DPP No.- 41 to 44

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

DPP. NO.-41

[SINGLE CORRECT CHOICE TYPE] **[8 × 3 = 24]**

Q.1 If $\int \frac{x^2+x+1}{\sqrt{3x^2+4x+6}}dx = F(x)+c$, where c is constant of integration and $F(0)=0$, then $F(-1)$ equals

(A) $\frac{-\sqrt{5}}{6}$ (B) $\frac{1}{6}$ (C) $\frac{\sqrt{5}}{6}$ (D) $\frac{-1}{6}$

Q.2 $\lim\limits_{x\to 0} \frac{1}{\sin x}\int_0^{ln(1+x)}(1-\tan 2y)^{1/y}dy$ equals

(A) e^{-2} (B) e (C) e^2 (D) e^4

Q.3 Let $f: X \to Y$ be defined as $f(x)=\sin x+\cos x+2\sqrt{2}$. If f is invertible then $X \to Y$, is

(A) $\left[\frac{-3\pi}{4},\frac{-\pi}{4}\right]\to\left[\sqrt{2},3\sqrt{2}\right]$ (B) $\left[\frac{-\pi}{4},\frac{3\pi}{4}\right]\to\left[\sqrt{2},3\sqrt{2}\right]$

(C) $\left[\frac{-3\pi}{4},\frac{3\pi}{4}\right]\to\left[\sqrt{2},3\sqrt{2}\right]$ (D) $\left[\frac{\pi}{4},\frac{5\pi}{4}\right]\to\left[\sqrt{2},3\sqrt{2}\right]$

Q.4 The value of $\lim\limits_{x\to 0}\frac{\sin^3 x - x^3\,\text{sgn}\left(1-\left[\frac{x}{\sin^{-1}x}\right]\right)}{x\tan^2 x\ \sin(\pi\cos x)}$ is equal to

[Note : [k] denotes greatest integer less than or equal to k and sgn(k) denotes signum function of k.]

(A) $\frac{1}{\pi}$ (B) $\frac{-1}{\pi}$ (C) $\frac{1}{6\pi}$ (D) $-\frac{1}{6\pi}$

Q.5 The graph of quadratic polynomial $f(x) = (x-a)(x-b)$ where $a, b > 0$ and $a \neq b$, then the graph does not pass through

(A) first quadrant (B) second quadrant (C) third quadrant (D) fourth quadrant

Q.6 Let P(x) be a polynomial satisfying $P(x)-P'(x)=x^2+2x+1$, then $P(-1)$ is equal to

(A) 0 (B) 2 (C) –2 (D) 4

Q.7 $\int_{\pi/2}^{\pi}\left(x^{\sin x}\right)(1+x\cos x\,ln\,x+\sin x)dx$ is equal to

(A) $\frac{\pi^2}{2}$ (B) $\frac{\pi}{2}$ (C) $\frac{4\pi-\pi^2}{4}$ (D) $\frac{\pi}{2}-1$

Q.8 If $\cos^{-1}\left(\frac{a+x}{2}\right)-\sin^{-1}ax=\frac{3\pi}{2}$, then the value of $\sin\left(\frac{\pi}{x^2+a^2+2(x+a)}\right)$ equals

(A) 0 (B) –1 (C) 1 (D) $\frac{1}{2}$

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

DPP. NO.-42

[COMPREHENSION TYPE] [8 × 3 = 24]

Paragraph for question nos. 1 & 2

Let f be a differentiable function satisfying the functional rule

$$f(x+y)=f(x)\,f(y)-\left(e^{x}f(y)+e^{y}f(x)\right)+2e^{x+y}-\frac{1}{(x+1)(y+1)}-\frac{1}{x+y+1}$$

where $x, y \in R$ and $x>-1, y>-1,\ f(0)\neq 3$ and $f'(0)=2$.

Q.1 The number of solution(s) of the equation $(x+1)\,f(x)=x^2+3x+1$ is(are)
(A) 0 (B) 1 (C) 2 (D) 3

Q.2 If g is the inverse function of f then number of solution(s) of the equation $f(x)=g(x)$ is(are)
(A) 0 (B) 1 (C) 2 (D) 3

Paragraph for question nos. 3 to 5

Let $f(x)$ be a polynomial function of degree 2 satisfying

$$\int \frac{f(x)}{x^3-1}\,dx = \ln\left|\frac{x^2+x+1}{x-1}\right|+\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+C,$$

where C is indefinite integration constant.

Q.3 The value of $f(1)$ is equal to
(A) 1 (B) 2 (C) – 1 (D) – 3

Q.4 Let $\int \frac{1-6\,\text{cosec}\,x}{6+f(\sin x)}\,d(\sin x)=g(x)+K$, where $g(x)$ contains no constant term.

Then $\lim_{t\to\frac{\pi}{2}} g(t)$ is equal to (where K is indefinite integration constant.)
(A) $\ln 1$ (B) $\ln 2$ (C) $\ln 3$ (D) $\ln 4$

Q.5 Let $\int \frac{5+f(\sin x)+f(\cos x)}{\sin x+\cos x}\,dx = h(x)+\lambda$, where $h(1)=-1$.

The value of $\tan^{-1}(h(2))+\tan^{-1}(h(3))$ is equal to (where λ is indefinite integration constant.)
(A) $\frac{\pi}{4}$ (B) $\frac{-\pi}{4}$ (C) $\frac{3\pi}{4}$ (D) $\frac{-3\pi}{4}$

Paragraph for question nos. 6 to 8

Let $f(x)=\begin{cases}\lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n^2-n+1}\right)x, & x\neq 0\\ 0, & x=0\end{cases}$ and $g(x)=|x|+|x^2-1|$, for all $x\in R$.

