Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-05

[SINGLE CORRECT CHOICE TYPE] [2 × 3 = 6]

Q.1 Area of the parallelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$ and $y = nx + 1$ equals

(A) $\frac{|m+n|}{(m-n)^2}$ (B) $\frac{2}{|m+n|}$ (C) $\frac{1}{|m+n|}$ (D) $\frac{1}{|m-n|}$

[JEE 2001 (Screening)]

Q.2 Number of elements in the domain of the function $f(x) = \frac{1}{\sqrt{{}^{10}C_{x-1} - 3\cdot{}^{10}C_x}}$, is

(A) 2 (B) 3 (C) 4 (D) 10

[COMPREHENSION TYPE] [2 × 3 = 6]

Paragraph for question nos. 3 & 4

A straight line $L : 4x - 4y + 3 = 0$ is rotated in clockwise about the point where the line cuts the y-axis and a circle S_1 whose centre is $\left(\lambda, \frac{3}{4}\right)$ touches both the lines L and L_1 (L_1 is the line obtained after rotation) and the x-axis.

Q.3 The value of $[\lambda]$ is equal to

(A) 1 (B) 2 (C) 8 (D) 9

[**Note :** [k] denotes greatest integral value of k.]

Q.4 If area of the triangle formed by the lines L_1, angle bisector between L & L_1 and the x-axis is $\frac{p}{q}$, $p, q \in N$ then least value of $(p + q)$ equals

(A) 3 (B) 17 (C) 41 (D) 56

[INTEGER TYPE] [2 × 5 = 10]

Q.5 If the number of arrangements of 4 alike apples, 5 alike mangoes, 1 banana and 1 orange in which all the apples are together or all the mangoes are together is K, then find the sum of digits in K.

Q.6 In ΔABC, if $\sin A (\sin A + \cos B - \sin B) + \cos A (\cos A + \sin B + \cos B) = 1 + \sin C$ and $a = 4$, $b = 3$, then find the area of the ΔABC.

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-06

[SINGLE CORRECT CHOICE TYPE] [3 × 3 = 9]

Q.1 If $x^2 - 3x + 2$ is a factor of $x^4 - Ax^2 + B$, then $(A + B)$ is equal to
(A) 1 (B) –1 (C) 9 (D) –9

Q.2 In the expansion of $(ax + b)^{2000}$, if the coefficients of x^2 and x^3 are equal then $b = ka$, where $k \in N$, then k is not divisible by
(A) 11 (B) 9 (C) 37 (D) 111

Q.3 The domain of $f(x) = \dfrac{\log (x^2 + 5x + 6)}{[x] - 1}$, is

[**Note:** where [y] denotes greatest integer function less than or equal to y.]
(A) $(-\infty, -3) \cup (-2, \infty)$ (B) $(-\infty, -3) \cup [-2, \infty)$
(C) $(-\infty, 1) \cup (-2, \infty)$ (D) none

[MULTIPLE CORRECT CHOICE TYPE] [2 × 4 = 8]

Q.4 The first two terms of a geometric progression add up to 12 . The sum of third and fourth terms is 48 . If the terms are alternately positive and negative, then
(A) common ratio of G.P. is equal to –2. (B) common ratio of G.P. is equal to 2.
(C) sixth term of G.P. is equal to 128. (D) sixth term of G.P. is equal to 384.

Q.5 The equation of straight line which is equidistant from the points A(2, – 2), B(6, 1) and C(–3, 4) can be
(A) $2x + 6y - 5 = 0$ (B) $12x + 10y - 43 = 0$
(C) $6x - 8y - 11 = 0$ (D) $6x - 8y + 11 = 0$

[INTEGER TYPE] [2 × 5 = 10]

Q.6 Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = \{1, 2, 3, 4\}$. If N be the number of 4 element subsets Y of X such that $10 \in Y$ and the intersection of Y and A is not empty, then find the value of $\dfrac{(N-4)}{10}$.

Q.7 In a triangle ABC, with usual notation, if $a^2b^2c^2 (\sin 2A + \sin 2B + \sin 2C) = \lambda (\Delta)^x$ where Δ is the area of the triangle and $x \in Q$, find (λx).

