DPP No.-37 to 40

DPP. NO.-37

[SINGLE CORRECT CHOICE TYPE] [4 × 3 = 12]

Q.1 If $\int \frac{x^4+1}{x(x^2+1)^2}\,dx = A\,ln\,|x| + \frac{B}{1+x^2} + c$, where c is the constant of integration then

(A) A = 1 ; B = – 1 (B) A = – 1 ; B = 1 (C) A = 1 ; B = 1 (D) A = – 1 ; B = – 1

Q.2 If $\lim_{x\to 0} \frac{p\sin 2x + (1-\cos 2x)}{x+\tan x} = 1$, then p is equal to

(A) 0 (B) 1 (C) 2 (D) –2

Q.3 If $f(x) = \frac{(x+1)(x+2)^2}{(x+3)^3(x+4)^4}$ then f'(0) is equal to

(A) 0 (B) f(0) (C) 2f(0) (D) 4f(0)

Q.4 If number of significant digit before decimal in 2^{64} is a and number of zero's after decimal and before first significant digit in 3^{-50} is b, then b – a is
(here $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$)

(A) 3 (B) 4 (C) 6 (D) 1

[SUBJECTIVE] [2 × 5 = 10]

Q.5 (a) $\lim_{x\to 0} \tan^{-1}\frac{a}{x^2}$, where $a \in R$; (b) Plot the graph of the function $f(x) = \lim_{t\to 0}\left(\frac{2x}{\pi}\tan^{-1}\frac{x}{t^2}\right)$

[Ans. (a) π/2 if a > 0 ; 0 if a = 0 and –π/2 if a < 0; (b) f(x) = | x |]

Q.6 Let f be a function defined implicitly by the equation $\frac{1-e^{f(x)}}{1+e^{f(x)}} = x$ and g be the inverse of f.

If $g''(ln\,3) - g'(ln\,3) = \frac{p}{q}$ where p and q relative prime numbers then find the value of (p + q).

[3 + 3 = 6]

Q.1
(a) $\int \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right)dx$. [Ans. $(x+1)\tan^{-1}\frac{2(x+1)}{3} - \frac{3}{4}\ell n(4x^2+8x+13) + C$]

(b) $\int \frac{x^2+x}{(e^x+x+1)^2}dx$ [Ans. $C - \ln(1+(x+1)e^{-x}) - \frac{1}{1+(x+1)e^{-x}}$]

[SINGLE CORRECT CHOICE TYPE] **[5 × 3 = 15]**

Q.2$_{qe}$ Number of integral values of a so that the graph of $f(x) = x^2 - 2(4a - 1)x + (15a^2 - 2a - 7)$ is completely above the x-axis is
(A) 1 (B) 2 (C) 3 (D) 4

Q.3 If $\lim_{x\to\infty} \frac{x^2+3x+5}{4x+1+x^k}$ exists then
(A) $k = 2$ (B) $k < 2$ (C) $k > 2$ (D) $k \geq 2$

Q.4 If $f(x) = x^x \cdot e^x$ then $f'(1)$ equals
(A) 1 (B) e (C) 2e (D) $\frac{e}{2}$

Q.5 If $f: [1, 10] \to Q$ (where Q is the set of all rational numbers) is a continuous function and $f(1) = \lim_{x\to 0}\left(\left[\frac{x}{\sin x}\right] + \left[\frac{\tan x}{x}\right]\right)$, then the value of $\sum_{r=1}^{10} \tan^{-1}(\tan f(r))$ is equal to
[**Note:** where [y] denotes greatest integer function less than or equal to y.]
(A) 10 (B) 20 (C) $20 - 10\pi$ (D) $10\pi - 20$

Q.6 List -I contains the function and List-II contains their derivatives at $x = 0$.
Select the correct answer using the codes given below the list.

	List-I		**List-II**
(P)	$f(x) = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$	(1)	2
(Q)	$g(x) = \cos^{-1}(2x^2 - 1)$	(2)	3
(R)	$h(x) = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$	(3)	– 2
(S)	$k(x) = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$	(4)	non-existent

Codes:	**P**	**Q**	**R**	**S**		**P**	**Q**	**R**	**S**
(A)	3	4	4	1	(B)	3	4	4	2
(C)	4	3	2	1	(D)	2	4	4	2

DPP. NO.-39

[SINGLE CORRECT CHOICE TYPE] [8 × 3 = 24]

Q.1 The value of integral $\int e^x\left(\frac{2\tan x}{1+\tan x}+\cot^2\left(x+\frac{\pi}{4}\right)\right)dx$ is equal to

(A) $e^x \tan\left(\frac{\pi}{4}-x\right)+C$
(B) $e^x \tan\left(x-\frac{\pi}{4}\right)+C$
(C) $e^x \tan\left(\frac{3\pi}{4}-x\right)+C$
(D) $e^x \tan\left(x-\frac{3\pi}{4}\right)+C$

where C is constant of integration.

