\sqrt{2}+\sqrt{170}}{2}$

$\perp$ distance $= \frac{\sqrt{170}}{\sqrt{1+49}} = \sqrt{\frac{170}{50}} = \sqrt{\frac{17}{5}}$

$sr^2 = 17$

$sr^2 - 17 = 10$

(B) $|x| + |y| = |\alpha = |$

(C) $x + 2y + 4 = 0$

$4x + 2y - 1 = 0$

acute angle bisector

$\frac{x+2y+4}{\sqrt{5}} = -\left(\frac{4x+2y-1}{\sqrt{16+4}}\right)$

$2(x+2y+4) = -(4x+2y-1)$

$6x + 6y + 7 = 0$

$\Rightarrow m = 7$

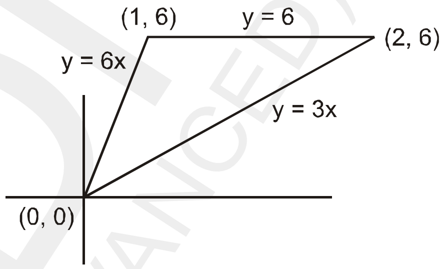
(D) $y^2 - 9xy + 18x^2 = 0$

$\left(\frac{y}{x}\right)^2 - 9\left(\frac{y}{x}\right) + 18 = 0$

$\left(\frac{y}{x}\right) = \frac{9 \pm \sqrt{81-72}}{2} = \frac{9 \pm 3}{2} = 6, 3$

$y = 6x$

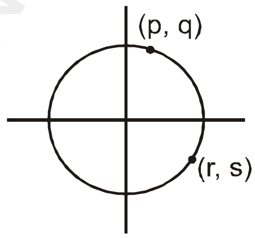
$y = 3x$



Area $= \frac{1}{2}\begin{vmatrix} 2 & 6 & 1 \\ 1 & 6 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{2}(12 - 6) = 3$

DPP NO. - 55

1.



$p^2 + q^2 = 1$

$r^2 + s^2 = 1$

let the point (p, r) be $(\cos\theta , \sin\theta)$

$P = \cos\theta \quad q = \sin\theta$

$(3\cos\theta - 4\cos^3\theta)^2 + (3\sin\theta - 4\sin^3\theta)^2$

$\cos^2 3\theta + \sin^2 3\theta = 1$

2. range of $qs + qr$

$p = \cos\theta$, $q = \sin\theta$

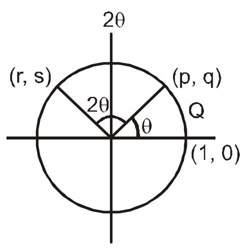
$r = \cos\phi$, $s = \sin\phi$

$\cos\theta \sin\phi + \sin\theta \cos\phi \quad \Rightarrow \sin(\theta + \phi)$

range is [–1 ,1]

3. $(p, q) \equiv (\cos\theta , \sin\theta)$

$(r, s) \equiv (\cos 3\theta , \sin 3\theta)$



$sp^3 + rq^3$

$= \sin3\theta\,[\cos^3\theta] + \cos 3\theta \sin^3\theta$

$= (3\sin\theta - 4\sin^3\theta)\cos^3\theta + (4\cos^3\theta - 3\cos\theta)\sin^2\theta$

$= 3\sin\theta\cos\theta\,(\cos2\theta) = \frac{3}{4}\sin4\theta$

4. $\sqrt{2}\, r_1 = a$

$r_1 = \frac{a}{\sqrt{2}}$

$\sqrt{2}\, r_2 = \frac{a}{\sqrt{2}} \quad \Rightarrow \quad r_2 = \frac{a}{2}$

$r_3 = \frac{a}{2\sqrt{2}}$

$2 = a + \frac{a}{\sqrt{2}} + \frac{a}{2} + \frac{a}{2\sqrt{2}} + \frac{a}{4} + \ldots\ldots$

$2 = a\left[1 + \frac{1}{\sqrt{2}} + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^3 + \ldots\ldots\right]$

$2 = a\left(\frac{1}{1-\frac{1}{\sqrt{2}}}\right) = \frac{a\sqrt{2}}{(\sqrt{2}-1)}$

