# DPP No.-33 to 36

**Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)**

# DPP. NO.-33

**COMPREHENSION TYPE]** **[8 × 3 = 24]**

**Paragraph for question nos. 1 to 3**

Let the function f be defined in (0, 1). Two more functions are given as $g(x) = e^x$ and $h(x) = ln|x|$.

Q.1 The number of solutions of the equation $g(x) \cdot h(g(x)) = 1$ is

(A) 0 (B) 1 (C) 2 (D) 3

Q.2 Domain of $f(g(x)) + f(h(x))$ is

(A) $(-e, -1)$ (B) $(-e, -1) \cup (1, e)$ (C) $[-e, -1]$ (D) $(-\infty, 0)$

Q.3 Let f be one-one function with range (1, 2) and $gof(x)$ is defined then domain of $f^{-1}og^{-1}(x)$ is

(A) (0, 1) (B) $(e, e^2)$ (C) $(\sqrt{e}, e)$ (D) (1, e)

**Paragraph for question nos. 4 & 5**

Let $f: [0, \infty) \to [2, \infty)$ be a derivable increasing function which is also surjective and satisfying

$$f^2(x) - f^2(y) = 3f(x) - 3f(y) + \sqrt{x} - \sqrt{y}$$

Q.4 The value of $\lim_{x \to 0^+} \dfrac{2f(x) - 3 - e^{\sqrt{x}}}{\sqrt{x}}$ equals

(A) 0 (B) 1 (C) 2 (D) 4

Q.5 If g is the inverse function of f then $\dfrac{d}{dx}\left(\dfrac{4}{g(x)}\right)$ at $x = 3$ is equal to

(A) 6 (B) 3 (C) – 6 (D) – 3

**Paragraph for question nos. 6 to 8**

Let f(x) be a polynomial function satisfying the following conditions

$$\lim_{x \to \infty} \frac{f(x)}{|x|^3} = 0; \lim_{x \to \infty}\left(\sqrt{f(x)} - x\right) = -1 \text{ and } f(0) = 0$$

[**Note:** [k] and {k} denotes greatest integer function less than or equal to k and fractional part function of k respectively.]

Q.6 The value of $\lim_{x \to 0} (1 + f(x))^{\operatorname{cosec} x}$ is equal to

(A) e (B) $e^{-1}$ (C) $e^{-2}$ (D) $e^{\frac{-1}{2}}$

Q.7 Number of points where $|f(|x|)|$ is non derivable, is

(A) 1 (B) 2 (C) 3 (D) 5

Q.8 If minimum value of f(x) occur at $x = a$, then the value of $\lim_{x \to a^-} \dfrac{[x + \{x\}]}{\{x + [x]\}}$ is equal to

(A) 0 (B) 1 (C) 2 (D) 3

## Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

# DPP. NO.-34

### [SINGLE CORRECT CHOICE TYPE] [4 × 3 = 12]

Q.1 Let $f(x)=\dfrac{(x^3-2x^2+x)(\pi^x-1)+(1+x^2+x^4)}{(1-x+x^2)}$ and $g(x)=\sin^2 x$. If $\displaystyle\sum_{r=0}^{1}\left.\frac{d^2g}{dx^2}\right|_{x=f'(r)}=a(\cos\alpha)(\cos\beta)$, where $a, \alpha, \beta \in N$, then the value of $(a+\alpha+\beta)$ is equal to

(A) 4 (B) 6 (C) 8 (D) 10

Q.2 $\displaystyle\int\frac{4x^2+3x+2}{\sqrt[3]{1+x+\frac{1}{x}}}\,dx$ equals

(A) $x^2\left(1+x+\dfrac{1}{x}\right)^{\frac{2}{3}}+c$ (B) $\dfrac{3}{2}\cdot x^2\left(1+x+\dfrac{1}{x}\right)^{\frac{2}{3}}+c$

(C) $\dfrac{3}{2}x\cdot\left(1+x+\dfrac{1}{x}\right)^{\frac{2}{3}}+c$ (D) $x\left(1+x+\dfrac{1}{x}\right)^{\frac{2}{3}}+c$

Q.3 If $\tan^{-1}x+\cos^{-1}\left(\dfrac{y}{\sqrt{1+y^2}}\right)=\tan^{-1}4$ where $x, y \in N$, then the number of possible values of $x$, is

(A) 0 (B) 1 (C) 2 (D) 3

Q.4 The function $f(x)$ is defined by $f(x)=\begin{cases}\left(x^2+e^{\frac{1}{2-x}}\right)^{-1}, & x\neq 2\\ k, & x=2\end{cases}$.