Q.2 If a, b, c, d are positive real numbers such that $a+b+c+d=2$, then $M=(a+b)(c+d)$ satisfies the relation :

(A) $0 \le M \le 1$ (B) $1 \le M \le 2$ (C) $2 \le M \le 3$ (D) $3 \le M \le 4$

Q.3 $\int_0^1 \frac{x-e^{2x}}{x^2-e^{2x}}\,dx$ equals

(A) 2 (B) $\ln(e^2-1)$ (C) $\frac{1}{2}\ln(e^2-1)$ (D) $e-1$

Q.4 For the curve $xy+y^2=1$, $\frac{d^2y}{dx^2}$ is equal to

(A) $\frac{-y}{x+2y}$ (B) $\frac{2xy+2y}{(x+2y)^3}$ (C) $\frac{2}{(x+2y)^3}$ (D) $\frac{2x+2y^2}{(x+2y)^3}$

Q.5 If $\frac{\cos 3x}{\cos x}=\frac{1}{3}$ for some angle x, $0\le x\le\frac{\pi}{2}$, then the value of $\frac{\sin 3x}{\sin x}$ for same x, is

(A) $\frac{7}{3}$ (B) $\frac{5}{3}$ (C) 1 (D) $\frac{2}{3}$

Q.6 The values for A, B and C respectively if $\lim_{x\to 1}\frac{Ax^4+Bx^3+1}{(x-1)\sin \pi x}$ exists and is equal to C are

(A) $-4, 3, \frac{6}{\pi}$ (B) $-4, 3, -\frac{6}{\pi}$ (C) $3, -4, \frac{6}{\pi}$ (D) $3, -4, \frac{-6}{\pi}$

Q.7 $\int \frac{dx}{x(x+1)(\ln(x+1)-\ln x)^{11}}$ equals

(A) $\frac{1}{10(\ln(x+1)-\ln x)^{10}}+C$
(B) $\frac{(\ln(x+1)-\ln x)^{10}}{10}+C$
(C) $\frac{1}{11(\ln(x+1)-\ln x)^{11}}+C$
(D) $\frac{(\ln(x+1)-\ln x)^{11}}{11}+C$

where C is constant of integration.

Q.8 If $f(x) = \begin{cases} \sqrt{1-x} & 0 \le x \le 1 \\ (7x-6)^{-1/3} & 1 < x \le 2 \end{cases}$, then $\int_0^2 f(x)\,dx$ is equal to

(A) $\frac{31}{6}$ (B) $\frac{32}{21}$ (C) $\frac{1}{42}$ (D) $\frac{55}{42}$

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

DPP. NO.-40

[SINGLE CORRECT CHOICE TYPE] [4 × 3 = 12]

Q.1 $\int \frac{\cot^{-1}(e^x)}{e^x} dx$ is equal to :

(A) $\frac{1}{2} \ln (e^{2x} + 1) - \frac{\cot^{-1}(e^x)}{e^x} + x + c$ (B) $\frac{1}{2} \ln (e^{2x} + 1) + \frac{\cot^{-1}(e^x)}{e^x} + x + c$

(C) $\frac{1}{2} \ln (e^{2x} + 1) - \frac{\cot^{-1}(e^x)}{e^x} - x + c$ (D) $\frac{1}{2} \ln (e^{2x} + 1) + \frac{\cot^{-1}(e^x)}{e^x} - x + c$

Q.2 $I = \int_0^1 \frac{(1-x^2)}{(1+x^2)\sqrt{1+x^4}} dx$ is equal to

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{4\sqrt{2}}$ (C) $\frac{\pi}{2\sqrt{2}}$ (D) $\frac{\pi\sqrt{2}}{2}$

Q.3 The graph of the function, $\cos x \cos (x+2) - \cos^2 (x+1)$ is
(A) a straight line passing through $(0, -\sin^2 1)$ with slope 2
(B) a straight line passing through (0 , 0)
(C) a parabola with vertex $(1 , -\sin^2 1)$
(D) a straight line passing through the point $\left(\frac{\pi}{2}, -\sin^2 1\right)$ & parallel to the x–axis.

Q.4 If $(6x^2 - 5xy + y^2)(x^2 - y^2 + 2x) = 0$ and $l = \lim_{x\to\infty} \frac{dy}{dx}$, then sum of all possible values of l is

(A) 3 (B) 5 (C) 6 (D) 7

[INTEGER TYPE] [2 × 5 = 10]

Q.5 If a, b, c and d are real constants such that $\lim_{x\to 0} \frac{ax^2 + \sin(bx) + \sin(cx) + \sin(dx)}{3x^2 + 5x^4 + 7x^6} = 8$,
then find the value of (a + b + c + d).