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-07

[SINGLE CORRECT CHOICE TYPE] [5 × 3 = 15]

Q.1 $\left(\tan\frac{3\pi}{8}\right)^{2013}+\left(-\cot\frac{3\pi}{8}\right)^{2013}$ is

(A) even integer (B) odd integer
(C) rational which is not an integer (D) irrational

Q.2 Number of zeroes at the end of $\lfloor 495$, is
(A) 119 (B) 120 (C) 121 (D) 122

Q.3 A geometric sequence has four positive terms a_1, a_2, a_3, a_4. If $\frac{a_3}{a_1}=9$ and $a_1+a_2=\frac{4}{3}$, then a_4 equals

(A) 3 (B) 9 (C) 27 (D) $3\sqrt{3}$

Q.4 In a triangle ABC, Let $\angle C=\frac{\pi}{2}$. If 'r' is the inradius and 'R' is the circumradius of the triangle, then $2(r+R)$ is equal to
(A) a + b (B) b + c (C) c + a (D) a + b + c
[JEE '2000 (Screening) 1]

Q.5 Sum of integers in domain of function $f(x)=\sqrt{(6x-8-x^2)\,ln^2(|x-4|)}$ is
(A) 10 (B) 9 (C) 14 (D) 15

[INTEGER TYPE] [2 × 5 = 10]

Q.6 The parallelogram is bounded by the lines $y=ax+c$; $y=ax+d$; $y=bx+c$ and $y=bx+d$ and has the area equal to 18. The parallelogram bounded by the lines $y=ax+c$; $y=ax-d$; $y=bx+c$ and $y=bx-d$ has area 72. Given that a, b, c and d are positive integers, find the smallest possible value of $(a+b+c+d)$.

Q.7 Let the variable line $ax+by+c=0$, where a, b, c are in arithmetic progression be normal to a family of circles. If r be the radius of the circle of the family which intersects the circle $x^2+y^2-4x-4y-1=0$ orthogonally, then find the value of r^2.

Dpp. No.-08

[SINGLE CORRECT CHOICE TYPE] [4 × 3 = 12]

Q.1 If all the letters of the word "NIDHI" are arranged in alphabetical order then the rank of the word NIDHI, is

(A) 43 (B) 54 (C) 55 (D) 56

Q.2 If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$ equals

(A) $(n-1)a_n$ (B) $n\,a_n$ (C) $n\,a_n/2$ (D) can not be determined [JEE'98]

Q.3 In ΔABC (right angled at B), length of the median BF is equal to 5 and D is the foot of the perpendicular from F on BA. If $\tan A = \frac{4}{3}$ then the area of parallelogram BDEF (taken in order), is

(A) 12 sq. units (B) 18 sq. units (C) 24 sq. units (D) 32 sq. units

Q.4 The domain of the function $f(x) = \sqrt{\log_2\left(\frac{1}{\log_2 x}\right)}$ is given by

(A) $\{x : x > 0\}$ (B) $\{x : 0 < x < 1\}$ (C) $\{x : 1 < x \le 2\}$ (D) $\{x : x > 1\}$

[MULTIPLE CORRECT CHOICE TYPE] [2 × 4 = 8]

Q.5 Consider a quadratic polynomial $P(x) = x^2 + bx + c$, if the equation $x^2 + bx + c = 2$ has only one real solution $x = 1$, then

(A) $2c + b = 4$ (B) Range of $P(x) = [2, \infty)$

(C) $P(x)$ is symmetric about the line $x = 1$. (D) $P(x) = 0$ has no real roots.

Q.6 The centre of a circle C lies on the line $2x - 2y + 9 = 0$ and the circle C cuts orthogonally the circle $x^2 + y^2 = 4$. The circle C passes through fixed points

(A) $(-3, 3)$ (B) $\left(\frac{-1}{2}, \frac{1}{2}\right)$ (C) $(-4, 4)$ (D) $(-2, 2)$

[MATRIX TYPE] [2+2+2=6]

Q.7

	Column – I	Column – II
(A)	In ΔABC, if $R = 4r$ then $(\cos A + \cos B + \cos C)$ is less than	(P) 1
(B)	In ΔABC, if $\frac{a^2b^2c^2(\Sigma \sin 2A)}{(rr_1r_2r_3)^{3/2}} = \lambda$ then $\frac{\lambda}{8}$ is equal to	(Q) 2
(C)	In ΔABC, if $r_1 = 2, r_2 = 3, r_3 = 4$ and area of the ΔABC is Δ, then $\frac{\sqrt{26}}{8}\Delta$ is greater than or equal to	(R) 3
		(S) 4

[**Note :** All the symbols used have usual meaning in ΔABC.]