Q.6 Which one of the following statement is **correct**?
(A) $f(x)$ is continuous at $x=0$.
(B) $f(x)$ is non-differentiable at $x=0$.
(C) $f(x)$ has non-removable type of discontinuity at $x=0$.
(D) $f(x)$ has removable type of discontinuity at $x=0$.

Q.7 Number of points where $g(x)$ is non-derivable, is

(A) 0 (B) 1 (C) 2 (D) 3

Q.8 Number of points of non-differentiability of $g(f(x))$, is

(A) 0 (B) 1 (C) 2 (D) 3

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

DPP. NO.-43

[MULTIPLE CORRECT CHOICE TYPE] [6 × 4 = 24]

Q.1 On the interval $I = [-2, 2]$, the function $f(x) = \begin{cases} (x+1)\, e^{-\left[\frac{1}{|x|} + \frac{1}{x}\right]} & (x \neq 0) \\ 0 & (x = 0) \end{cases}$

(A) is continuous for all values of $x \in I$
(B) is continuous for $x \in I - (0)$
(C) assumes all intermediate values from $f(-2)$ & $f(2)$
(D) has a maximum value equal to $3/e$.

Q.2 If $f(x) = \cos^{-1}(2x^2 - 1)$ and $g(x) = \sqrt{1-x^2}$, then

(A) $\left.\dfrac{d(f(x))}{d(g(x))}\right|_{x=\frac{3}{5}} = \dfrac{10}{3}$ (B) $\left.\dfrac{d(f(x))}{d(g(x))}\right|_{x=\frac{3}{5}} = \dfrac{-10}{3}$

(C) $\left.\dfrac{d(f(x))}{d(g(x))}\right|_{x=\frac{-3}{5}} = \dfrac{-10}{3}$ (D) $\left.\dfrac{d(f(x))}{d(g(x))}\right|_{x=\frac{-3}{5}} = \dfrac{10}{3}$

Q.3 Let P(x) be the polynomial $x^3 + ax^2 + bx + c$, where a, b, c ∈ R. If $P(-3) = P(+2) = 0$ and $P'(-3) < 0$, which of the following is a possible value of 'c'?
(A) – 27 (B) – 18 (C) – 6 (D) – 3

Q.4 $\int \sqrt{1 + \csc x}\ dx$ equals

(A) $2\sin^{-1}\sqrt{\sin x} + c$ (B) $\sqrt{2}\cos^{-1}\sqrt{\cos x} + c$
(C) $c - 2\sin^{-1}(1 - 2\sin x)$ (D) $\cos^{-1}(1 - 2\sin x) + c$

Q.5 Let $f(x) = \begin{cases} 2, & \text{if } 0 \le x \le 1 \\ 3, & \text{if } 1 < x \le 2 \end{cases}$. Define $g(x) = \int_0^x f(t)\, dt$, for $0 \le x \le 2$, then

(A) g is continuous at $x = 1$ (B) g is not differentiable at $x = 1$
(C) g is discontinuous at $x = 1$ (D) g is differentiable at $x = 1$

Q.6 If $f(x) = \begin{cases} \lim\limits_{t \to 0} \left(1 + t\, \ln(x^2 - 1)\right)^{\frac{1}{t}}, & |x| > 1 \\ \alpha x^3 - |x-2| - |x+2| + \beta, & |x| \le 1 \end{cases}$

is continuous for all $x \in R$, then
(A) $\alpha + \beta = 4$
(B) $\alpha + \beta = 0$
(C) the number of solutions of the equation $2\{f(x)\} = |x|$ is 5.
(D) the number of solutions of the equation $2\{f(x)\} = |x|$ is 6.
[**Note :** {k} denotes fractional part function of k.]

DPP. NO.-44

[SINGLE CORRECT CHOICE TYPE] [3 × 3 = 9]

Q.1 If the circles, $x^2+y^2+2x+2ky+6=0$ & $x^2+y^2+2ky+k=0$ intersect orthogonally, then 'k' is : [JEE '2000]

(A) 2 or – 3/2 (B) – 2 or – 3/2 (C) 2 or 3/2 (D) – 2 or 3/2

Q.2 The value of $\lim_{x\to 0}\left(\sin^{-1}[\sin x]+\cos^{-1}[\cos x]-2\tan^{-1}[\tan x]\right)$ is equal to

[**Note :** [k] denotes largest integer function less than or equal to k.]

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{2}$ (D) non-existent

Q.3 The value of $\tan\left(\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{4}{4r^2+3}\right)\right)$ is equal to

(A) 1 (B) 2 (C) 3 (D) 4

[MATRIX TYPE] [3+3+3+3=12]

Q.4

	Column–I		Column–II
(A)	If $f(x)=\int_0^{g(x)}\frac{dt}{\sqrt{1+t^3}}$ where $g(x)=\int_0^{\cos x}(1+\sin t^2)dt$ then the value of $f'(\pi/2)$	(P)	3
(B)	If $f(x)$ is a non zero differentiable function such that $\int_0^x f(t)dt=(f(x))^2$ for all x, then $f(2)$ equals	(Q)	2
		(R)	1
(C)	If $\int_a^b(2+x-x^2)\,dx$, $(a<b)$ is maximum then $(a+b)$ is equal to	(S)	– 1
(D)	If $\lim_{x\to 0}\left(\frac{\sin 2x}{x^3}+a+\frac{b}{x^2}\right)=0$ then $(3a+b)$ has the the value equal to		

[INTEGER TYPE] [1 × 5 = 5]

Q.5 Let P(x) be a polynomial satisfying $\lim_{x\to\infty}\frac{x^3P(x)}{x^6+3x^2+7}=2$ if $P(1)=2$, $P(3)=10$ and $P(5)=26$, then find the value of $\frac{P(2)+|P(0)|}{10}$.