Q.6 Let f (x) and g (x) be twice differentiable functions satisfying f (x) = x g (x) and g '(x) = f (x), where $g(x) \neq 0 \ \forall \ x \in R$. If $f'(x) = g(x) \cdot h(x)$, then find the number of roots of the equation $h(x) = e^x$.

DPP No.-37 to 40
SOLUTION

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

DPP. NO.-37

[SINGLE CORRECT CHOICE TYPE] [4 × 3 = 12]

Q.1 If $\int \frac{x^4+1}{x(x^2+1)^2}\,dx = A\ln|x| + \frac{B}{1+x^2} + c$, where c is the constant of integration then

(A) A = 1 ; B = – 1 (B) A = – 1 ; B = 1 (C*) A = 1 ; B = 1 (D) A = – 1 ; B = – 1

[Sol.$_{15/inde/SC}$ add and subtract $2x^2$ in the numerator]

Q.2 If $\lim_{x\to 0}\frac{p\sin 2x + (1-\cos 2x)}{x+\tan x} = 1$, then p is equal to

(A) 0 (B*) 1 (C) 2 (D) –2

[Sol.$_{178/lcd/SC}$ $\lim_{x\to 0}\frac{p\sin 2x + (1-\cos 2x)}{x+\tan x} = 1 \quad \left(\frac{0}{0}\text{ form}\right)$

$$\therefore \lim_{x\to 0}\frac{2p\left(\frac{\sin 2x}{2x}\right)+\left(\frac{1-\cos 2x}{x}\right)}{\left(1+\frac{\tan x}{x}\right)} = 1 \Rightarrow \frac{2p+0}{1+1} = 1 \Rightarrow p = 1 \text{ **Ans.**]}$$

Q.3 If $f(x) = \frac{(x+1)(x+2)^2}{(x+3)^3(x+4)^4}$ then $f'(0)$ is equal to

(A*) 0 (B) f(0) (C) 2f(0) (D) 4f(0)

[Sol.$_{330/mod/SC}$ $\ln f(x) = \ln(x+1) + 2\ln(x+2) - 3\ln(x+3) - 4\ln(x+4)$

differentiating we get

$$\frac{f'(x)}{f(x)} = \frac{1}{x+1} + \frac{2}{x+2} - \frac{3}{x+3} - \frac{4}{x+4}$$

$$\Rightarrow \frac{f'(0)}{f(0)} = 1 + 1 - 1 - 1 \Rightarrow f'(0) = 0$$ **Ans.**]

Q.4 If number of significant digit before decimal in 2^{64} is a and number of zero's after decimal and before first significant digit in 3^{-50} is b, then b – a is

(here $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$)

(A*) 3 (B) 4 (C) 6 (D) 1

[Sol.$_{146/log/SC}$ $\log_{10} 2^{64} = 64 \times \log_{10} 2 = 19 + 0.264$

$\therefore$ a = 19 + 1 = 20

and $\log_{10} 3^{-50} = -50 \times \log_{10} 3 = -50 \times 0.4771 = -23.855 = -24 + 0.145$

$\therefore$ b = (–24 + 1) = 23

$\therefore$ b – a = 3.]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

[SUBJECTIVE] **[2 × 5 = 10]**

Q.5 (a) $\lim\limits_{x\to 0} \tan^{-1}\dfrac{a}{x^2}$, where $a \in R$; (b) Plot the graph of the function $f(x) = \lim\limits_{t\to 0}\left(\dfrac{2x}{\pi}\tan^{-1}\dfrac{x}{t^2}\right)$

[Ans. (a) $\pi/2$ if $a > 0$; 0 if $a = 0$ and $-\pi/2$ if $a < 0$; (b) $f(x) = |x|$]

[Sol.$_{81068/lcd/SUB}$ (a) $\lim\limits_{x\to 0} \tan^{-1}\dfrac{a}{x^2}$

for $a < 0$ $\quad l = -\dfrac{\pi}{2}$

for $a > 0$ $\quad l = \dfrac{\pi}{2}$

for $a = 0$ $\quad l = 0$

(b) $$f(x) = \lim_{t\to 0}\left(\frac{2x}{\pi}\tan^{-1}\frac{x}{t^2}\right) = \begin{bmatrix} x & x>0 \\ 0 & x=0 \\ -x & x<0 \end{bmatrix} = |x|$$ **Ans.**]

Q.6 Let f be a function defined implicitly by the equation $\dfrac{1-e^{f(x)}}{1+e^{f(x)}} = x$ and g be the inverse of f.

If $g''(\ln 3) - g'(\ln 3) = \dfrac{p}{q}$ where p and q relative prime numbers then find the value of $(p+q)$.