Dpp. No.-05 TO 08
(SOLUTION)

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-05 (SOLUTION)

[SINGLE CORRECT CHOICE TYPE] [2 × 3 = 6]

Q.1 Area of the parallelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$ and $y = nx + 1$ equals

(A) $\frac{|m+n|}{(m-n)^2}$ (B) $\frac{2}{|m+n|}$ (C) $\frac{1}{|m+n|}$ (D*) $\frac{1}{|m-n|}$

[JEE 2001 (Screening)]

[Sol.$_{244/st.line/SC}$ **[D]**

$y = mx$ $y = mx + 1$, $y = nx$ $y = nx + 1$

Area of $||gm = \left|\frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2}\right| = \left|\frac{1\times 1}{m-n}\right| = \frac{1}{|m-n|}$]

Q.2 Number of elements in the domain of the function $f(x) = \frac{1}{\sqrt{{}^{10}C_{x-1} - 3\cdot {}^{10}C_x}}$, is

(A*) 2 (B) 3 (C) 4 (D) 10

[Hint:$_{122/func}$ {9, 10} only]

[COMPREHENSION TYPE] [2 × 3 = 6]

Paragraph for question nos. 3 & 4

A straight line $L : 4x - 4y + 3 = 0$ is rotated in clockwise about the point where the line cuts the y-axis and a circle S_1 whose centre is $\left(\lambda, \frac{3}{4}\right)$ touches both the lines L and L_1 (L_1 is the line obtained after rotation) and the x-axis.

Q.3 The value of $[\lambda]$ is equal to

(A*) 1 (B) 2 (C) 8 (D) 9

[**Note :** [k] denotes greatest integral value of k.]

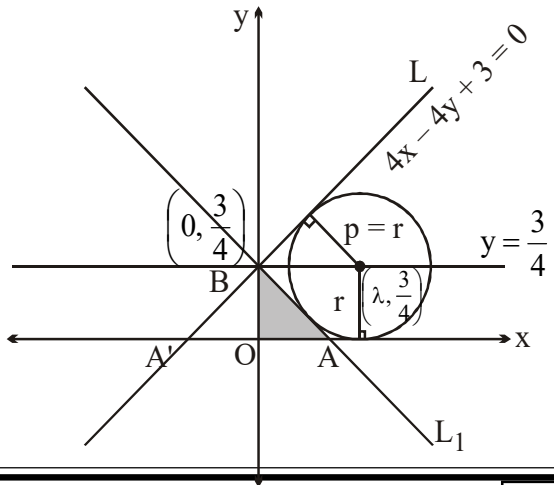
Q.4 If area of the triangle formed by the lines L_1, angle bisector between L & L_1 and the x-axis is $\frac{p}{q}$, p, q $\in N$ then least value of $(p + q)$ equals

(A) 3 (B) 17 (C*) 41 (D) 56

[Sol.$_{30481-82/cir}$ Using p = r

(i) $\frac{4\lambda - 3 + 3}{4\sqrt{2}} = r = \frac{3}{4} \Rightarrow \frac{\lambda}{\sqrt{2}} = \frac{3}{4}$

$\therefore \lambda = \frac{3}{2\sqrt{2}} \Rightarrow [\lambda] = 1$



(ii) $\Delta OAB = \frac{1}{2}\times\frac{3}{4}\times\frac{3}{4} = \frac{9}{32} \equiv \frac{p}{q}$

$\therefore (p+q) = 41$. **Ans.**

Note: slope of line L is 1, hence $\angle OA'B = 45°$
Hence slope of line L_1 is -1.]

[INTEGER TYPE] [2 × 5 = 10]

Q.5 If the number of arrangements of 4 alike apples, 5 alike mangoes, 1 banana and 1 orange in which all the apples are together or all the mangoes are together is K, then find the sum of digits in K.