DPP No.- 41 to 44

SOLUTION

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

DPP. NO.-41

[SINGLE CORRECT CHOICE TYPE] **[8 × 3 = 24]**

Q.1 If $\int \frac{x^2+x+1}{\sqrt{3x^2+4x+6}}\,dx = F(x)+c$, where c is constant of integration and $F(0)=0$, then $F(-1)$ equals

(A) $\frac{-\sqrt{5}}{6}$ (B) $\frac{1}{6}$ (C*) $\frac{\sqrt{5}}{6}$ (D) $\frac{-1}{6}$

[Sol.$_{149/inde/SC}$ $\int \frac{x^3+x^2+x}{\sqrt{3x^4+4x^3+6x^2}}\,dx = \frac{1}{12}\int \frac{dt}{\sqrt{t}} = \frac{2\sqrt{t}}{12}+c$

$= \frac{\sqrt{3x^4+4x^3+6x^2}}{6}+c$ **Ans.**]

Q.2 $\lim\limits_{x\to 0} \frac{1}{\sin x}\int_0^{\ln(1+x)} (1-\tan 2y)^{1/y}\,dy$ equals

(A*) e^{-2} (B) e (C) e^2 (D) e^4

[Sol.$_{417/def/SC}$ $l = \lim\limits_{x\to 0} \frac{\int_0^{\ln(1+x)} (1-\tan 2y)^{1/y}\,dy}{\frac{\sin x}{x}\cdot x}$

Using L'Hospital's Rule

$l = \lim\limits_{x\to 0} \frac{[1-\tan 2(\ln(1+x))]^{\frac{1}{\ln(1+x)}}}{(1+x)}$ (1^∞)

$= e^{-\lim\limits_{x\to 0}\frac{1}{\ln(1+x)}\tan 2(\ln(1+x))} = e^{-\lim\limits_{x\to 0}\frac{\tan(2\ln(1+x))}{2\ln(1+x)}\cdot 2} = e^{-2}$ (using $\lim\limits_{x\to 0}\frac{\tan\theta}{\theta}=1$) **Ans.**]

Q.3 Let $f: X \to Y$ be defined as $f(x) = \sin x + \cos x + 2\sqrt{2}$. If f is invertible then $X \to Y$, is

(A) $\left[\frac{-3\pi}{4}, \frac{-\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$ (B) $\left[\frac{-\pi}{4}, \frac{3\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$

(C) $\left[\frac{-3\pi}{4}, \frac{3\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$ (D*) $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$

[Sol.$_{173/func/SC}$ $f(x) = \underbrace{\sqrt{2}\cdot\sin\left(x+\frac{\pi}{4}\right)}_{\text{From } -1 \text{ to } 1} + 2\sqrt{2}$

$\therefore$ $Y = \text{Range of function} = \left[\sqrt{2}, 3\sqrt{2}\right]$. Clearly, f will be one-one also if $X = \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$ **Ans.**]

Q.4 The value of $\lim\limits_{x\to 0} \dfrac{\sin^3 x - x^3 \operatorname{sgn}\left(1-\left[\dfrac{x}{\sin^{-1} x}\right]\right)}{x\tan^2 x\ \sin(\pi\cos x)}$ is equal to

[Note : [k] denotes greatest integer less than or equal to k and sgn(k) denotes signum function of k.]

(A) $\frac{1}{\pi}$ (B*) $\frac{-1}{\pi}$ (C) $\frac{1}{6\pi}$ (D) $-\frac{1}{6\pi}$

[Sol.$_{473/lcd/SC}$ $\dfrac{x}{\sin^{-1} x} < 1 \Rightarrow \left[\dfrac{x}{\sin^{-1} x}\right] = 0 \Rightarrow \operatorname{sgn}\left(1-\left[\dfrac{x}{\sin^{-1} x}\right]\right) = 1$

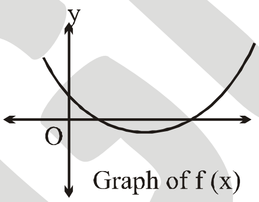
$$\lim_{x\to 0}\frac{\sin^3 x - x^3}{x.\tan^2 x.\sin(\pi\cos x)} = \lim_{x\to 0}\frac{(\sin x - x)(\sin^2 x + x\sin x + x^2)}{x\cdot\dfrac{\tan^2 x}{x^2}\cdot x^2\ \sin(\pi-\pi\cos x)}$$

$$= \lim_{x\to 0}\frac{(\sin x - x)\left(\dfrac{\sin^2 x}{x^2}+\dfrac{x\sin x}{x^2}+\dfrac{x^2}{x^2}\right)}{x\left(\dfrac{\sin(\pi-\pi\cos x)}{\pi-\pi\cos x}\right)\dfrac{\pi(1-\cos x)}{x^2}\cdot x^2} = -\frac{1}{6}\times\frac{3}{\pi}\times 2 = \frac{-1}{\pi}$$ **Ans.**]

Q.5 The graph of quadratic polynomial $f(x) = (x-a)(x-b)$ where $a, b > 0$ and $a \neq b$, then the graph does not pass through

(A) first quadrant (B) second quadrant (C*) third quadrant (D) fourth quadrant

[Sol.$_{405/qe/SC}$



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Q.6 Let P(x) be a polynomial satisfying $P(x) - P'(x) = x^2 + 2x + 1$, then $P(-1)$ is equal to

(A) 0 (B*) 2 (C) –2 (D) 4

[Sol.$_{319/mod/SC}$ Let degree of P(x) be n

$\therefore$ degree of P'(x) be $n-1$

$\because$ degree of L.H.S. = degree of R.H.S.