[Ans. 25]

[Sol.$_{50721/mod}$ $\dfrac{1-e^{f(x)}}{1+e^{f(x)}} = x \Rightarrow f(x) = \ln\left(\dfrac{1-x}{1+x}\right)$ Domain of f(x) is $(-1, 1)$

$$f'(x) = -\frac{1}{1-x} - \frac{1}{1+x} = \frac{2}{x^2-1} < 0 \Rightarrow f(x) \text{ is decreasing}$$

$$f(x) = \ln 3 \Rightarrow x = \frac{-1}{2}$$

$$g'(y) = \frac{1}{f'(x)} \Rightarrow g'(\ln 2) = \frac{1}{f'\left(\frac{-1}{2}\right)} = \frac{1}{\frac{-8}{3}} = \frac{-3}{8}$$

$$g''(y) = -\frac{f''(x)}{(f'(x))^3} \Rightarrow g''(\ln 2) = -\frac{f''\left(\frac{-1}{2}\right)}{\left(f'\left(\frac{-1}{2}\right)\right)^3}$$

$$f''(x) = -\frac{4x}{(x^2-1)^2}$$

$$f''\left(\frac{-1}{2}\right)=\frac{-4\cdot\left(\frac{-1}{2}\right)}{\frac{9}{16}}=\frac{32}{9}$$

$$\therefore\ g''(\ln 2)=-\frac{\left(\frac{32}{9}\right)}{\left(\frac{-8}{3}\right)^3}=\frac{32\times 3}{2^9}=\frac{3}{16}$$

$$\therefore\ g''(\ln 2)-g'(\ln 2)=\frac{3}{16}-\left(\frac{-3}{8}\right)=\frac{9}{16}$$

$\therefore\ p+q=25$ **Ans.**]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

DPP. NO.-38

Q.1 **[3 + 3 = 6]**

(a) $\int \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right)dx$. [Ans. $(x+1)\tan^{-1}\frac{2(x+1)}{3} - \frac{3}{4}\ell n(4x^2+8x+13) + C$]

[Sol.$_{81065/inde/SUB}$ $I = \int \sin^{-1}\left(\frac{2x+2}{\sqrt{(2x+2)^2+9}}\right)dx$ $\Rightarrow$ $2x+2 = t$ $\Rightarrow$ $dx = \frac{dt}{2}$

$\Rightarrow \frac{1}{2}\int \sin^{-1}\left(\frac{t}{\sqrt{t^2+9}}\right)dt$] $\Rightarrow$ $t = 3\tan\theta$; $q = \tan^{-1}(t/3)$

$\Rightarrow$ $dt = 3\sec^2\theta\, d\theta$

$\Rightarrow \frac{1}{2}\int \sin^{-1}(\sin\theta) 3\sec^2\theta d\theta = \frac{1}{2}\int \theta \cdot 3\sec^2\theta dx$

$= \frac{3}{2}\int \theta\cdot\sec^2\theta d\theta$ ... Intergrated by parts $= \frac{3}{2}\left[\theta\int\sec^2\theta d\theta - \int\left[\frac{d(\theta)}{d\theta}\int\sec^2\theta d\theta\right]d\theta\right]$

$= \frac{3}{2}\left[\theta\tan\theta - \int\tan\theta d\theta\right]$ $= (x+1)\tan^{-1}\frac{2(x+1)}{3} - \frac{3}{4}\ell n(4x^2+8x+13) + C$ Ans.]

(b) $\int \frac{x^2+x}{(e^x+x+1)^2}dx$ [Ans. $C - ln(1+(x+1)e^{-x}) - \frac{1}{1+(x+1)e^{-x}}$]

[Sol.$_{81076/inde/sub}$ $I = \int \frac{x(x+1)}{e^{2x}(1+xe^{-x}+e^{-x})^2}dx = \int \frac{e^{-2x}x(x+1)}{\left(1+(x+1)e^{-x}\right)^2}dx = \int \frac{(x+1)e^{-x}\cdot xe^{-x}}{(1+(x+1)e^{-x})^2}dx$

put $(x+1)e^{-x} = t$ $\Rightarrow$ $[-(x+1)e^{-x} + e^{-x}]\,dx = dt$ $\Rightarrow$ $e^{-x}(-x)\,dx = dt$

$I = \int \frac{t(-dt)}{(1+t)^2} = -\int \frac{t\,dt}{(1+t)^2} = -\int \frac{1+t-1}{(1+t)^2}dt = -\left(\int\frac{dt}{(1+t)} - \int\frac{1}{(1+t)^2}dt\right)$

$= C - ln(1+t) - \frac{1}{1+t}$

$= C - ln\left(1+(x+1)e^{-x}\right) - \frac{1}{1+(x+1)e^{-x}}$ Ans.]