[Ans. 0009]

[Sol.$_{50037/perm/OMR}$ No. of ways when Apples are together + no. of ways when mangoes are together – when Apples are also & Mangoes are also together

$\frac{8!}{5!}+\frac{7!}{4!}-4! \Rightarrow k = 522 \Rightarrow 9.$]

Q.6 In ΔABC, if $\sin A(\sin A + \cos B - \sin B) + \cos A(\cos A + \sin B + \cos B) = 1 + \sin C$ and $a = 4$, $b = 3$, then find the area of the ΔABC.

[Ans. 6]

[Sol.$_{50049/sot/OMR}$ $\sin A(\sin A + \cos B - \sin B) + \cos A(\cos A + \sin B + \cos B) = 1 + \sin C$

$\Rightarrow \sin A(\cos B - \sin B) + \cos A(\sin B + \cos B) = \sin C$

$\Rightarrow \sin(A+B) + \cos(A+B) = \sin C$

$\therefore \cos(A+B) = 0$

$\Rightarrow A + B = \frac{\pi}{2}$

$\therefore C = \frac{\pi}{2}$

$\Rightarrow \Delta = \frac{1}{2}ab\sin C = \frac{1}{2}\times 4\times 3 = 6$. **Ans.**]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-06

[SINGLE CORRECT CHOICE TYPE] [3 × 3 = 9]

Q.1 If $x^2 - 3x + 2$ is a factor of $x^4 - Ax^2 + B$, then $(A + B)$ is equal to

(A) 1 (B) –1 (C*) 9 (D) –9

[Sol.$_{351/qe/SC}$ $1 - A + B = 0$

$16 - 4A + B = 0$

$\therefore$ $A = 5, B = 4$]

Q.2 In the expansion of $(ax + b)^{2000}$, if the coefficients of x^2 and x^3 are equal then $b = ka$, where $k \in N$, then k is not divisible by

(A*) 11 (B) 9 (C) 37 (D) 111

[Sol.$_{128/bin/SC}$ $T_{r+1} = {}^nC_r\, a^{n-r} \cdot x^{n-r} \cdot b^r$ where $n = 2000$

coefficient of x^2 = coefficient of x^3

${}^{2000}C_2 \cdot a^2 \cdot b^{1998} = {}^{2000}C_3 \cdot a^3 \cdot b^{1997}$

$\Rightarrow$ $b = 666a$ $\Rightarrow$ $k = 666$ which is not divisible by 11. **Ans.**]

Q.3 The domain of $f(x) = \dfrac{\log (x^2 + 5x + 6)}{[x] - 1}$, is

[**Note:** where [y] denotes greatest integer function less than or equal to y.]

(A) $(-\infty, -3) \cup (-2, \infty)$ (B) $(-\infty, -3) \cup [-2, \infty)$

(C) $(-\infty, 1) \cup (-2, \infty)$ (D*) none

[Sol.$_{161/func/SC}$ Correct domain is $(-\infty, -3) \cup (-2, 1) \cup [2, \infty)$.]

[MULTIPLE CORRECT CHOICE TYPE] [2 × 4 = 8]

Q.4 The first two terms of a geometric progression add up to 12 . The sum of third and fourth terms is 48 . If the terms are alternately positive and negative, then

(A*) common ratio of G.P. is equal to –2. (B) common ratio of G.P. is equal to 2.

(C) sixth term of G.P. is equal to 128. (D*) sixth term of G.P. is equal to 384.

[Sol.$_{40555/seq/MORE}$

Let $a, ar, ar^2, \ldots\ldots$ $(r < 0)$

Now, $a + ar = 12$ ……(1)

Also $ar^2 + ar^3 = 48$ ……(2)

So, $\dfrac{\text{equation (2)}}{\text{equation (1)}} \Rightarrow \dfrac{ar^2(1+r)}{a(1+r)} = 4$

$\Rightarrow$ $r^2 = 4$ (As $r \neq -1$)

$\therefore$ $r = -2$ (Reject $r = 2$)

So, $a = -12$

$\therefore$ $t_6 = ar^5 = -12\,(-2)^5 = 12 \times 32 = 384$. **Ans.**]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Q.5 The equation of straight line which is equidistant from the points A(2, – 2), B(6, 1) and C(–3, 4) can be

(A*) $2x + 6y - 5 = 0$ (B*) $12x + 10y - 43 = 0$

(C) $6x - 8y - 11 = 0$ (D*) $6x - 8y + 11 = 0$

[Sol.$_{40534/st.line/MORE}$ **Method-I**

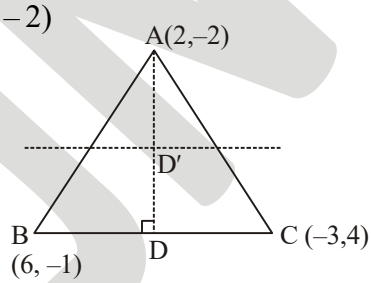
Let m_1, m_2 and m_3 be the slopes of BC, CA and AB respectively,

then $m_1 = \frac{-1}{3}$, $m_2 = \frac{-6}{5}$ and $m_3 = \frac{3}{4}$.