$\therefore$ $n = 2$

Let $P(x) = ax^2 + bx + c$

$\therefore$ $ax^2 + bx + c - (2ax + b) = x^2 + 2x + 1$

$\therefore$ $a = 1,\ b - 2a = 2 \Rightarrow b = 4$

and $c - b = 1 \Rightarrow c = 5$

$\therefore \quad P(x) = x^2 + 4x + 5$

$\therefore \quad P(-1) = 2$ **Ans.**]

Q.7 $\int_{\pi/2}^{\pi}\left(x^{\sin x}\right)(1 + x\cos x \ln x + \sin x)dx$ is equal to

(A) $\frac{\pi^2}{2}$ (B) $\frac{\pi}{2}$ (C*) $\frac{4\pi-\pi^2}{4}$ (D) $\frac{\pi}{2} - 1$

[Sol.$_{21/def/SC}$ Integrand is $(x^{\sin x} \cdot x)'$

$\therefore \quad \int\left(x^{\sin x}\cdot x\right)' = x^{\sin x}\cdot x\Big|_{\pi/2}^{\pi} = \pi^0 \cdot \pi - \frac{\pi}{2}\cdot\frac{\pi}{2} = \pi - \frac{\pi^2}{4} = \frac{4\pi-\pi^2}{4}$ **Ans.**]

Q.8 If $\cos^{-1}\left(\frac{a+x}{2}\right) - \sin^{-1} ax = \frac{3\pi}{2}$, then the value of $\sin\left(\frac{\pi}{x^2+a^2+2(x+a)}\right)$ equals

(A) 0 (B) –1 (C*) 1 (D) $\frac{1}{2}$

[Sol.$_{377/itf/SC}$ $\cos^{-1}\left(\frac{a+x}{2}\right) = \pi$; $\sin^{-1} ax = \frac{-\pi}{2}$

$a + x = -2$ $ax = -1$

$x - \frac{1}{x} = -2 \Rightarrow x^2 + 2x = 1$

Also, $x = \frac{-1}{a} \Rightarrow \left(\frac{-1}{a}\right)^2 - \frac{2}{a} = 1$

$1 = a^2 + 2a$

$\Rightarrow \quad \sin\left(\frac{\pi}{x^2+2x+a^2+2a}\right) = \sin\frac{\pi}{2} = 1$]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

DPP. NO.-42

[COMPREHENSION TYPE] **[8 × 3 = 24]**

Paragraph for question nos. 1 & 2

Let f be a differentiable function satisfying the functional rule

$$f(x+y) = f(x)\,f(y) - \left(e^{x}f(y) + e^{y}f(x)\right) + 2e^{x+y} - \frac{1}{(x+1)(y+1)} - \frac{1}{x+y+1}$$

where $x, y \in R$ and $x > -1, y > -1,\ f(0) \neq 3$ and $f'(0) = 2$.

Q.1 The number of solution(s) of the equation $(x+1)\,f(x) = x^2 + 3x + 1$ is(are)
(A) 0 (B*) 1 (C) 2 (D) 3

Q.2 If g is the inverse function of f then number of solution(s) of the equation $f(x) = g(x)$ is(are)
(A) 0 (B*) 1 (C) 2 (D) 3

[Sol.$_{30432\text{-}33/mod}$ Particully differentiating w.r.t. taking y as a constant

$$f'(x+y) = f'(x)\,f(y) - e^{x}f(y) - e^{y}f'(x) + 2e^{x+y} + \frac{1}{(x+1)^2(y+1)} + \frac{1}{(x+y+1)^2}$$

Putting $y = 0$

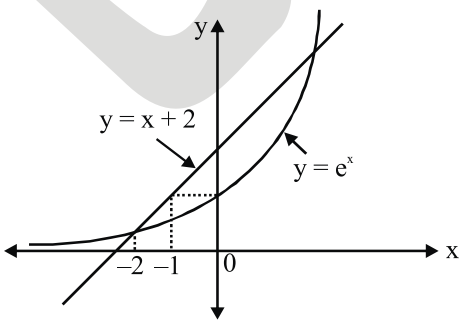
$$f'(x) = f'(x)\,f(0) - e^{x}f(0) - f'(x) + 2e^{x} + \frac{2}{(1+x)^2}$$

$$2f'(x) = 2e^{x} + \frac{2}{(x+1)^2} \qquad \{\because f(0) = 0\}$$

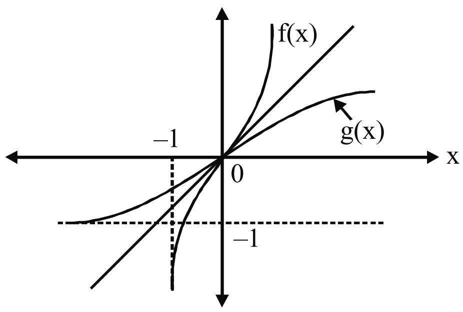
$$f(x) = e^{x} - \frac{1}{x+1} + c$$

$$f(x) = e^{x} - \frac{1}{x+1}$$

(i) $(x+1)f(x) = x^2 + 3x + 1$
$\Rightarrow (x+1)e^{x} - 1 = x^2 + 3x + 1$
$\Rightarrow (x+1)e^{x} = x^2 + 3x + 2$
$e^{x} = x + 2$



(ii) Clearly f(x) and g(x) meet each other at the origin only.



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Paragraph for question nos. 3 to 5

Let $f(x)$ be a polynomial function of degree 2 satisfying

$$\int \frac{f(x)}{x^3-1}\,dx = \ln\left|\frac{x^2+x+1}{x-1}\right| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C,$$

where C is indefinite integration constant.