If it is continuous from right at the point $x=2$ then $k$ is equal to

(A) 0 (B) $\dfrac{1}{4}$ (C) $\dfrac{-1}{2}$ (D) 1

Q.5 Let $f(x)=ax^2+bx+c$ where $a\neq 0$. If $f(0)=2016$, $f(d^2)=2017$, $f(d)=2018$ where $d\in(0, 1)$ and sum of the roots of $f(x)$ is 0, then the value of 'd' is

(A) $\dfrac{1}{\sqrt{3}}$ (B) $\dfrac{1}{\sqrt{2}}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{1}{3}$

## [MATRIX TYPE]

**[3+3+3+3=12]**

Q.6

| | **Column-I** | **Column-II** |
|---|---|---|
| (A) | Let $f(x) = \|x+4\| + \|x+1\| + \|x\| + \|x-5\| + \|x-9\|$, then the largest positive integral value of k for which the equation $f(x) = k$ has no solution is | (P) 10 |
| (B) | Let $f(x) = \begin{cases} \dfrac{e^{\sin^{-1}\frac{x}{\sqrt{x^2+1}}} - e^{\sin^{-1}x}}{e^{\tan x} - e^{\sin x}}, & \text{if } x \neq 0 \\ p, & \text{if } x = 0 \end{cases}$. If $f(x)$ is continuous at $x = 0$ and the value of $\left(\cot^{-1}(p) + \sin^{-1}(p^2)\right) = \frac{k\pi}{8}$, $(k \in N)$, then $k$ equals | (Q) 18 |
| | | (R) 25 |
| (C) | Let $\{a_n\}$ be a geometric sequence with $a_7 = 50$ and $a_{11} = 250\sqrt[3]{5}$. If the value of $a_3$ is $2\sqrt[3]{k}$ $(k \in N)$, then the value of $k$, is | (S) 40 |
| (D) | Let $k_1$ and $k_2$ are two values of k for which the equation $4x^2 - 4(5x+1) + k^2 = 0$ has one root equals to two more than the other, then the value of $\left(k_1^2 + k_2^2\right)$, is | (T) 50 |

**Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)**

# DPP. NO.-35

Q.1 Evaluate $\lim\limits_{n\to\infty}\left(\frac{\sqrt{n^2+n}-1}{n}\right)^{2\sqrt{n^2+n}-1}$ **[3]**

Q.2 Evaluate $\lim\limits_{x\to\frac{1}{\sqrt{2}}}\frac{\cos^{-1}\left(2x\sqrt{1-x^2}\right)}{x-\frac{1}{\sqrt{2}}}$ [Ans. does not exists] **[3]**

## [SINGLE CORRECT CHOICE TYPE] [3 × 3 = 9]

Q.3 Range of $f(x)=\cos^{-1}\left(\sqrt{x^2+x+1}\right)$ is

(A) $\left[0,\frac{\pi}{6}\right]$ (B) $\left[0,\frac{\pi}{3}\right]$ (C) $\left[\frac{\pi}{6},\frac{\pi}{2}\right]$ (D) $\left[\frac{\pi}{6},\frac{\pi}{3}\right]$

Q.4 $\int\frac{\cos x\,dx}{\sqrt{\sin^2 x+2\cos^2 x}}$ is equal to

(A) $\sin^{-1}\left(\frac{\sin x}{2}\right)+C$ (B) $ln\left(\sin x+\sqrt{2-\sin^2 x}\right)+C$

(C) $\sin^{-1}\left(\frac{\sin x}{\sqrt{2}}\right)+C$ (D) $ln\left(\sin x-\sqrt{2-\sin^2 x}\right)+C$

(Where C is constant of integration)

Q.5 In a $\Delta ABC$, sides a, b, c are in G.P. and angles A, B, C are in A.P. If area of $\Delta ABC = 3\sqrt{3}$ then $(R+r)$ is equal to

(A) – 2 (B) 0 (C) 2 (D) 3

## [INTEGER TYPE] [1 × 5 = 5]

Q.6 Let [k] denotes the greatest integer less than or equal to k.

If number of positive integral solutions of the equation $\left[\frac{x}{[\pi^2]}\right]=\left[\frac{x}{\left[11\frac{1}{2}\right]}\right]$ is n,

then find the value of $\sqrt{n-8}$.

## Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

## DPP. NO.-36

### [SINGLE CORRECT CHOICE TYPE] [2 × 3 = 6]

Q.1 If $x^2 + y^2 = 1$, then

(A) $yy'' - 2(y')^2 + 1 = 0$ (B) $yy'' + (y')^2 + 1 = 0$
(C) $yy'' - (y')^2 - 1 = 0$ (D) $yy'' + 2(y')^2 + 1 = 0$

[ JEE 2000, Screening, 1 out of 35 ]

Q.2 Range of the function $f(x) = 4\tan^{-1}x + 3\sin^{-1}x + \sec^{-1}x$ is

(A) $\left\{\frac{-3\pi}{2}, \frac{-5\pi}{2}\right\}$ (B) $\left\{\frac{3\pi}{2}, \frac{5\pi}{2}\right\}$ (C) $\left(\frac{-3\pi}{2}, \frac{5\pi}{2}\right)$ (D) $\left\{\frac{-3\pi}{2}, \frac{5\pi}{2}\right\}$

### [MULTIPLE CORRECT CHOICE TYPE] [3 × 4 = 12]

Q.3 Consider, $P = \frac{x^2 - 2x}{x^2 + x + 1}$, $Q = \frac{y - 1}{y^2 + y + 1}$ and $R = \frac{2}{z^2 + z + 1}$ where $x, y, z \in R$.

If $k = [P + Q + R] - ([P] + [Q] + [R])$ then the possible value(s) of k is(are)

(A) 0 (B) 1 (C) 2 (D) 3

**[Note :** $[\lambda]$ denotes the greatest integer less than or equal to $\lambda$.]

Q.4 Given that $(x-2)^2 + (y-2)^2 = 1$ then maximum value of

(A) $x + y$ is $4 + \sqrt{2}$ (B) $x - y$ is $\sqrt{2}$ (C) $\frac{x}{y}$ is $\frac{4+\sqrt{7}}{3}$ (D) $\frac{y}{x}$ is $\frac{4+\sqrt{7}}{3}$

Q.5 The value(s) of p for which the inequality $\log_{(p^2+1)}(3x^2 - 2x - p + 6) \geq \cos^2\theta\,(1 - 4\sin^2\theta)^2 - \cos^2 3\theta$ is satisfied for all real values of x and $\theta$ is(are)

(A) 0 (B) 1 (C) 4 (D) 5

### [MATRIX TYPE] [2+2+2+2=8]

Q.6

| Column–I | Column-II |
|---|---|
| (A) $\lim_{x\to 1} \frac{x^3 - 1}{\ln x}$ is | (P) 2 |
| (B) $\lim_{x\to 0} \frac{x(\cos x - \cos 2x)}{2\sin x - \sin 2x}$ is | (Q) 3 |
| (C) $\lim_{x\to 0} \frac{\tan x\sqrt{\tan x} - \sin x\sqrt{\sin x}}{x^3 \cdot \sqrt{x}}$ is | (R) $\frac{3}{2}$ |
| (D) If $f(x) = \cos(x\cos\frac{1}{x})$ and $g(x) = \frac{\ln(\sec^2 x)}{x\sin x}$ are both continuous at $x = 0$ then $f(0) + g(0)$ equals | (S) $\frac{3}{4}$ |

# DPP No.-33 to 36

# SOLUTION

## Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

## DPP. NO.-33

**[COMPREHENSION TYPE]** **[8 × 3 = 24]**

**Paragraph for question nos. 1 to 3**

Let the function f be defined in (0, 1). Two more functions are given as $g(x) = e^x$ and $h(x) = \ln|x|$.

Q.1 The number of solutions of the equation $g(x) \cdot h(g(x)) = 1$ is

(A) 0 (B*) 1 (C) 2 (D) 3

Q.2 Domain of $f(g(x)) + f(h(x))$ is

(A*) $(-e, -1)$ (B) $(-e, -1) \cup (1, e)$ (C) $[-e, -1]$ (D) $(-\infty, 0)$

Q.3 Let f be one-one function with range $(1, 2)$ and $gof(x)$ is defined then domain of $f^{-1}og^{-1}(x)$ is

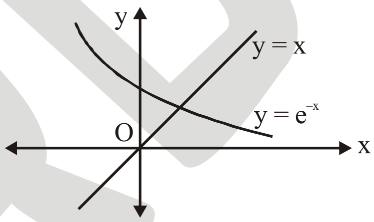
(A) (0, 1) (B*) $(e, e^2)$ (C) $(\sqrt{e}, e)$ (D) (1, e)