Let AD be the altitude, then equation of AD will be $(y + 2) = 3(x - 2)$

$\Rightarrow 3x - y - 8 = 0$

which intersects BC i.e. $x + 3y - 9 = 0$ at $D\left(\frac{33}{10}, \frac{19}{10}\right)$.



Now mid point of AD, $D'\left(\frac{53}{20}, \frac{-1}{20}\right)$

Equation of a straight line parallel to BC and passing through D' will be

$2(x + 3y) - 5 = 0$

Similarly, the equation of a straight line parallel to AC and AB and passing through the mid-point of altitude of B and C respectively will be $12x + 10y - 43 = 0$ and $6x - 8y + 11 = 0$ respectively.

Method-II

Perpendicular distances of points A, B and C from the lines given by

Option (A) : $\frac{13}{2\sqrt{10}}, \frac{13}{2\sqrt{10}}, \frac{13}{2\sqrt{10}}$, respectively, Option (B) : $\frac{39}{2\sqrt{61}}, \frac{39}{2\sqrt{61}}, \frac{39}{2\sqrt{61}}$ respectively.

Option (C) : $\frac{17}{10}, \frac{17}{10}, \frac{61}{10}$ respectively, Option (D) : $\frac{39}{10}, \frac{39}{10}, \frac{39}{10}$ respectively

$\therefore$ Clearly options (A), (B) and (D) give equidistant lines. Ans. (A), (B), (D).]

[INTEGER TYPE] [2 × 5 = 10]

Q.6 Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = \{1, 2, 3, 4\}$. If N be the number of 4 element subsets Y of X such that $10 \in Y$ and the intersection of Y and A is not empty, then find the value of $\frac{(N-4)}{10}$.

[Ans. 0007]

[Sol.$_{50028/perm/OMR}$ S_1 : 1, 2, 3, 4 (4)

S_2 : 5, 6, 7, 8, 9 (5)

Number of 4 elements subsets Y : ${}^9C_3 - {}^5C_3 = 84 - 10 = 74$. **Ans.**]

Q.7 In a triangle ABC, with usual notation, if $a^2b^2c^2(\sin 2A + \sin 2B + \sin 2C) = \lambda(\Delta)^x$ where Δ is the area of the triangle and $x \in Q$, find (λx). [Ans. 0096]

[Sol.$_{50749/sot/OMR}$ $a^2b^2c^2\ 4 \sin A \sin B \sin C = 4\ a^2\ b^2\ c^2\ \frac{abc}{8R^3}$ [Using $\sum \sin 2A = \prod \sin A$]

$\frac{1}{2}\left(\frac{abc}{4R}\right)^3 \cdot 64 = 32(\Delta)^3$ $\left(\text{Using } \frac{abc}{4R} = \Delta\right)$

$\therefore \lambda = 32$ and $x = 3$

$\Rightarrow (\lambda x) = 96$. **Ans.**]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-07

[SINGLE CORRECT CHOICE TYPE] [5 × 3 = 15]

Q.1 $\left(\tan\frac{3\pi}{8}\right)^{2013}+\left(-\cot\frac{3\pi}{8}\right)^{2013}$ is

(A*) even integer (B) odd integer
(C) rational which is not an integer (D) irrational

[Sol.$_{99/bin/SC}$ $\left(\tan\frac{3\pi}{8}\right)^{2013}+\left(-\cot\frac{3\pi}{8}\right)^{2013}=\left(\sqrt{2}+1\right)^{2013}-\left(\sqrt{2}-1\right)^{2013}$

$=2\left[{}^{2013}C_1\left(\sqrt{2}\right)^{2012}+{}^{2013}C_3\left(\sqrt{2}\right)^{2010}+{}^{2013}C_5\left(\sqrt{2}\right)^{2008}+.....+{}^{2013}C_{2013}\left(\sqrt{2}\right)^{0}\right]$