Q.3 The value of $f(1)$ is equal to

(A) 1 (B) 2 (C) – 1 (D*) – 3

Q.4 Let $\int \frac{1-6\operatorname{cosec} x}{6+f(\sin x)}\,d(\sin x) = g(x)+K$, where $g(x)$ contains no constant term.

Then $\lim\limits_{t\to\frac{\pi}{2}} g(t)$ is equal to (where K is indefinite integration constant.)

(A) $\ln 1$ (B) $\ln 2$ (C*) $\ln 3$ (D) $\ln 4$

Q.5 Let $\int \frac{5+f(\sin x)+f(\cos x)}{\sin x+\cos x}\,dx = h(x)+\lambda$, where $h(1)=-1$.

The value of $\tan^{-1}(h(2))+\tan^{-1}(h(3))$ is equal to (where λ is indefinite integration constant.)

(A) $\frac{\pi}{4}$ (B) $\frac{-\pi}{4}$ (C) $\frac{3\pi}{4}$ (D*) $\frac{-3\pi}{4}$

[Sol.$_{\text{30404-5-6/inde}}$

(i) $$\int \frac{f(x)}{x^3-1}\,dx = \ln\left|\frac{x^2+x+1}{x-1}\right| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$$

Differentiating both sides, we get

$$\frac{f(x)}{x^3-1} = \frac{x-1}{x^2+x+1}\,\frac{(x-1)(2x+1)-(x^2+x+1).1}{(x-1)^2} + \frac{2}{\sqrt{3}}\cdot\frac{1}{1+\left(\frac{2x+1}{\sqrt{3}}\right)^2}\cdot\frac{2}{\sqrt{3}}$$

$$= \frac{2x^2 - x - 1 - x^2 - x - 1}{x^3 - 1} + \frac{4}{3} \cdot \frac{3}{3 + (4x^2 + 4x + 1)} = \frac{x^2 - 2x - 2}{x^3 - 1} + \frac{1}{x^2 + x + 1}$$

$f(x) = (x^2 - 2x - 2) + (x - 1) = x^2 - x - 3 \Rightarrow f(1) = -3$ **Ans.**

(ii) $I = \int \frac{1 - 6\operatorname{cosec} x}{6 + f(\sin x)} d(\sin x) = \int \frac{1 - \frac{6}{\sin x}}{6 + \sin^2 x - \sin x - 3} d(\sin x)$

$\int \frac{1 - \frac{6}{\sin x}}{\sin^2 x - \sin x + 3} d(\sin x)$ (Put $\sin x = t$)

$$= \int \frac{1 - \frac{6}{t}}{t^2 - t + 3} dt = \int \frac{\frac{1}{t^2} - \frac{6}{t^3}}{1 - \frac{1}{t} + \frac{3}{t^2}} dt = \ln\left(1 - \frac{1}{t} + \frac{3}{t^2}\right) + K = \ln\left(1 - \frac{1}{\sin x} + \frac{3}{\sin^2 x}\right) + K$$

$\Rightarrow \quad g(x) = \ln\left(1 - \frac{1}{\sin x} + \frac{3}{\sin^2 x}\right)$

$\therefore \quad g(t) = \ln\left(1 - \frac{1}{\sin t} + \frac{3}{\sin^2 t}\right)$

Now, $\lim_{t \to \frac{\pi}{2}} g(t) = \ln 3$ **Ans.**

(iii) $I = \int \frac{5 + f(\sin x) + f(\cos x)}{\sin x + \cos x} dx = \int \frac{5 + \sin^2 x - \sin x - 3 + \cos^2 x - \cos x - 3}{\sin x + \cos x} dx$

$= \int -dx = -x + \lambda$

$\therefore$ $h(x) = -x$ (since $h(1) = -1$)

Hence $\tan^{-1}(h(2)) + \tan^{-1}(h(3)) = \tan^{-1}(-2) + \tan^{-1}(-3) = \frac{-3\pi}{4}$ **Ans.**]

Paragraph for question nos. 6 to 8

Let $f(x) = \begin{cases} \lim_{n \to \infty}\left(\sqrt{n^2 + n + 1} - \sqrt{n^2 - n + 1}\right)x, & x \neq 0 \\ 0, & x = 0 \end{cases}$

and $g(x) = |x| + |x^2 - 1|$, for all $x \in R$.

Q.6 Which one of the following statement is **correct**?

(A*) $f(x)$ is continuous at $x = 0$.

(B) $f(x)$ is non-differentiable at $x = 0$.

(C) $f(x)$ has non-removable type of discontinuity at $x = 0$.
(D) $f(x)$ has removable type of discontinuity at $x = 0$.

Q.7 Number of points where $g(x)$ is non-derivable, is
(A) 0 (B) 1 (C) 2 (D*) 3

Q.8 Number of points of non-differentiability of $g(f(x))$, is
(A) 0 (B) 1 (C) 2 (D*) 3

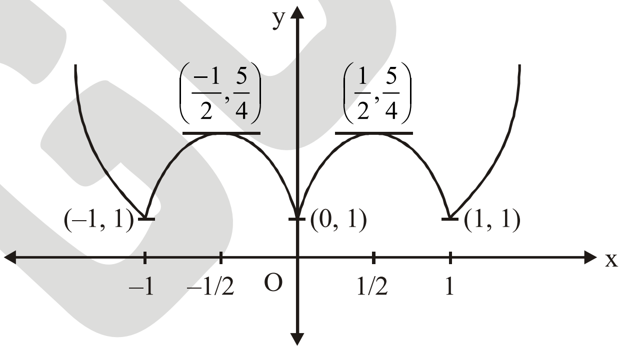
[Sol.$_{30001\text{-}2\text{-}3/\text{lcd}}$ $\lim\limits_{n\to\infty}\left(\sqrt{n^2+n+1}-\sqrt{n^2-n+1}\right)$ $(\infty-\infty)$

$$= \lim_{n\to\infty}\left(\frac{2n}{\sqrt{n^2+n+1}+\sqrt{n^2-n+1}}\right) = 1$$

$\therefore$ $f(x) = x \ \forall\ x \in R$
$g(x) = |x| + |x-1|\,|x+1| \quad \forall\ x \in R$