[SINGLE CORRECT CHOICE TYPE] [5 × 3 = 15]

Q.2$_{qe}$ Number of integral values of a so that the graph of $f(x) = x^2 - 2(4a-1)x + (15a^2 - 2a - 7)$ is completely above the x-axis is

(A*) 1 (B) 2 (C) 3 (D) 4

[Sol.$_{445/dpp}$ $x^2 - 2(4a-1)x + (15a^2 - 2a - 7) > 0$

if (i) coefficient of $x^2 > 0$ and (ii) discriminant < 0

Now, $D = 4(4a-1)^2 - 4(15a^2 - 2a - 7) < 0$

$\Rightarrow$ $16a^2 + 1 - 8a - 15a^2 + 2a + 7 < 0$

$\Rightarrow \quad a^2 - 6a + 8 < 0$
$\Rightarrow \quad (a-2)(a-4) < 0$
$\Rightarrow \quad 2 < a < 4.$
$\therefore$ The only integer satisfying the conditions is 3.
Hence number of integral values of a are 1. **Ans.**]

Q.3 If $\lim_{x\to\infty} \dfrac{x^2+3x+5}{4x+1+x^k}$ exists then
(A) $k = 2$ (B) $k < 2$ (C) $k > 2$ (D*) $k \ge 2$

[Sol.$_{45/lcd/SC}$ $l = \lim_{x\to\infty} \dfrac{1+\frac{3}{x}+\frac{5}{x^2}}{\frac{4}{x}+\frac{1}{x^2}+x^{k-2}}$

for existence of limit $k - 2 \ge 0 \quad \Rightarrow \quad k \ge 2$ Ans.]

Q.4 If $f(x) = x^x \cdot e^x$ then $f'(1)$ equals
(A) 1 (B) e (C*) 2e (D) $\dfrac{e}{2}$

[Sol.$_{328/mod/SC}$ $f(x) = x^x \cdot e^x = e^{x\log x} \cdot e^x = e^{x(\log x + 1)}$

$f'(x) = e^{x(\log x + 1)} \cdot \left((\log x + 1) + \dfrac{x}{x}\right) = x^x \cdot e^x (\log x + 2)$

$f'(1) = 2 \cdot (1) \cdot e = 2e$. **Ans.**]

Q.5 If $f : [1, 10] \to Q$ (where Q is the set of all rational numbers) is a continuous function
and $f(1) = \lim_{x\to 0}\left(\left[\dfrac{x}{\sin x}\right] + \left[\dfrac{\tan x}{x}\right]\right)$, then the value of $\sum_{r=1}^{10} \tan^{-1}(\tan f(r))$ is equal to
[**Note:** where [y] denotes greatest integer function less than or equal to y.]
(A) 10 (B) 20 (C*) $20 - 10\pi$ (D) $10\pi - 20$

[Sol.$_{608/lcd/SC}$ $\because$ $f(1) = 2$ and $f(x) = $ constant

$\therefore \sum_{r=1}^{10} \tan^{-1}(\tan(f(r))) = \sum_{r=1}^{10} \tan^{-1}(\tan 2) = \sum_{r=1}^{10} (2-\pi) = 20 - 10\pi$ **Ans.**]

Q.6 List -I contains the function and List-II contains their derivatives at $x = 0$.
Select the correct answer using the codes given below the list.

	List-I		**List-II**
(P)	$f(x) = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$	(1)	2
(Q)	$g(x) = \cos^{-1}(2x^2 - 1)$	(2)	3
(R)	$h(x) = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$	(3)	– 2
(S)	$k(x) = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$	(4)	non-existent

Codes :

	P	**Q**	**R**	**S**
(A)	3	4	4	1
(B*)	3	4	4	2
(C)	4	3	2	1
(D)	2	4	4	2

[Sol.$_{90001/mod/MTC}$

(P) $f(x) = \cos^{-1}\left(\frac{2x}{1+x^2}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$\therefore \quad f(x) = \frac{\pi}{2} - 2\tan^{-1}x, \ -1 \le x\ 1,$ So, $f'(x)_{\text{at } x=0} = \frac{-2}{1+x^2} = -2.$

(Q) $g(x) = \cos^{-1}(2x^2 - 1)$

$\therefore \quad g'(x) = \frac{-1}{\sqrt{1-(2x^2-1)^2}}\, 4(x)$ So, $g'(0) =$ non-existent.

(R) $h(x) = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \begin{cases} \frac{\pi}{2} + 2\tan^{-1}x, & -\infty < x < 0 \\ \frac{\pi}{2} - 2\tan^{-1}x, & x \ge 0 \end{cases}$

As $h'(0^-) = 2$ and $h'(0^+) = -2$
So, $h'(0) =$ non-existent.

(S) $k(x) = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = 3\tan^{-1}x, \quad \frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

$\therefore \quad k'(x) = \frac{3}{1+x^2}.$ So , $k'(0) = 3$ Ans.]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

DPP. NO.-39

[SINGLE CORRECT CHOICE TYPE] [8 × 3 = 24]

Q.1 The value of integral $\int e^x\left(\frac{2\tan x}{1+\tan x}+\cot^2\left(x+\frac{\pi}{4}\right)\right)dx$ is equal to

(A) $e^x \tan\left(\frac{\pi}{4}-x\right)+C$ (B*) $e^x \tan\left(x-\frac{\pi}{4}\right)+C$

(C) $e^x \tan\left(\frac{3\pi}{4}-x\right)+C$ (D) $e^x \tan\left(x-\frac{3\pi}{4}\right)+C$

where C is constant of integration.