= which is an even integer $\Rightarrow$ **(A)**]

Q.2 Number of zeroes at the end of $\lfloor\underline{495}$, is

(A) 119 (B) 120 (C*) 121 (D) 122

[Sol.$_{167/perm/SC}$ Number of zeroes at the end of $(495)! = 99 + 19 + 3 = 121$ **Ans.**]

Q.3 A geometric sequence has four positive terms a_1, a_2, a_3, a_4. If $\frac{a_3}{a_1}=9$ and $a_1+a_2=\frac{4}{3}$, then a_4 equals

(A) 3 (B*) 9 (C) 27 (D) $3\sqrt{3}$

[Sol.$_{247/seq/SC}$ $\frac{ar^2}{a}=9 \Rightarrow r=3$

$a+ar=\frac{4}{3} \quad \therefore \quad a\cdot 4=\frac{4}{3} \quad \Rightarrow \quad a=\frac{1}{3}$

$\therefore \quad a_4=ar^3=\frac{1}{3}\cdot 3^3=3^2=9$. **Ans.**]

Q.4 In a triangle ABC, Let $\angle C=\frac{\pi}{2}$. If 'r' is the inradius and 'R' is the circumradius of the triangle, then $2(r+R)$ is equal to

(A*) a + b (B) b + c (C) c + a (D) a + b + c

[JEE '2000 (Screening) 1]

[Sol.$_{146/sot/SC}$ $\angle C=\frac{\pi}{2} \quad \therefore \quad a^2+b^2=c^2$

$\therefore \quad 2(r+R)$

$2\left[(s-c)\tan\frac{c}{2}+\frac{c}{2\sin c}\right]=2\left[\frac{a+b-c}{2}\tan 45^\circ+\frac{c}{2\sin 90^\circ}\right]$

$$2\left[\frac{a+b-c}{2}+\frac{c}{2}\right]=2\left(\frac{a+b}{2}\right)=a+b.$$ **Ans.**]

Q.5 Sum of integers in domain of function $f(x)=\sqrt{(6x-8-x^2)\,ln^2(|x-4|)}$ is

(A*) 10 (B) 9 (C) 14 (D) 15

[Sol.$_{293/func/SC}$ $(6x-8-x^2)\,ln^2|x-4|\geq 0$

$\Rightarrow$ $6x-8-x^2\geq 0,\ x=5,\ x\neq 4$

$x^2-6x+8\leq 0$

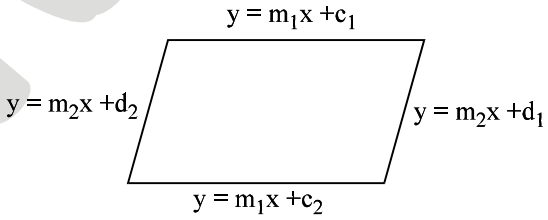
$x\in[2,4]$

$\therefore$ $D_f=[2,4)\cup\{5\}$]

[INTEGER TYPE] [2 × 5 = 10]

Q.6 The parallelogram is bounded by the lines $y=ax+c$; $y=ax+d$; $y=bx+c$ and $y=bx+d$ and has the area equal to 18. The parallelogram bounded by the lines $y=ax+c$; $y=ax-d$; $y=bx+c$ and $y=bx-d$ has area 72. Given that a, b, c and d are positive integers, find the smallest possible value of $(a+b+c+d)$. [Ans. 16]

[Sol.$_{50711/st.line}$ Area of the parallelogram $=\dfrac{|c_1-c_2||d_1-d_2|}{|m_1-m_2|}$ (for the figure as shown)



hence Area of the parallelogram formed by the 1st set of 4 lines

$$=\frac{|c-d|^2}{|a-b|}=18 \quad ....(1)$$

and Area of the parallelogram formed by the 2nd set of 4 lines

$$=\frac{|c+d|^2}{|a-b|}=72 \quad ....(2)$$

from (1) $|c-d|^2=|a-b|\cdot 18$

and from (2) $|c+d|^2=|a-b|\cdot 72$. Since $a, b, c, d\in N$,

Hence RHS must be a perfect square for some least value of $|a-b|$ which obviously can be equal to 2 (think ?) $\Rightarrow$ $|a-b|=2$

and $\boxed{a=1 \text{ and } b=3}$

hence $c-d=6$
and $c+d=12$ $\Rightarrow$ $\boxed{c=9 \text{ and } d=3}$

$\therefore$ $(a+b+c+d)=16$ Ans.]