[Sol.30421-22-23/func

**(i)** $g(x)\ h(g(x)) = 1$

$e^x \ \ln|e^x| = 1 \quad \Rightarrow \quad xe^x = 1 \quad \Rightarrow \quad x = e^{-x}$

Clearly equation has only one solution




(ii) $f(g(x)) + f(h(x)) = f(e^x) + f(\ln|x|)$

$\because$ f is defined in (0, 1)

$\therefore \quad 0 < e^x < 1 \quad$ and $\quad 0 < \ln|x| < 1$

$x \in (-\infty, 0)$ and $\quad 1 < |x| < e$

$x \in (-e, -1) \cup (1, e)$

$\therefore$ Domian is $(-e, -1)$

(iii) $f : (0, 1) \to (1, 2)$

$gof(x) = g[f(x)] = e^{f(x)}$

Domain of $gof(x)$ is (0, 1)

and range of $gof(x) = e^{f(x)}$ is $(e, e^2)$

$\therefore$ Domain of $(gof)^{-1} = f^{-1}og^{-1}$ is $(e, e^2)$. ]

**Paragraph for question nos. 4 & 5**

Let $f : [0, \infty) \to [2, \infty)$ be a derivable increasing function which is also surjective and satisfying

$$f^2(x) - f^2(y) = 3f(x) - 3f(y) + \sqrt{x} - \sqrt{y}$$

Q.4 The value of $\lim\limits_{x \to 0^+} \dfrac{2f(x) - 3 - e^{\sqrt{x}}}{\sqrt{x}}$ equals

(A) 0 (B*) 1 (C) 2 (D) 4

Q.5 If g is the inverse function of f then $\dfrac{d}{dx}\left(\dfrac{4}{g(x)}\right)$ at $x = 3$ is equal to

(A) 6 (B) 3 (C) – 6 (D*) – 3

[Sol.$_{30440\text{-}41/mod}$ $f^2(x) - f^2(y) = 3\,f(x) - 3\,f(y) + \sqrt{x} - \sqrt{y}$

$$2\,f(x)\,f'(x) = 3\,f'(x) + \frac{1}{2\sqrt{x}}$$

$$\Rightarrow (2f(x)-3)\,f'(x) = \frac{1}{2\sqrt{x}}$$

Integrating

$$\Rightarrow \frac{1}{2}\cdot\frac{(2f(x)-3)^2}{2} = \frac{1}{2}\cdot 2\sqrt{x} + C$$

$$(2f(x)-3)^2 = 4\sqrt{x} + 4c \qquad \{f(0)=2\}$$

$x = 0,\ 4c = 1$

$$\therefore \quad 2\,f(x) - 3 = \sqrt{4\sqrt{x}+1}$$

$$f(x) = \frac{1}{2}\left(3+\sqrt{4\sqrt{x}+1}\right)$$

(i) $$\lim_{x\to0^+}\frac{2f(x)-3-e^{\sqrt{x}}}{\sqrt{x}} = \lim_{x\to0^+}\frac{\sqrt{4\sqrt{x}+1}-e^{\sqrt{x}}}{\sqrt{x}} = \lim_{x\to0^+}\frac{4\sqrt{x}+1-e^{2\sqrt{x}}}{\sqrt{x}\left(\sqrt{4\sqrt{x}+1}+e^{\sqrt{x}}\right)}$$

$$= \lim_{x\to0^+}\left(4-\frac{(e^{2\sqrt{x}}-1)}{\sqrt{x}}\right)\cdot\frac{1}{\left(\sqrt{4\sqrt{x}+1}+e^{\sqrt{x}}\right)} = (4-2)\cdot\frac{1}{2} = 1$$

(ii) $$\frac{d}{dx}\left(\frac{4}{g(x)}\right) = \frac{-4\,g'(x)}{g^2(x)}$$

$$\left.\frac{d}{dx}\left(\frac{4}{g(x)}\right)\right|_{x=3} = \frac{-4\,g'(3)}{g^2(3)}$$

For $f(x) = 3 \Rightarrow x = 4$

$\therefore\ g(3) = 4$

$$g'(3) = \frac{1}{f'(4)},\quad f'(x) = \frac{1}{2\cdot2\sqrt{4\sqrt{x}+1}}\cdot\frac{4}{2\sqrt{x}}$$

$$f'(4) = \frac{1}{2\cdot3\cdot2} = \frac{1}{12}$$

$$\therefore \quad \left.\frac{d}{dx}\left(\frac{4}{g(x)}\right)\right|_{x=3} = \frac{-4\times12}{16} = -3.$$ **Ans.** ]

**Paragraph for question nos. 6 to 8**

Let f(x) be a polynomial function satisfying the following conditions

$$\lim_{x\to\infty}\frac{f(x)}{|x|^3}=0;\ \lim_{x\to\infty}\left(\sqrt{f(x)}-x\right)=-1 \text{ and } f(0)=0$$

[**Note:** [k] and {k} denotes greatest integer function less than or equal to k and fractional part function of k respectively.]