[Sol.$_{68/inde/SC}$ Let $I=\int e^x\left(\frac{2\tan x}{1+\tan x}+\tan^2\left(x-\frac{\pi}{4}\right)\right)dx=\int e^x\left(\frac{2\tan x}{1+\tan x}+\sec^2\left(x-\frac{\pi}{4}\right)-1\right)dx$

$=\int e^x\left(\frac{\tan x-1}{1+\tan x}+\sec^2\left(x-\frac{\pi}{4}\right)\right)dx=\int e^x\left(\tan\left(x-\frac{\pi}{4}\right)+\sec^2\left(x-\frac{\pi}{4}\right)\right)dx$

$=e^x\tan\left(x-\frac{\pi}{4}\right)+C$]

Q.2 If a, b, c, d are positive real numbers such that $a+b+c+d=2$, then $M=(a+b)(c+d)$ satisfies the relation :

(A*) $0 \le M \le 1$ (B) $1 \le M \le 2$ (C) $2 \le M \le 3$ (D) $3 \le M \le 4$

[Sol.$_{202/seq/SC}$ [JEE 2000, Screening, 1 out of 35]]

Q.3 $\int_0^1 \frac{x-e^{2x}}{x^2-e^{2x}}dx$ equals

(A) 2 (B) $ln(e^2-1)$ (C*) $\frac{1}{2}ln(e^2-1)$ (D) e – 1

[Sol.$_{530/def/SC}$ $\frac{1}{2}\int_0^1\frac{2x-2e^{2x}}{x^2-e^{2x}}dx$

Putting $x^2-e^{2x}=t$

$(2x-2e^{2x})\,dx=dt$

$\frac{1}{2}\int_{-1}^{1-e^2}\frac{dt}{t}=\frac{1}{2}\left(ln|t|\right)_{-1}^{1-e^2}=\frac{1}{2}\left(ln\left(e^2-1\right)-0\right)=\frac{1}{2}ln\left(e^2-1\right)$. **Ans.**]

Q.4 For the curve $xy + y^2 = 1$, $\frac{d^2y}{dx^2}$ is equal to

(A) $\frac{-y}{x+2y}$ (B) $\frac{2xy+2y}{(x+2y)^3}$ (C*) $\frac{2}{(x+2y)^3}$ (D) $\frac{2x+2y^2}{(x+2y)^3}$

[Sol.$_{191/mod/SC}$ $xy + y^2 = 1$

$$\Rightarrow x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-y}{x+2y}$$

$$\frac{d^2y}{dx^2} = \frac{(x+2y)\left(\frac{-dy}{dx}\right) + y\left(1+\frac{2dy}{dx}\right)}{(x+2y)^2} = \frac{1}{(x+2y)^2}\left\{\frac{(x+2y)y}{x+2y} + y + (2y)\left(\frac{-y}{x+2y}\right)\right\}$$

$$= \frac{1}{(x+2y)^2}\left\{y + y - \frac{2y^2}{x+2y}\right\} = \frac{1}{(x+2y)^2}\left\{\frac{2xy+4y^2-2y^2}{x+2y}\right\}$$

$$= \frac{1}{(x+2y)^2}\left\{\frac{2y(x+y)}{x+2y}\right\} = \frac{2(xy+y^2)}{(x+2y)^3} = \frac{2}{(x+2y)^3}$$ **Ans.**]

Q.5 If $\frac{\cos 3x}{\cos x} = \frac{1}{3}$ for some angle x, $0 \le x \le \frac{\pi}{2}$, then the value of $\frac{\sin 3x}{\sin x}$ for same x, is

(A*) $\frac{7}{3}$ (B) $\frac{5}{3}$ (C) 1 (D) $\frac{2}{3}$

[Sol.$_{43/ph-1/SC}$ Consider, $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x} = \frac{\sin 3x\cos x - \cos 3x \sin x}{\sin x\cos x} = \frac{\sin 2x}{\sin x\cos x} = 2\cdot\frac{\sin 2x}{\sin 2x} = 2$

so $\frac{\sin 3x}{\sin x} - \frac{1}{3} = 2$

or $\frac{\sin 3x}{\sin x} = 2 + \frac{1}{3} = \frac{7}{3}$ Ans.]