Q.7 Let the variable line $ax + by + c = 0$, where a, b, c are in arithmetic progression be normal to a family of circles. If r be the radius of the circle of the family which intersects the circle $x^2 + y^2 - 4x - 4y - 1 = 0$ orthogonally, then find the value of r^2. [Ans.8]

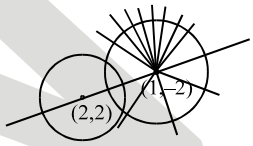
[Sol.$_{50704/cir/OMR}$ As a, b, c are in A.P., so $ax + by + c = 0$ represents a family of lines passing through the fixed point (1 ,–2). Since each member of the family is normal to a circle hence its centre must be (1, –2).

So, the family of circles with centre (1, – 2) will be given by

$(x - 1)^2 + (y + 2)^2 = r^2 \Rightarrow x^2 + y^2 - 2x + 4y + (5 - r^2) = 0$

And using condition of orthogonality, we get

$2[(-1)(-2) + (2)(-2)] = -1 + 5 - r^2 \Rightarrow r^2 = 8$ **Ans.**]



Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-08

[SINGLE CORRECT CHOICE TYPE] [4 × 3 = 12]

Q.1 If all the letters of the word "NIDHI" are arranged in alphabetical order then the rank of the word NIDHI, is

(A) 43 (B) 54 (C*) 55 (D) 56

[Sol.$_{171/perm/SC}$ NIDHI

DHIIN

D					12
H					12
I					24
N	D				3
N	H				3
NIDHI					55th]

Q.2 If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$ equals

(A) $(n-1)a_n$ (B) $n\,a_n$ (C*) $n\,a_n / 2$ (D) can not be determined

[JEE'98]

[Sol.$_{119/bin/SC}$ $a_n = \frac{1}{{}^nC_0} + \frac{1}{{}^nC_1} + \frac{1}{{}^nC_2} + ...$

$$S = \frac{0}{{}^nC_0} + \frac{1}{{}^nC_1} + \frac{2}{{}^nC_2} + \frac{3}{{}^nC_3} + ... + \frac{n}{{}^nC_n}$$

$$S = \frac{n}{{}^nC_0} + \frac{n-1}{{}^nC_n} + ...$$

$$2S = n\left[\frac{1}{{}^nC_0} + \frac{1}{{}^nC_1} + \frac{1}{{}^nC_2} + ...\right] = \frac{na_n}{2}$$ **Ans.**]

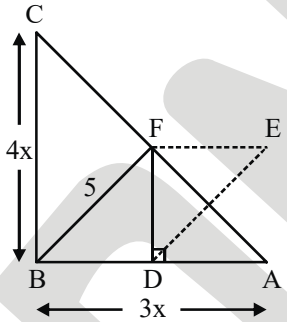
Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Q.3 In ΔABC (right angled at B), length of the median BF is equal to 5 and D is the foot of the perpendicular from F on BA. If $\tan A = \frac{4}{3}$ then the area of parallelogram BDEF (taken in order), is

(A*) 12 sq. units (B) 18 sq. units (C) 24 sq. units (D) 32 sq. units

[Sol.$_{235/st.line/SC}$ $16x^2 + 9x^2 = 100$

$\Rightarrow \quad x = 2$



$BD = 3$ and $DF = 4$

Area of parallel = Base × Height
$= 3 \times 4$
$= 12$ sq. units.]