(i) Clearly, $f(x)$ is continuous and differentiable for all $x \in R$.
(ii) Clearly, $g(x)$ is non-derivable at 3points viz. $x = -1, 0, 1$.
(iii) As, $g(f(x)) = g(x)$
$\Rightarrow$ $g(f(x))$ is non-derivable at 3 points viz. $x = -1, 0, 1$.

$$g(x) = \begin{cases} x^2 - x - 1 & ; \quad -\infty < x < -1 \\ -x^2 - x + 1 & ; \quad -1 \le x < 0 \\ -x^2 + x + 1 & ; \quad 0 \le x < 1 \\ x^2 + x - 1 & ; \quad 1 \le x < \infty \end{cases}$$



Note : g(x) is an even function also.]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

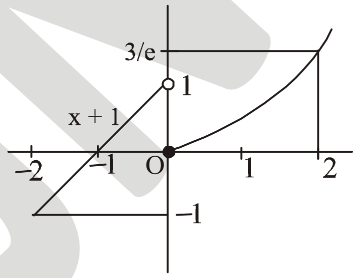
DPP. NO.-43

[MULTIPLE CORRECT CHOICE TYPE] [6 × 4 = 24]

Q.1 On the interval $I = [-2, 2]$, the function $f(x) = \begin{cases} (x+1)\, e^{-\left[\frac{1}{|x|}+\frac{1}{x}\right]} & (x \neq 0) \\ 0 & (x = 0) \end{cases}$

(A) is continuous for all values of $x \in I$
(B*) is continuous for $x \in I - (0)$
(C*) assumes all intermediate values from $f(-2)$ & $f(2)$
(D*) has a maximum value equal to $3/e$.

[Sol.$_{40030/lcd/MORE}$ $f(x) = \begin{cases} (x+1)e^{-2/x} & \text{if } x > 0 \\ x+1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \end{cases}$



the graph of $f(x)$ is
hence f can assume all values for $f(-2)$ to $f(2)$]

Q.2 If $f(x) = \cos^{-1}(2x^2 - 1)$ and $g(x) = \sqrt{1-x^2}$, then

(A*) $\left.\dfrac{d(f(x))}{d(g(x))}\right|_{x=\frac{3}{5}} = \dfrac{10}{3}$ (B) $\left.\dfrac{d(f(x))}{d(g(x))}\right|_{x=\frac{3}{5}} = \dfrac{-10}{3}$

(C) $\left.\dfrac{d(f(x))}{d(g(x))}\right|_{x=\frac{-3}{5}} = \dfrac{-10}{3}$ (D*) $\left.\dfrac{d(f(x))}{d(g(x))}\right|_{x=\frac{-3}{5}} = \dfrac{10}{3}$

[Sol.$_{40553/mod/MORE}$ $f(x) = \cos^{-1}(2x^2 - 1) = \cos^{-1}(\cos 2\theta) = \begin{cases} 2\theta; & \theta \in \left[0, \dfrac{\pi}{2}\right] \Rightarrow x \in [0, 1] \\ 2\pi - 2\theta; & \theta \in \left(\dfrac{\pi}{2}, \pi\right] \Rightarrow x \in [-1, 0) \end{cases}$

$g(x) = \sqrt{1-x^2} = \sqrt{1-\cos^2\theta} = \sin\theta$

At $x = \dfrac{3}{5}$

$$\frac{d(f(x))}{d(g(x))} = \frac{2}{\cos\theta} = \frac{2}{3/5} = \frac{10}{3}$$

At $x = \dfrac{-3}{5}$

$$\frac{d(f(x))}{d(g(x))} = \frac{-2}{\cos\theta} = \frac{-2}{-3/5} = \frac{10}{3}.$$]

Q.3 Let P(x) be the polynomial $x^3 + ax^2 + bx + c$, where a, b, c $\in$ R. If P(–3) = P(+2) = 0 and P'(–3) < 0, which of the following is a possible value of 'c'?

(A*) – 27 (B) – 18 (C) – 6 (D) – 3

[Sol.$_{40515/mod/MORE}$ $P(x) = x^3 + ax^2 + bx + c$

$P'(x) = 3x^2 + 2ax + b$

$P(-3) = 0 \Rightarrow -27 + 9a - 3b + c = 0$ ....(1)

$P(2) = 0 \Rightarrow 8 + 4a + 2b + c = 0$ ....(2)

hence (1) – (2) gives

$5a - 5b - 35 = 0$

$a - b = 7$ ....(3)

$P'(-3) = 27 - 6a + b < 0 \Rightarrow 27 - 6(a - b) - 5b < 0 \Rightarrow 27 - 42 - 5b < 0$

$\Rightarrow -15 - 5b < 0 \Rightarrow 3 + b > 0 \Rightarrow b > -3$, hence $a - 7 = b > -3$

$a - 7 > -3$ [from (3)]

$a > 4$ ....(4)

$\therefore$ from (2)

$8 + 16 - 6 + c_{max} = 0$

$c_{max} = -18 \Rightarrow c < -18$

Aliter: $P(x) \equiv (x + 3)(x - 2)(x - \alpha)$

where a is the third root i.e. $P(x) = x^3 + (1 - \alpha)x^2 + (-6 - \alpha)x + 6\alpha$ now proceed.]