Q.6 The value of $\lim_{x\to 0}(1+f(x))^{\text{cosec}\,x}$ is equal to

(A) e (B) $e^{-1}$ (C*) $e^{-2}$ (D) $e^{\frac{-1}{2}}$

Q.7 Number of points where $|f(|x|)|$ is non derivable, is

(A) 1 (B) 2 (C*) 3 (D) 5

Q.8 If minimum value of f(x) occur at x = a, then the value of $\lim_{x\to a^-}\frac{[x+\{x\}]}{\{x+[x]\}}$ is equal to

(A) 0 (B*) 1 (C) 2 (D) 3

[Sol.30023-24-25/lcd(d) $\because \lim_{x\to\infty}\frac{f(x)}{|x|^3}=0 \Rightarrow f(x)$ is either linear or quadratic

Lets $f(x)=ax^2+bx$ $\because f(0)=0$

$$\therefore \lim_{x\to\infty}\left(\sqrt{f(x)}-x\right)=\lim_{x\to\infty}\frac{(a-1)x^2+bx}{x\left(\sqrt{a+\frac{b}{x}}+1\right)}=-1$$

$$\Rightarrow a-1=0 \text{ and } \frac{b}{\sqrt{a}+1}=-1 \Rightarrow b=-2$$

$\therefore f(x)=x^2-2x$

and Minimum of f(x) occur at x = 1

$$\therefore \lim_{x\to 1^-}\frac{[x+\{x\}]}{\{x+[x]\}}=\frac{[1^-+1^-]}{\{1^-+0\}}=\frac{1}{1}=1.$$ ]

**Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)**

## DPP. NO.-34

**[SINGLE CORRECT CHOICE TYPE]** **[4 × 3 = 12]**

Q.1 Let $f(x) = \dfrac{(x^3 - 2x^2 + x)(\pi^x - 1) + (1 + x^2 + x^4)}{(1 - x + x^2)}$ and $g(x) = \sin^2 x$.

If $\displaystyle\sum_{r=0}^{1} \left.\frac{d^2g}{dx^2}\right|_{x=f'(r)}$ = a (cos α) (cos β), where a, α, β ∈ N, then the value of (a + α + β) is equal to

(A) 4 (B) 6 (C) 8 (D*) 10

[Sol.$_{333/mod/SC}$ $f(x) = \underbrace{\dfrac{x(\pi^x - 1)(x-1)^2}{1 - x + x^2}}_{\phi(x)} + 1 + x + x^2$

$f'(x) = \phi'(x) + 1 + 2x$ $\{\phi'(0) = \phi'(1) = 0\}$
$f'(0) = 1,\ f'(1) = 3$
$g(x) = \sin^2 x \Rightarrow g'(x) = \sin 2x \Rightarrow g''(x) = 2\cos 2x$
Now, $g''(x)|_{x = f'0)} + g''(x)|_{x = f'(1)} = g''(1) + g''(3)$
$= 2\cos 2 + 2\cos 6 = 2(2\cos 4 \cdot \cos 2)$
$= 4\cos 2\cos 4 \equiv a\cos\alpha\cos\beta$
∴ a + α + β = 4 + 2 + 4 = 10 **Ans.**]

Q.2 $\displaystyle\int \frac{4x^2 + 3x + 2}{\sqrt[3]{1 + x + \frac{1}{x}}}\,dx$ equals

(A) $x^2\left(1 + x + \dfrac{1}{x}\right)^{\frac{2}{3}} + c$

(B*) $\dfrac{3}{2} \cdot x^2\left(1 + x + \dfrac{1}{x}\right)^{\frac{2}{3}} + c$

(C) $\dfrac{3}{2}x \cdot \left(1 + x + \dfrac{1}{x}\right)^{\frac{2}{3}} + c$

(D) $x\left(1 + x + \dfrac{1}{x}\right)^{\frac{2}{3}} + c$

[Sol.$_{150/inde/SC}$ $\displaystyle\int \frac{4x^2 + 3x + 2}{\sqrt[3]{1 + x + \frac{1}{x}}}\,dx = \int \frac{4x^3 + 3x^2 + 2x}{\sqrt[3]{x^3 + x^4 + x^2}}\,dx$