Q.4 The domain of the function $f(x) = \sqrt{\log_2\left(\frac{1}{\log_2 x}\right)}$ is given by

(A) $\{x : x > 0\}$ (B) $\{x : 0 < x < 1\}$ (C*) $\{x : 1 < x \le 2\}$ (D) $\{x : x > 1\}$

[Sol.$_{303/func/SC}$ $\log_2\left(\frac{1}{\log_2 x}\right) \ge 0$]

$\frac{1}{\log_2 x} \ge 1$; also $\log_2 x > 0, \quad x > 1$

$\log_x 2 \ge 1$

$2 \ge x \quad \Rightarrow \quad x \le 2$

$\therefore \quad 1 < x \le 2 \quad \Rightarrow \quad$ **(C)**]

[MULTIPLE CORRECT CHOICE TYPE] [2 × 4 = 8]

Q.5 Consider a quadratic polynomial $P(x) = x^2 + bx + c$, if the equation $x^2 + bx + c = 2$ has only one real solution $x = 1$, then

(A*) $2c + b = 4$ (B*) Range of $P(x) = [2, \infty)$

(C*) $P(x)$ is symmetric about the line $x = 1$. (D*) $P(x) = 0$ has no real roots.

[Sol.$_{40055/qe/MORE}$ $x^2 + bx + c = 2$ has one only one real solution

$\Rightarrow P_{min} = 2$ at $x = 1 \Rightarrow P(x) = (x-1)^2 + 2 = x^2 + 3 - 2x$]

Q.6 The centre of a circle C lies on the line $2x - 2y + 9 = 0$ and the circle C cuts orthogonally the circle $x^2 + y^2 = 4$. The circle C passes through fixed points

(A) $(-3, 3)$ (B*) $\left(\frac{-1}{2}, \frac{1}{2}\right)$ (C*) $(-4, 4)$ (D) $(-2, 2)$

[Sol.$_{40537/cir/More}$ $C : x^2 + y^2 + 2gx + 2fy + c = 0$

$x^2 + y^2 = 4$

$2(g_1g_2 + f_1f_2) = C_1 + C_2$

$2(0 + 0) = C - 4 \Rightarrow C = 4$

also $2x - 2y + 9 = 0$

$2(-g) - 2(-f) + 9 = 0$

$2f = 2g - 9$

$\therefore \quad x^2 + y^2 + 2gx + (2g - 9)y + 4 = 0$

$\therefore \quad (x^2 + y^2 - 9y + 4) + 2g(x + y) = 0$

$\therefore \quad x^2 + y^2 - 9y + 4 = 0$ and $x + y = 0$

$\therefore \quad x^2 + x^2 + 9x + 4 = 0 \Rightarrow 2x^2 + 9x + 4 = 0 \Rightarrow (2x + 1)(x + 4) = 0 \Rightarrow x = \frac{-1}{2}, -4.$

$\therefore \quad$ Point $\left(\frac{-1}{2}, \frac{1}{2}\right), (-4, 4)$. **Ans.**]

[MATRIX TYPE] [2+2+2=6]

Q.7

	Column – I	Column – II
(A)	In ΔABC, if $R = 4r$ then $(\cos A + \cos B + \cos C)$ is less than	(P) 1
(B)	In ΔABC, if $\frac{a^2b^2c^2(\Sigma \sin 2A)}{(rr_1r_2r_3)^{3/2}} = \lambda$ then $\frac{\lambda}{8}$ is equal to	(Q) 2
(C)	In ΔABC, if $r_1 = 2, r_2 = 3, r_3 = 4$ and area of the ΔABC is Δ, then $\frac{\sqrt{26}}{8}\Delta$ is greater than or equal to	(R) 3
		(S) 4

[**Note :** All the symbols used have usual meaning in ΔABC.]

[Ans : (A) Q, R, S; (B) S; (C) P, Q, R]

[Sol.$_{92014/sot/MTC}$

(A) $R = 4r$

$r = 4R \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$1 = 16 \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$1 = 4(\cos A + \cos B + \cos C - 1)$

$\frac{1}{4} + 1 = \cos A + \cos B + \cos C \Rightarrow \cos A = \frac{5}{4}$

(B) $\frac{a^2b^2c^2(4\sin A \sin B \sin C)}{\Delta^3}$

$\frac{4(ab\sin C)(bc\sin A)(ac\sin B)}{\Delta^3} = \frac{4(2\Delta)^3}{\Delta^3} = 32$

$\therefore \quad \lambda = 32$

Hence, $\frac{\lambda}{8} = 4$

(C) $s = \sqrt{\Sigma r_1 r_2} = \sqrt{26}$

$$\Delta = \frac{r_1 r_2 r_3}{s} = \frac{2\times3\times4}{\sqrt{26}} = \frac{24}{\sqrt{26}}$$

$$\frac{\sqrt{26}}{8}\Delta = 3 \quad]$$