$a = \sqrt{2}(\sqrt{2}-1)$

$a = 2 - \sqrt{2}$

5. Let the centre be (h, h)

$3h + 6h > 10$

$9h > 10$

$h > 10/9$

line $x + 2y = 3$ is tangent to the circle

$\frac{|h + 2h - 3|}{\sqrt{5}} = 4\sqrt{5}$

$|3h - 3| = 20$

$|h - 1| = \frac{20}{3}$

$h > 1 \qquad \& \qquad h < 1$

$h = 1 + \frac{20}{3}$

$h = \frac{23}{3} \qquad\qquad h = 1 - \frac{20}{3}$

$\Rightarrow \quad h = \frac{-17}{3}$

but $h > \frac{10}{9}$

so centre is $\left(\frac{23}{3}, \frac{23}{3}\right)$

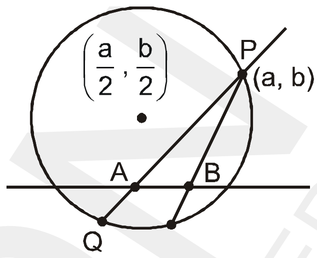
Equation of circle is -

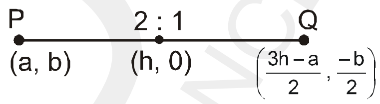
$\left(x - \frac{23}{3}\right)^2 + \left(y - \frac{23}{3}\right)^2 = 80$

6. $x^2 + y^2 - ax - by = 0$

P (a, b) is on the circle

Let $A \equiv (h, 0)$





Q satisfies the circle

$\left(\frac{3h-a}{2}\right)^2 + \frac{b^2}{4} - \frac{a}{2}(3h - 0) + \frac{b^2}{2} = 0$

$9h^2 + a^2 - 6ah + b^2 - 2a(3h - a) + 2b^2 = 0$

$9h^2 + a^2 - 6ah + b^2 - 6ah + 2a^2 + 2b^2 = 0$

$9h^2 - 12ah + 3a^2 + 3b^2 = 0$

$D > 0$

$144a^2 - 12 \times 9\,(a^2 + b^2) > 0$

$36a^2 - 108b^2 > 0$

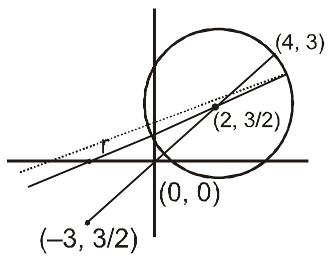
$a^2 > 3b^2$

7. $x^2 + 2xy + 3x + 6y = 0$

$x(x + 3) + 2y(x + 3) = 0$

$(x + 3)(x + 2y) = 0$

normal point of intersection is the (– 3 , 3/2)



size of the circle in such that it just contain the circle $x(x - 4) + y(y - 3) = 0$

radius of circle be r

$r = (2 + 3) + \frac{5}{2}$

$r = \frac{15}{2}$

$(x + 3)^2 + (y - 3/2)^2 = \left(\frac{15}{2}\right)^2$

$x^2 + y^2 + 6x - 3y - 45 = 0$

DPP NO. - 56

1. $f(x) = x^2 + Px + q = 0$

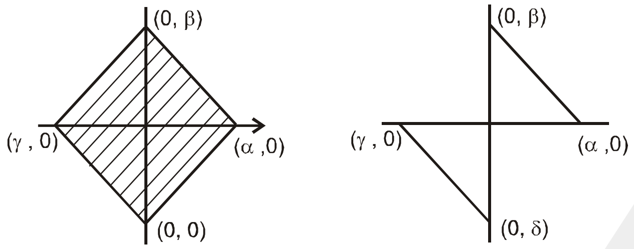
$\alpha + \beta = -P$ ...... (i)

$\alpha\beta = q$ ...... (ii)

$x^2 + rs + s = 0$

$\gamma + \delta = -r$ ...... (iii)

$\gamma\delta = S$ ...... (iv)