Q.6 The values for A, B and C respectively if $\lim_{x\to 1}\frac{Ax^4+Bx^3+1}{(x-1)\sin \pi x}$ exists and is equal to C are

(A) $-4, 3, \frac{6}{\pi}$ (B) $-4, 3, -\frac{6}{\pi}$ (C) $3, -4, \frac{6}{\pi}$ (D*) $3, -4, \frac{-6}{\pi}$

[Sol.$_{179/mod/SC}$ $\lim_{x\to 1}\frac{Ax^4+Bx^3+1}{(x-1)\sin \pi x} = C$ $\left(\frac{0}{0}\text{ form}\right)$

$A + B + 1 = 0$ ....(1)

Put $x-1=h \Rightarrow x=h+1$

$$\lim_{h\to 0}\frac{A(h+1)^4+B(h+1)^3+1}{h\left(\frac{-\sin \pi h}{\pi h}\right)\pi h}=\lim_{h\to 0}\frac{A(h+1)^4+B(h+1)^3+1}{-\pi h^2}\quad\left(\frac{0}{0}\text{ form}\right)$$

By L'Hopital's Rule

$$=\frac{-1}{\pi}\lim_{h\to 0}\frac{4A(h+1)^3+3B(h+1)^2}{2h}\quad\left(\text{Again }\frac{0}{0}\text{ form}\right)$$

So, $4A+3B=0$ ....(2)

Solve equation (1) and (2), we get $A=3$, $B=-4$

$$\frac{-1}{\pi}\lim_{h\to 0}\frac{12\left[(h+1)^3-(h+1)^2\right]}{2h}=\frac{-1}{\pi}\lim_{h\to 0}\frac{6\left[3(h+1)-2(h+1)\right]}{1}=\frac{-6}{\pi}.\ \textbf{Ans.}]$$

Q.7 $\displaystyle\int\frac{dx}{x(x+1)(\ln(x+1)-\ln x)^{11}}$ equals

(A*) $\dfrac{1}{10(\ln(x+1)-\ln x)^{10}}+C$

(B) $\dfrac{(\ln(x+1)-\ln x)^{10}}{10}+C$

(C) $\dfrac{1}{11(\ln(x+1)-\ln x)^{11}}+C$

(D) $\dfrac{(\ln(x+1)-\ln x)^{11}}{11}+C$

where C is constant of integration.

[Sol.$_{70/inde/SC}$ Let $I=\displaystyle\int\frac{dx}{x(1+x)(\ln(x+1)-\ln x)^{11}}$

Put $\ln(x+1)-\ln x=t \Rightarrow \dfrac{dx}{x(1+x)}=-dt$

So, $I=-\displaystyle\int\frac{dt}{t^{11}}=\frac{1}{10}\left(\frac{1}{t^{10}}\right)=\frac{1}{10(\ln(x+1)-\ln x)^{10}}+C$ **Ans.**]

Q.8 If $f(x)=\begin{cases}\sqrt{1-x} & 0\le x\le 1\\ (7x-6)^{-1/3} & 1<x\le 2\end{cases}$, then $\displaystyle\int_0^2 f(x)\,dx$ is equal to

(A) $\dfrac{31}{6}$ (B) $\dfrac{32}{21}$ (C) $\dfrac{1}{42}$ (D*) $\dfrac{55}{42}$

[Sol.$_{75/def/SC}$ Answer is $\dfrac{55}{42}$]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

DPP. NO.-40

[SINGLE CORRECT CHOICE TYPE] **[4 × 3 = 12]**

Q.1 $\int \frac{\cot^{-1}(e^x)}{e^x}\,dx$ is equal to :

(A) $\frac{1}{2}\ln(e^{2x}+1) - \frac{\cot^{-1}(e^x)}{e^x} + x + c$ (B) $\frac{1}{2}\ln(e^{2x}+1) + \frac{\cot^{-1}(e^x)}{e^x} + x + c$

(C*) $\frac{1}{2}\ln(e^{2x}+1) - \frac{\cot^{-1}(e^x)}{e^x} - x + c$ (D) $\frac{1}{2}\ln(e^{2x}+1) + \frac{\cot^{-1}(e^x)}{e^x} - x + c$

[Sol.$_{6/inde/SC}$ IBP taking e^{-x} as the II function and $\cot^{-1}(e^x)$ as the 1st function.]

Q.2 $I = \int_0^1 \frac{(1-x^2)}{(1+x^2)\sqrt{1+x^4}}\,dx$ is equal to

(A) $\frac{\pi}{4}$ (B*) $\frac{\pi}{4\sqrt{2}}$ (C) $\frac{\pi}{2\sqrt{2}}$ (D) $\frac{\pi\sqrt{2}}{2}$

[Sol.$_{528/def/SC}$ $I = -\int_0^1 \frac{x^2-1}{x^2\left(x+\frac{1}{x}\right)\sqrt{x^2+\frac{1}{x^2}}}\,dx = -\int_0^1 \frac{\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)\sqrt{\left(x+\frac{1}{x}\right)^2-2}}\,dx$