Put $x^3 + x^4 + x^2 = t^3$

$\displaystyle\int 3t\,dt = \frac{3}{2}t^2 + c$ **Ans.**]

Q.3 If $\tan^{-1}x + \cos^{-1}\left(\frac{y}{\sqrt{1+y^2}}\right) = \tan^{-1}4$ where $x, y \in N$, then the number of possible values of $x$, is

(A) 0 (B*) 1 (C) 2 (D) 3

[Sol.$_{183/itf/SC}$ $\tan^{-1}x + \cot^{-1}y = \tan^{-1}4$

$$\tan^{-1}\left(\frac{1}{y}\right) = \tan^{-1}\left(\frac{4-x}{1+4x}\right)$$

$$\therefore \quad y = \frac{4x+1}{4-x} \quad \Rightarrow \quad 4y - xy = 4x + 1 \quad \Rightarrow \quad x(y+4) = 4y - 1 \quad \Rightarrow \quad x = \frac{4y-1}{y+4}$$

$$x = \frac{4(y+4-4)-1}{y+4} = 4 - \frac{17}{y+4}$$

$y + 4 = 1 \Rightarrow y = -3$ (Rejected)
$y + 4 = 17 \Rightarrow y = 13, x = 3$
$\therefore$ 1 solution **Ans.**]

Q.4 The function $f(x)$ is defined by $f(x) = \begin{cases} \left(x^2 + e^{\frac{1}{2-x}}\right)^{-1}, & x \neq 2 \\ k, & x = 2 \end{cases}$.

If it is continuous from right at the point $x = 2$ then $k$ is equal to

(A) 0 (B*) $\frac{1}{4}$ (C) $\frac{-1}{2}$ (D) 1

[Sol.$_{801/lcd/SC}$ $\because k = \lim_{x \to 2^+} \frac{1}{x^2 + e^{\frac{1}{2-x}}} = \frac{1}{4 + e^{-\infty}} = \frac{1}{4+0} = \frac{1}{4}$. ]

Q.5 Let $f(x) = ax^2 + bx + c$ where $a \neq 0$. If $f(0) = 2016$, $f(d^2) = 2017$, $f(d) = 2018$ where $d \in (0, 1)$ and sum of the roots of $f(x)$ is 0, then the value of 'd' is

(A) $\frac{1}{\sqrt{3}}$ (B*) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

[Sol.$_{470/func/SC}$ sum of the roots = 0 $\Rightarrow$ b = 0
$f(0) = c = 2016$
$f(d^2) = ad^4 + c = 2017 \Rightarrow ad^4 = 1$
$f(d) = ad^2 + c = 2018 \Rightarrow ad^2 = 2$

on dividing $d^2 = \frac{1}{2} \Rightarrow d = \frac{1}{\sqrt{2}}$. **Ans.**]

## [MATRIX TYPE]

[3+3+3+3=12]

Q.6

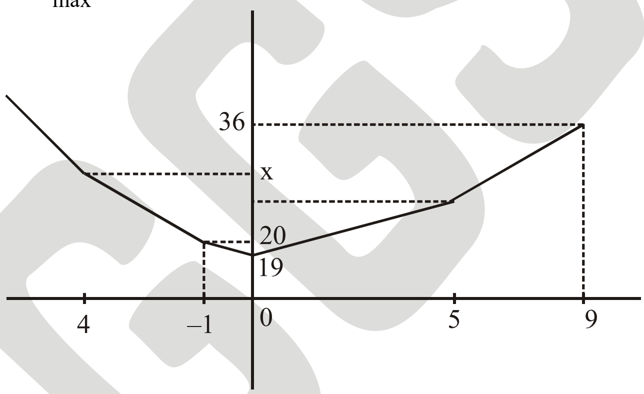
| Column-I | | Column-II | |
|---|---|---|---|
| (A) | Let $f(x) = \lvert x+4 \rvert + \lvert x+1 \rvert + \lvert x \rvert + \lvert x-5 \rvert + \lvert x-9 \rvert$, then the largest positive integral value of k for which the equation $f(x) = k$ has no solution is | (P) | 10 |
| (B) | Let $f(x) = \begin{cases} \dfrac{e^{\sin^{-1}\frac{x}{\sqrt{x^2+1}}} - e^{\sin^{-1}x}}{e^{\tan x} - e^{\sin x}}, & \text{if } x \neq 0. \\ p, & \text{if } x = 0 \end{cases}$ If $f(x)$ is continuous at $x = 0$ and the value of $\left(\cot^{-1}(p) + \sin^{-1}(p^2)\right) = \frac{k\pi}{8}$, $(k \in N)$, then $k$ equals | (Q) | 18 |
| | | (R) | 25 |
| (C) | Let $\{a_n\}$ be a geometric sequence with $a_7 = 50$ and $a_{11} = 250\sqrt[3]{5}$. If the value of $a_3$ is $2\sqrt[3]{k}$ $(k \in N)$, then the value of k, is | (S) | 40 |
| (D) | Let $k_1$ and $k_2$ are two values of k for which the equation $4x^2 - 4(5x+1) + k^2 = 0$ has one root equals to two more than the other, then the value of $\left(k_1^2 + k_2^2\right)$, is | (T) | 50 |