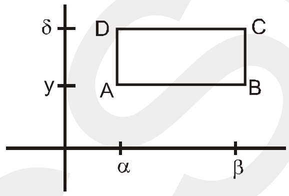
$$\Delta = \frac{1}{2}\left|\begin{vmatrix}\gamma & 0\\ \alpha & 0\end{vmatrix} + \begin{vmatrix}\alpha & 0\\ 0 & \beta\end{vmatrix} + \begin{vmatrix}0 & \beta\\ 0 & \delta\end{vmatrix} + \begin{vmatrix}0 & \delta\\ \gamma & 0\end{vmatrix}\right|$$

$$= \frac{1}{2}|\alpha\beta - \gamma\delta| = \frac{1}{2}|q - s|$$

2. Centre of circle circumscribing rectangle ABCD is -

$$\left(\frac{\alpha+\beta}{2}, \frac{\gamma+\delta}{2}\right)$$

$$= \left(\frac{-p}{2}, \frac{-q}{2}\right)$$



3. circle $f(x) + f(y) = 0$

$x^2 + y^2 + px + r y + q + s = 0$

$$\text{Radius} = \sqrt{\frac{p^2+t^2}{4} - (q+s)}$$

centre $(-p/2, -r/2)$

director circle $(x+ p/2)^2 + (y+r/2)^2$

$$= 2\left[\frac{p^2+r^2}{4} - (q+s)\right]$$

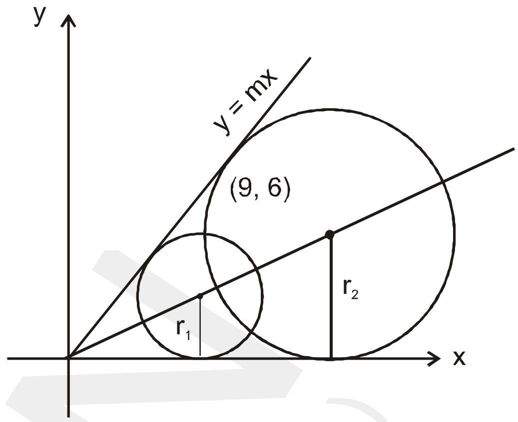
$$x^2 + y^2 + px + ry + q + s = \frac{p^2+r^2}{4} - (q + s)$$

$$f(x) + g(y) = \frac{p^2+r^2}{4} - (q + s)$$

4. Centre of circles will lie on angle bisector of lines $y = mx$ and $y = 0$

$$\frac{mx - y}{\sqrt{1+m^2}} = \pm y$$

since $m > 0$, centre will lie on



$$\frac{mx - y}{\sqrt{1+m^2}} = y$$

$$x = \frac{(1+\sqrt{1+m^2})}{m}\, y \quad \text{.............. (i)}$$

$$\text{Let } c_1 = \left(\frac{1+\sqrt{1+m^2}}{m} r_1 , r_1\right)$$

$$\text{let } \frac{1+\sqrt{1+m^2}}{m} = k$$

$$c_2 = \left(\frac{1+\sqrt{1+m^2}}{m} r_2 , r_2\right)$$

$c_1 = (kr_1 , r_1)$

$c_2 = (kr_2 , r_2)$

$(kr_1 - 9)^2 + (r_1 - 6)^2 = r_1^2$

$\therefore\ k_2r_1^2 - r_1 (18k + 12) + 117 = 0$

similarly $k^2r^2_2 - r_2 (18k + 12) + 117 = 0$

$\therefore$ hence r_1 , r_2 ....will the root of equaiton

$k^2r^2 - r (18k + 12) + 117 = 0$

$$r_1r_2 = \frac{117}{k^2} = 68$$

$$\Rightarrow k^2 = \frac{117}{68} \quad \Rightarrow k = \sqrt{\frac{117}{68}}$$

$$\Rightarrow \frac{1+\sqrt{1+m^2}}{m} = \sqrt{\frac{117}{68}}$$

$$\sqrt{1+m^2} = m\sqrt{\frac{117}{68}} - 1$$

$$1 + m^2 = m^2 \frac{117}{68} - 2m\sqrt{\frac{117}{68}} + 1$$

$$m^2\left(\frac{117}{68} - 1\right) - 2m\sqrt{\frac{117}{68}} = 0$$

$$\therefore\ m = \frac{2\times 68}{49}\sqrt{\frac{117}{68}} = \frac{2}{49}\sqrt{117\times 68}$$