Q.4 $\int \sqrt{1 + \csc x}\, dx$ equals

(A*) $2 \sin^{-1} \sqrt{\sin x} + c$ (B) $\sqrt{2} \cos^{-1} \sqrt{\cos x} + c$

(C) $c - 2 \sin^{-1} (1 - 2 \sin x)$ (D*) $\cos^{-1} (1 - 2 \sin x) + c$

[Sol.$_{40512/inde/MORE}$ $I = \dfrac{\sqrt{(1+\sin x)(1-\sin x)}}{\sqrt{\sin x (1-\sin x)}} = \dfrac{\cos x}{\sqrt{\sin x (1-\sin x)}} = \dfrac{\cos x}{\sqrt{\frac{1}{4} - \left(\frac{1}{2} - \sin x\right)^2}} = \int \dfrac{-dt}{\sqrt{\left(\frac{1}{2}\right)^2 - t^2}}$]

Q.5 Let $f(x) = \begin{cases} 2, & \text{if } 0 \le x \le 1 \\ 3, & \text{if } 1 < x \le 2 \end{cases}$.

Define $g(x) = \int_0^x f(t)\, dt$, for $0 \le x \le 2$, then

(A*) g is continuous at x = 1 (B*) g is not differentiable at x = 1

(C) g is discontinuous at x = 1 (D) g is differentiable at x = 1

[Sol.$_{40872/def/MORE}$ $g(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 3x - 1, & 1 < x \le 2 \end{cases}$]

Q.6 If $f(x) = \begin{cases} \lim\limits_{t \to 0} \left(1 + t\, ln(x^2 - 1)\right)^{\frac{1}{t}}, & |x| > 1 \\ \alpha x^3 - |x - 2| - |x + 2| + \beta, & |x| \le 1 \end{cases}$

is continuous for all $x \in R$, then

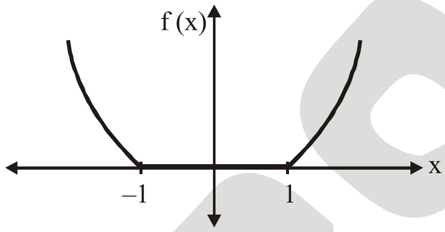
(A*) $\alpha + \beta = 4$
(B) $\alpha + \beta = 0$
(C*) the number of solutions of the equation $2\{f(x)\} = |x|$ is 5.
(D) the number of solutions of the equation $2\{f(x)\} = |x|$ is 6.
[**Note :** $\{k\}$ denotes fractional part function of k.]

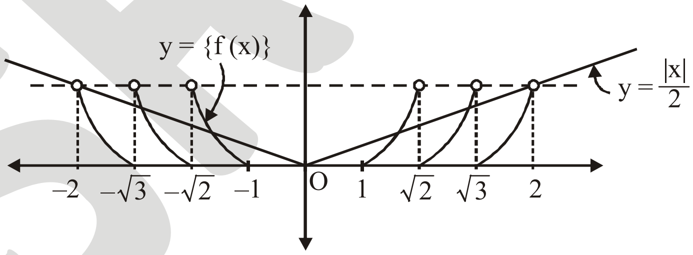
[Sol.$_{40139/lcd/MORE}$ $f(x) = \begin{cases} x^2 - 1; & |x| > 1 \\ \alpha x^3 + x - 2 - x - 2 + \beta; & |x| \le 1 \end{cases}$

$$f(x) = \begin{cases} x^2 - 1; & |x| > 1 \\ \alpha x - 4 + \beta; & |x| \le 1 \end{cases}$$

For the continuity,
at x = 1 at x = –1
$\alpha + \beta - 4 = 0$ $-\alpha + \beta - 4 = 0$
$\therefore \alpha = 0, \beta = 4$

$$\therefore f(x) = \begin{cases} x^2 - 1; & |x| > 1 \\ 0; & |x| \le 1 \end{cases}$$





From the graph it is clear that $\{f(x)\} = \frac{1}{2}|x|$ has 5 distinct solutions.]

DPP. NO.-44

[SINGLE CORRECT CHOICE TYPE] **[3 × 3 = 9]**

Q.1 If the circles, $x^2 + y^2 + 2x + 2ky + 6 = 0$ & $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then 'k' is :

(A*) 2 or – 3/2 (B) – 2 or – 3/2 (C) 2 or 3/2 (D) – 2 or 3/2

[Sol.$_{221/cir/SC}$ [JEE '2000 (Screening), 1]

$2(1)(0) + 2(k)(k) = 6 + k$

$2k^2 - k - 6 = 0$

$2k^2 - 4k + 3k - 6 = 0$

$2k(k - 2) + 3(k - 2) = 0$

$(k - 2)(2k + 3) = 0$

$k = 2$ or $k = -\frac{3}{2}$.]

Q.2 The value of $\lim_{x\to 0} \left(\sin^{-1}[\sin x] + \cos^{-1}[\cos x] - 2\tan^{-1}[\tan x]\right)$ is equal to

[**Note :** [k] denotes largest integer function less than or equal to k.]