[Ans. (A) Q; (B) P; (C) R; (D) T]

[Sol.$_{1133/mtc}$

(A) For no solution, $k < 19$

$\therefore k_{max} = 18$




(B) $$\lim_{x\to 0} \frac{e^{\tan^{-1}x} - e^{\sin^{-1}x}}{e^{\tan x} - e^{\sin x}} = \lim_{x\to 0} \frac{\tan^{-1}x - \sin^{-1}x}{\tan x - \sin x} = \lim_{x\to 0} \left(\frac{\tan^{-1}x - \sin^{-1}x}{x^3}\right)\left(\frac{x^3}{\tan x - \sin x}\right)$$

$$= \lim_{x\to 0} \frac{\left(x - \frac{x^3}{3} - \left(x + \frac{x^3}{3!}\right)\right)}{x^3} \cdot \frac{x^3}{\left(x + \frac{x^3}{3}\right) - \left(x - \frac{x^3}{3!}\right)} = \frac{-1}{2} \times 2 = -1$$

Hence p = – 1

$\therefore \quad \left(\cot^{-1}(p)+\sin^{-1}(p^2)\right)=\frac{3\pi}{4}+\frac{\pi}{2}=\frac{5\pi}{4}=\frac{10\pi}{8} \quad \Rightarrow \quad k = 10$ **Ans.**

(C) $a_3=\frac{a_7^2}{a_{11}}=\frac{50^2}{250\sqrt[3]{5}}=\frac{10}{\sqrt[3]{5}}=2\sqrt[3]{25} \quad \Rightarrow \quad k = 25$ **Ans.**

(D) $4x^2 - 4(5x + 1) + k^2 = 0$

$4x^2 - 20x + (k^2 - 4) = 0$

two roots are $\alpha, \ \alpha + 2$

$\therefore \quad 2\alpha + 2 = \frac{20}{4} = 5 \quad \Rightarrow \quad \alpha + 1 = \frac{5}{2} \quad \Rightarrow \quad \alpha = \frac{5}{2} - 1 \Rightarrow \alpha = \frac{3}{2}$

$\therefore \quad \alpha(\alpha + 2) = \frac{k^2-4}{4}$

$\frac{3}{2}\left(\frac{3}{2}+2\right)=\frac{k^2-4}{4} \quad \Rightarrow \quad \frac{3}{2}\cdot\frac{7}{2}=\frac{k^2-4}{4} \quad \Rightarrow \quad 21 = k^2 - 4$

$\Rightarrow \quad k^2 = 25 \quad \Rightarrow \quad k = \pm 5$

$\Rightarrow \quad \left(k_1^2 + k_2^2\right) = 50$ **Ans.**]

**Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)**

**DPP. NO.-35**

Q.1 Evaluate $\lim_{n\to\infty}\left(\frac{\sqrt{n^2+n-1}}{n}\right)^{2\sqrt{n^2+n-1}}$ [Ans. $e^{-1}$] **[3]**

[Sol.$_{80002/lcd/SUB}$ $L = e^{\lim_{n\to\infty} 2\sqrt{n^2+n-1}\left(\frac{\sqrt{n^2+n-1}}{n}-1\right)} = e^{l}$, where $l = \lim_{n\to\infty}\left(2\sqrt{n^2+n-1}\right)\left(\frac{\sqrt{n^2+n}-(1+n)}{n}\right)$

$$= \lim_{n\to\infty}\frac{\not{n}\left[2\sqrt{1+\frac{1}{n}-\frac{1}{n}}\right]}{\not{n}}\cdot\lim_{n\to\infty}\left(\sqrt{n^2+n}-(n+1)\right)$$