$$= \frac{2}{49}\sqrt{13\times 5\times 17\times 4} = \frac{12}{49}\sqrt{221}$$

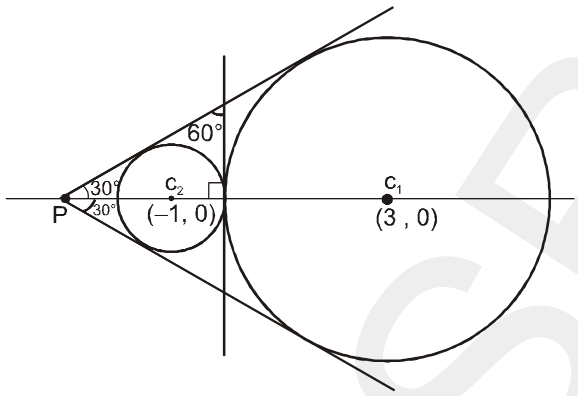
$m = \frac{12}{49}\sqrt{221}$

5. $c_1 : x^2 + y^2 - 6x = 0$ centre = (3, 0) radius = 3
$c_2 : x^2 + y^2 + 2x = 0$ centre = (–1, 0) radius = 1

$\frac{Pc_1}{Pc_2} = \frac{r_1}{r_2} = \frac{3}{1}$

co-ordinate of P = (–3, 0)
direct common tangent will pass through P.
Let slope of direct common tangent is m
$y - 0 = m(x + 3)$
$y = mx + 3m$
$mx - y + 3m = 0$

$\left|\frac{-m-0+3m}{\sqrt{1+m^2}}\right| = 1$



$\frac{|2m|}{\sqrt{1+m^2}} = 1$

$4m^2 = 1 + m^2$
$3m^2 = 1$

$m = \pm\frac{1}{\sqrt{3}}$

Equation of common tangents

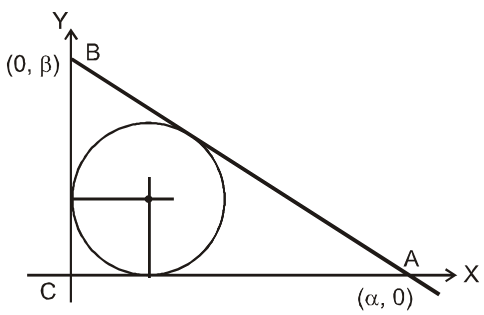
$y = \frac{1}{\sqrt{3}}x + \sqrt{3}$ .... (i)

$y = -\frac{1}{\sqrt{3}}x - \sqrt{3}$ .... (ii)

$x = 0$ .... (iii)

(i) , (ii) and (iii) form equileteral Δ

6.



Circum centre of ΔABC lie on the mid point of AB.

Equation of AB $\Rightarrow \frac{x}{\alpha} + \frac{y}{\beta} = 1$

$(h, k) \equiv \left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

$h = \frac{\alpha}{2}$ .....(i)

$k = \frac{\beta}{2}$ .....(ii)

Centre of inscribed circle $\equiv$ (2, 2)
radius = 2

$\Rightarrow \left|\frac{\frac{2}{\alpha}+\frac{2}{\beta}-1}{\sqrt{\frac{1}{\alpha^2}+\frac{1}{\beta^2}}}\right| = 2$ .... (iii)

$\Rightarrow \left(\frac{2}{\alpha}+\frac{2}{\beta}-1\right)^2 = 4\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)$

$1 + \frac{4}{\alpha^2} + \frac{4}{\beta^2} - \frac{4}{\alpha} - \frac{4}{\beta} + \frac{8}{\alpha\beta} = \frac{4}{\alpha^2} + \frac{4}{\beta^2}$

$1 + \frac{8}{\alpha\beta} = \frac{4}{\alpha} + \frac{4}{\beta}$

$\Rightarrow 1 + \frac{8}{4hk} = \frac{4}{2h} + \frac{4}{2k}$ $\Rightarrow 1 + \frac{2}{hk} = \frac{2}{h} + \frac{2}{k}$