(A) π (B*) $\frac{\pi}{2}$ (C) $\frac{3\pi}{2}$ (D) non-existent

[Sol.$_{362/lcd/SC}$ L.H.L. = $\lim_{x\to 0^-} (\sin^{-1}[\sin x] + \cos^{-1}[\cos x] - 2\tan^{-1}[\tan x])$

$= \sin^{-1}(-1) + \cos^{-1}(0) - 2\tan^{-1}(-1) = -\frac{\pi}{2} + \frac{\pi}{2} - 2\left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$

R.H.L. = $\lim_{x\to 0^+} (\sin^{-1}[\sin x] + \cos^{-1}[\cos x] - 2\tan^{-1}[\tan x])$

$= \sin^{-1}0 + \cos^{-1}0 - 2\tan^{-1}0 = 0 + \frac{\pi}{2} - 0 = \frac{\pi}{2}$

$\therefore$ Given limit $= \frac{\pi}{2}$ **Ans.**]

Q.3 The value of $\tan\left(\sum_{r=1}^{\infty} \tan^{-1}\left(\frac{4}{4r^2+3}\right)\right)$ is equal to

(A) 1 (B*) 2 (C) 3 (D) 4

[Sol.$_{199/itf/SC}$ $\tan^{-1}\left(\frac{4}{4r^2+3}\right) = \tan^{-1}\left(\frac{1}{r^2+\frac{3}{4}}\right) = \tan^{-1}\left(\frac{1}{1+\left(r+\frac{1}{2}\right)\left(r-\frac{1}{2}\right)}\right) = \tan^{-1}\left(r+\frac{1}{2}\right) - \tan^{-1}\left(r-\frac{1}{2}\right)$

$$\therefore \quad \sum_{r=1}^{n} \tan^{-1}\left(\frac{4}{4r^2+3}\right) = \left(\tan^{-1}\left(n+\frac{1}{2}\right) - \tan^{-1}\frac{1}{2}\right)$$

$$\Rightarrow \quad \sum_{r=1}^{\infty} \tan^{-1}\left(\frac{4}{4r^2+3}\right) = \cot^{-1}\frac{1}{2} = (\tan^{-1} 2)$$

$\therefore$ $\tan(\tan^{-1} 2) = 2$ **Ans.**]

[MATRIX TYPE]

[3+3+3+3=12]

Q.4

	Column–I		Column–II
(A)	If $f(x) = \int_0^{g(x)} \frac{dt}{\sqrt{1+t^3}}$ where $g(x) = \int_0^{\cos x} (1+\sin t^2)dt$ then the value of $f'(\pi/2)$	(P)	3
(B)	If $f(x)$ is a non zero differentiable function such that $\int_0^x f(t)dt = (f(x))^2$ for all x, then $f(2)$ equals	(Q)	2
		(R)	1
(C)	If $\int_a^b (2+x-x^2)\,dx$, $(a<b)$ is maximum then $(a+b)$ is equal to	(S)	– 1
(D)	If $\lim_{x\to 0}\left(\frac{\sin 2x}{x^3}+a+\frac{b}{x^2}\right) = 0$ then $(3a+b)$ has the the value equal to		

[Ans. (A) S; (B) R; (C) R; (D) Q]

[Sol.$_{92003/def/MTC}$ (A) $f'(x) = \frac{g'(x)}{\sqrt{1+g^3(x)}}$ and $g'(x) = [1+\sin(\cos^2 x)](-\sin x)$

hence $f'(x) = \frac{[1+\sin(\cos^2 x)](-\sin x)}{\sqrt{1+g^3(x)}}$

$$f'\left(\frac{\pi}{2}\right) = \frac{1+0}{\sqrt{1+g^3(\pi/2)}} = \frac{-1}{1+0} = -1 \quad \text{as} \quad g\left(\frac{\pi}{2}\right) = 0$$

$$\therefore \quad f'\left(\frac{\pi}{2}\right) = -1 \quad \text{Ans.}$$

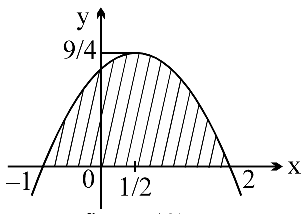


figure (C)

(C) Maximum when $a = -1$; $b = 2$
$\Rightarrow$ $a+b = 1$

(D) If $\lim_{x\to 0} \frac{\sin 2x}{x^3} + a + \frac{b}{x^2} = 0 \quad \Rightarrow \quad \lim_{x\to 0} \frac{\sin 2x + ax^3 + bx}{x^3} = 0$

for limit to exist $2 + b = 0 \Rightarrow \boxed{b = -2}$

$\therefore \quad \lim_{x\to 0} \dfrac{\sin 2x + ax^3 - 2x}{x^3} = 0$

apply LHS rule, $\lim_{x\to 0} \dfrac{2\cos 2x + 3ax^2 - 2}{3x^2} = 0$

$\therefore \quad a = \lim_{x\to 0} \dfrac{2(1-\cos 2x)}{3x^2} \Rightarrow \boxed{a = \dfrac{4\sin^2 x}{3x^2} = \dfrac{4}{3}}$

$\therefore \quad 3a + b = 3 \cdot \dfrac{4}{3} - 2 = 2$ Ans.]

[INTEGER TYPE] **[1 × 5 = 5]**

Q.5 Let P(x) be a polynomial satisfying $\lim_{x\to\infty} \dfrac{x^3P(x)}{x^6 + 3x^2 + 7} = 2$ if $P(1) = 2$, $P(3) = 10$ and $P(5) = 26$, then find the value of $\dfrac{P(2) + |P(0)|}{10}$. [Ans. 4]

[Sol.$_{50106/lcd/OMR}$ Let $f(x) = P(x) - (x^2 + 1) = A(x-1)(x-3)(x-5)$ also P(x) should be of degree 3

$\Rightarrow \quad P(x) = A(x-1)(x-3)(x-5) + x^2 + 1$

$\lim_{x\to\infty} \dfrac{x^3P(x)}{x^6 + 3x^2 + 7} = 2 \Rightarrow A = 2$

$P(x) = 2(x-1)(x-3)(x-5) + x^2 + 1$

Hence, $P(2) = 11$, $P(0) = -29$

$\therefore \quad \dfrac{|P(0)| + P(2)}{10} = 4$. **Ans.**]