$x + \frac{1}{x} = t \Rightarrow \left(1-\frac{1}{x^2}\right)dx = dt$

$I = \int_2^\infty \frac{dt}{t\sqrt{t^2-2}} = \frac{1}{\sqrt{2}}\sec^{-1}\frac{t}{\sqrt{2}}\Big]_2^\infty = \frac{1}{2}\left[\frac{\pi}{2}-\frac{\pi}{4}\right] = \frac{\pi}{4\sqrt{2}}$. **Ans.**]

Q.3 The graph of the function, $\cos x\cos(x+2) - \cos^2(x+1)$ is
(A) a straight line passing through $(0, -\sin^2 1)$ with slope 2
(B) a straight line passing through $(0, 0)$
(C) a parabola with vertex $(1, -\sin^2 1)$
(D*) a straight line passing through the point $\left(\frac{\pi}{2}, -\sin^2 1\right)$ & parallel to the x–axis.

[Sol.$_{167/st.line/SC}$ [JEE'97, 2]]

Q.4 If $(6x^2 - 5xy + y^2)(x^2 - y^2 + 2x) = 0$ and $l = \lim_{x\to\infty}\frac{dy}{dx}$, then sum of all possible values of l is

(A) 3 (B*) 5 (C) 6 (D) 7

[Sol.$_{291/mod/SC}$ $y = 3x$, $y = 2x$ and $x^2 - y^2 + 2x = 0$

$\Rightarrow (x+1)^2 - y^2 = 1$

$2(x+1) = 2y\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{x+1}{y} \Rightarrow \dfrac{dy}{dx} = \lim\limits_{x\to\infty} \dfrac{x+1}{\pm\sqrt{x^2+2x}} \Rightarrow \dfrac{dy}{dx} = \pm 1$

$l = \lim\limits_{x\to\infty} \dfrac{dy}{dx}$ → 3, 2, 1, −1

$\therefore$ Required sum = 5 **Ans.**]

[INTEGER TYPE] [2 × 5 = 10]

Q.5 If a, b, c and d are real constants such that $\lim\limits_{x\to 0} \dfrac{ax^2 + \sin(bx) + \sin(cx) + \sin(dx)}{3x^2 + 5x^4 + 7x^6} = 8$, then find the value of $(a + b + c + d)$. [Ans. 24]

[Sol.$_{\text{50770/lcd/OMR}}$

$$\lim_{x\to 0} \frac{ax^2 + (b+c+d)x - \frac{1}{3!}(b^3+c^3+d^3)x^3 + \frac{1}{5!}(b^5+c^5+d^5)x^5 - \frac{1}{7!}(b^7+c^7+d^7)x^7 + ...}{3x^2+5x^4+7x^6}$$

Dividing N^r and D^r by x^2, we have

for existence of limit $b + c + d = 0$; then $L = \dfrac{a}{3} = 8 \Rightarrow a = 24$.

Hence $(a + b + c + d) = 24$. **Ans.**]

Q.6 Let f (x) and g (x) be twice differentiable functions satisfying f (x) = x g (x) and g ' (x) = f (x), where $g(x) \neq 0 \ \forall \ x \in R$. If $f'(x) = g(x) \cdot h(x)$, then find the number of roots of the equation $h(x) = e^x$. [Ans. 1]

[Sol.$_{\text{50720/mod}}$ $g'(x) = f(x) = x\,g(x)$

$\dfrac{g'(x)}{g(x)} = x \Rightarrow ln(g(x)) = \dfrac{x^2}{2} + C \Rightarrow g(x) = k \cdot e^{\frac{x^2}{2}}$

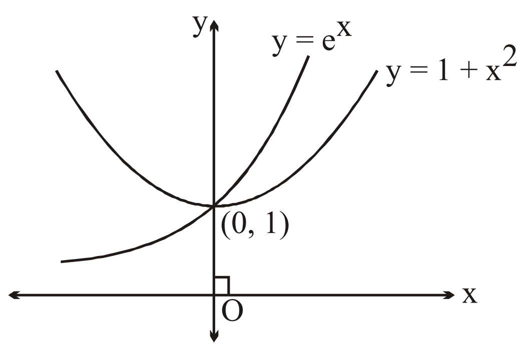
$f(x) = x\,g(x) = k\,x\,e^{\frac{x^2}{2}}$

$\therefore \quad f'(x) = k\,e^{\frac{x^2}{2}}(1+x^2) = g(x)\,h(x)$

$\therefore \quad h(x) = 1 + x^2$

Now $1 + x^2 = e^x$

only one solution. **Ans.**



Aliter: $f(x) = x\,g(x)$

$f'(x) = x\,g'(x) + g(x)$

$f'(x) = x\,f(x) + g(x)$

$g(x)\,h(x) = x^2\,g(x) + g(x)$

$\Rightarrow \quad h(x) = x^2 + 1 \ (g(x) \neq 0)$]