$\Rightarrow \frac{hk+2}{hk} = \frac{2(h+k)}{hk}$

$hk + 2 = 2h + 2k$
from (iii)

$\frac{2}{\alpha} + \frac{2}{\beta} - 1 = \pm 2\frac{\sqrt{\alpha^2+\beta^2}}{\alpha\beta}$

$\Rightarrow 2\beta + 2\alpha - \alpha\beta = \pm 2\sqrt{\alpha^2+\beta^2}$

$\Rightarrow 4k + 4h - 4hk = \pm 4\sqrt{h^2+k^2}$

$\Rightarrow h + k - hk = \pm\sqrt{h^2+k^2}$

$x + y - xy = \pm\sqrt{x^2+y^2}$

k = ± 1 **Ans.**

7. $E = \sqrt{(\tan C - \sin A)^2 + (\cot C - \cos B)^2}$,
is the distance
b/w pt. P(tan C, cot C) and pt. (sin A, cos B)
Minimum dist. of P from circle is PQ – r
(where Q $\equiv$ origin)
$\sin^2A + \cos^2B = 1$
$[(\tan C - \sin A)^2 + (\cot C - \cos B)^2]_{min}$
$= (\tan^2C + \cot^2C) - (\sin^2A + \cos^2B)$
$= \tan^2C + \cot^2C - 1$

DPP NO. - 57

1. log2, log(2^x –1) , log(2^x + 3) → AP

$(2^x - 1)\, 2 = 2(2^x + 3)$

$y^2 - 2y + 1 = 2y + 6$

$y^2 - 4y - 5 = 0$

$y = 5, -1$

$2^x = 5$

$x = \log_2 5$

2. $\log_3(3^x - 8) + x - 2 = 0$

$\log_3(3^x - 8) = 2 - x$

$3^x - 8 = 3^{2-x}$

$3^x - 8 = \dfrac{9}{3^x}$

$\Rightarrow (3^x)^2 - 8\ 3^x - 9 = 0$

$3^x = t \quad t > 0$

$\Rightarrow t^2 - 8t - 9 = 0 \quad \Rightarrow t^2 - 9t + t - 9 = 0$

$\Rightarrow t(t - 9) + 1(t - 9) = 0$

$t = 9$

$3^x = 9$

$x = 2$

$\cos\dfrac{22\pi}{3}$ first term = 3

$\cos\left(6\pi + \dfrac{4}{3}\pi\right) \Rightarrow \cos\dfrac{4\pi}{3}$

$\cos\left(\pi + \dfrac{\pi}{3}\right) = -\cos\dfrac{\pi}{3} = -\dfrac{1}{2}$

$S = \dfrac{9}{1-r} = \dfrac{2}{1+\dfrac{1}{2}} = \dfrac{4}{3}$

3. $\dfrac{1}{2}\sum_{r=1}^{n}\dfrac{(2r+1)-1}{1.3.5.7.....(2r-1)(2r+1)}$

$\Rightarrow \dfrac{1}{2}\left[\sum_{r=1}^{n}\dfrac{1}{1.3.5.7.....(2r-1)} - \dfrac{1}{1.3.5.7....(2r+1)}\right]$

$\Rightarrow \dfrac{1}{2}\left[\dfrac{1}{1} - \dfrac{1}{1.3}\right]$

$\Rightarrow \dfrac{1}{2}\left[\dfrac{1}{1.3} - \dfrac{1}{1.3.5}\right]$

$\vdots \quad \vdots$

$\Rightarrow \dfrac{1}{2}\left[\dfrac{1}{1.3.5..(2r-1)} - \dfrac{1}{1.3.5.7.....(2r+1)}\right]$

$\Rightarrow \dfrac{1}{2}\left[1 - \dfrac{1}{1.3.5.7.....(2n+1)}\right]$

4. $BM = \ell$

Area of $\Delta = \dfrac{1}{2}(2\ell)(\ell) = 50$

$\Rightarrow \ell = 5\sqrt{2}$

5. $S_1 = x^2 + y^2 - 2x + 3y - 7 = 0$

$S_2 = x^2 + y^2 + 5x - 5y + 9 = 0$

$S_3 = x^2 + y^2 + 7x - 9y + 29 = 0$

$S_1 - S_2 = 0$

$\Rightarrow -7x + 8y - 16 = 0$ .... (1)

$S_2 - S_3 = 0$

$\Rightarrow -2x + 4y - 20 = 0$ .... (2)

$-3x + 24 = 0$

$x = 8 \quad y = 9$

centre $(8, 9) = (-g, -f)$

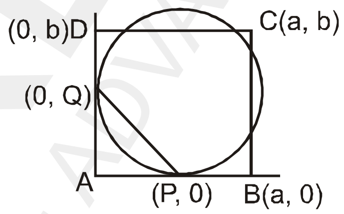
radius $= \sqrt{64 + 81 - 16 + 27 - 7}$

$= \sqrt{g^2 + f^2 - c}$

$g^2 - c + 81 = 149$

$\Rightarrow 2\sqrt{g^2 - c} = 4\sqrt{17}$

6.



Equation of circle be (p = q)

$(x - P^2) + (y - p)^2 = P^2$

$x^2 + y^2 - 2px - 2py + p^2 = 0$

This circle passes through (a, b)

$a^2 + b^2 - 2pa - 2pb + p^2 = 0$ ....(1)

PQ = x+ y = P and distance of c from PQ = 5

$(a + b - P)^2 = 50$

$a^2 + b^2 + P^2 + 2(ab - pa - pb) = 50$ .... (2)

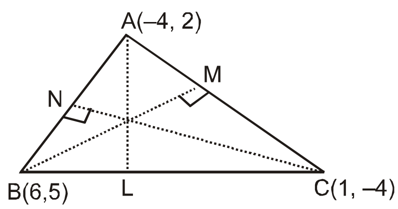
(1) – (2)

2ab = 50

$\Rightarrow$ ab = 25 = area of rectangle

7. Altitude AL

$m_{BC} = \dfrac{5+4}{6-1} = \dfrac{9}{5}$



$m_{AL} = -\dfrac{5}{9}$

Equation of altitude AL

$y - 2 = \dfrac{-5}{9}(x + 4)$

$9y - 18 = -5x + 20 \Rightarrow 5x + 9y + 2 = 0$

Altitude BM $5x - 6y = 0$

Altitube CN $10x + 3y + 2 = 0$

DPP NO. - 58

1. Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

S_1 : $x^2 + y^2 + 2x - 3 = 0$
common chord will pass through the centre of the circle S_1
$(2g - 2)x + 2fy + c + 3$
point $(-1, 0)$ $(-1, 0)$
$-2g + 2 + c + 3$ .... (1)
S_2 : $x^2 + y^2 - 1 = 0$
$2gx + 2fy + c + 1 = 0$
point $(0, 0)$ $(0, 0)$
$c = -1$
S_3 : $x^2 + y^2 + 2y - 3 = 0$
$2gx + (2fy - 2)y + c + 3 = 0$
$\downarrow (0, -1)$
$-2f + 2 + c + 3 = 0$
put $c = -1$
$-2f + 4 = 0 \Rightarrow f = 2$
$-2g + 2 + c + 3 = 0$
$g = 2$
$r = \sqrt{g^2 + f^2 - c} = \sqrt{4 + 4 + 1} = 3$

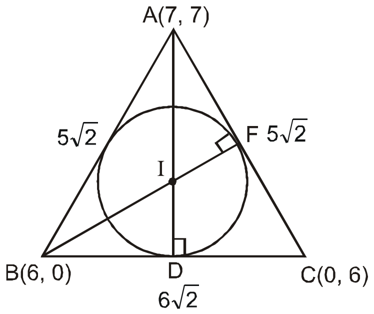
2. circle = radical centre
$S_1 - S_2 = 0 \Rightarrow 2x - 2 = 0 \Rightarrow x = 1$
$S_2 - S_3 = 0 \Rightarrow 2y - 2 = 0 \Rightarrow y = 1$
$C \equiv (1, 1)$
r = length of tangent from (1, 1) to S_2
$r = 1$
$a + b + r = 1 + 1 + 1 = 3$

3. S_1 and S_2 touch each other internally at (1, 0)
$(x - 1)^2 + y^2 + \lambda [x - 1] = 0$
$\downarrow (3, 2)$
$4 + 4 + \lambda (2) = 0$
$\lambda = -4$
$x^2 + y^2 - 2x + 1 - 4x + 4$
$x^2 + y^2 - 6x + 5 = 0$
$r = 2$

5. $\sum_{k=1}^{5} (x - k)^2$
$(x - 1)^2 + (x - 2)^2 + (x - 3)^2 + (x - 4)^2 + (x - 5)^2$
min. when $x = 3$

6. $I = \left[\frac{30\sqrt{2} + 42\sqrt{2}}{16\sqrt{2}}, \frac{30\sqrt{2} + 42\sqrt{2}}{16\sqrt{2}}\right]$

$I = \left[\frac{72}{16}, \frac{72}{16}\right]$



$I = \left[\frac{9}{2}, \frac{9}{2}\right]$
$D \equiv (3, 3)$

$r = \sqrt{\left(\frac{9}{2} - 3\right)^2 + \left(\frac{9}{2} - 3\right)^2}$

$= \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2}$

$= \frac{3}{2}\sqrt{2} = \frac{3}{\sqrt{2}}$

$\left(x - \frac{9}{2}\right)^2 + \left(y - \frac{9}{2}\right)^2 = \left(\frac{3}{\sqrt{2}}\right)^2$

$x^2 + y^2 - 9x - 9y + \frac{81}{2} - \frac{9}{2}$
$x^2 + y^2 - 9x - 9y + 36 = 0$

7. $\frac{x^{\ell n(y/z)} + y^{\ell n(z/x)} + z^{\ell n(x/y)}}{3} \geq 3\left[\frac{x^{\ell ny}}{x^{\ell nz}} \cdot \frac{y^{\ell nz}}{y^{\ell nx}} \cdot \frac{z^{\ell nx}}{z^{\ell ny}}\right]$

$\frac{x^{\ell n(y/z)} + y^{\ell n(z/x)} + z^{\ell n(x/y)}}{3} \geq 3$

because
$x^{\ell ny} = y^{\ell n x}$
$x^{\ell nz} = z^{\ell nx}$
$y^{\ell nz} = z^{\ell ny}$

DPP NO. - 59

1. (i) $\left(\frac{2^5}{x}\right) - {}^5C_1\left(\frac{2}{x}\right)^4\left(\frac{x}{2}\right)^1 + {}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{x}{2}\right)^2$

$- {}^5C_3\left(\frac{2}{x}\right)^2\left(\frac{x}{2}\right)^3 + {}^5C_4\left(\frac{2}{x}\right)^1\left(\frac{x}{2}\right)^4 - {}^5C_5\left(\frac{x}{2}\right)^5$

$= \left(\frac{2}{x}\right)^5 - 5\left(\frac{2}{x}\right)^3 + 10\left(\frac{2}{x}\right) - 10\left(\frac{x}{2}\right) + 5\left(\frac{x}{2}\right)^3 - \left(\frac{x}{2}\right)^5$

(ii) $(y^2)^4 + {}^4C_1(y^2)^3(2/y) + {}^4C_2(y^2)^2(2/y)^2 + {}^4C_3(y^2)(2/y)^3$

$+ {}^4C_4(2/y)^4 = y^8 + 8y^5 + 24y^2 + \frac{32}{y} + \frac{16}{y^4}$

2. 7^{th} term from end = $(18 - 7 + 2)^{th}$ term from begning

$= {}^{18}C_{12}\left(-\frac{1}{3\sqrt{x}}\right)^{12}(9x)^6 = {}^{18}C_6$

3. 7^{th} term from beginning $T_7 = {}^nC_6(2)^{\frac{n-6}{3}}\left(\frac{1}{3}\right)^2$

7^{th} term from the end $T_{n-5} = {}^nC_{n-6}(2)^2\left(\frac{1}{3}\right)^{\frac{n-6}{3}}$