$$= 2\cdot\lim_{n\to\infty}\left(\frac{(n^2+n)-(n+1)^2}{\sqrt{n^2+n}+(n+1)}\right)\text{(rationalisation)} = 2\cdot\lim_{n\to\infty}\frac{n^2+n-n^2-2n-1}{\sqrt{n^2+n}+n+1}$$

$$= 2\cdot\lim_{n\to\infty}\frac{-(n+1)}{n\left[\left(\sqrt{1+\frac{1}{n}}+1+\frac{1}{n}\right)\right]} = 2\cdot\lim_{n\to\infty}\frac{-n\left(1+\frac{1}{n}\right)}{n\left[\sqrt{1+\frac{1}{n}}+1+\frac{1}{n}\right]} = -2\left(\frac{1}{2}\right) = -1$$

$\therefore$ $L = e^{-1}$ **Ans.**]

Q.2 Evaluate $\lim_{x\to\frac{1}{\sqrt{2}}}\frac{\cos^{-1}\left(2x\sqrt{1-x^2}\right)}{x-\frac{1}{\sqrt{2}}}$ [Ans. does not exists] **[3]**

[Sol.$_{80007/lcd/SUB}$ does not exists

Put $x = \sin\theta$ $\theta\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

$$\sqrt{2}\lim_{\theta\to\pi/4}\frac{\cos^{-1}(2\sin\theta\cos\theta)}{(\sqrt{2}\sin\theta-1)} = \sqrt{2}\lim_{\theta\to\pi/4}\frac{\cos^{-1}(\sin 2\theta)}{\left(\sqrt{2}\sin^2\theta-1\right)}\cdot\left(\sqrt{2}\sin\theta+1\right) = -2\sqrt{2}\lim_{\theta\to\pi/4}\frac{\cos^{-1}(\sin 2\theta)}{\cos 2\theta}$$

$\theta = \frac{\pi}{4}-h \Rightarrow 2\theta = \frac{\pi}{2}-2h$

$$-2\sqrt{2}\cdot\lim_{h\to 0}\frac{\cos^{-1}(\sin 2h)}{\cos 2h} = -2\sqrt{2} = f\left(\frac{\pi}{4}^-\right) = f\left(\frac{1}{\sqrt{2}}^+\right)$$

$\theta = \frac{\pi}{4}+h \Rightarrow 2\theta = \frac{\pi}{2}+2h$

$$-2\sqrt{2}\cdot\lim_{h\to 0}\frac{\cos^{-1}(\cos 2h)}{-\sin 2h} = 2\sqrt{2} = f\left(\frac{\pi}{4}^+\right) = f\left(\frac{1}{\sqrt{2}}^-\right)$$

$\Rightarrow$ Limit does not exists. ]

## [SINGLE CORRECT CHOICE TYPE] [3 × 3 = 9]

Q.3 Range of $f(x) = \cos^{-1}\left(\sqrt{x^2+x+1}\right)$ is

(A*) $\left[0, \frac{\pi}{6}\right]$ (B) $\left[0, \frac{\pi}{3}\right]$ (C) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ (D) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

[Sol.$_{217/itf/SC}$ For domain point of view

$0 \le x^2 + x + 1 \le 1$, but $x^2 + x + 1 \ge \frac{3}{4}$ $\Rightarrow$ $\frac{\sqrt{3}}{2} \le \sqrt{x^2+x+1} \le 1$

$\Rightarrow$ $0 \le \cos^{-1}\left(\sqrt{x^2+x+1}\right) \le \frac{\pi}{6}$ **Ans.** ]

Q.4 $\int \frac{\cos x\, dx}{\sqrt{\sin^2 x + 2\cos^2 x}}$ is equal to

(A) $\sin^{-1}\left(\frac{\sin x}{2}\right) + C$ (B) $ln\left(\sin x + \sqrt{2 - \sin^2 x}\right) + C$

(C*) $\sin^{-1}\left(\frac{\sin x}{\sqrt{2}}\right) + C$ (D) $ln\left(\sin x - \sqrt{2 - \sin^2 x}\right) + C$

(Where C is constant of integration)

[Sol.$_{74/inde/SC}$ Let $I = \int \frac{\cos x\, dx}{\sqrt{1+\cos^2 x}} = \int \frac{\cos x\, dx}{\sqrt{2-\sin^2 x}}$ ;put $\sin x = t$

$$= \int \frac{dt}{\sqrt{2-t^2}} = \int \frac{dt}{\sqrt{2-t^2}} = \sin^{-1}\left(\frac{t}{\sqrt{2}}\right) + C = \sin^{-1}\left(\frac{\sin x}{\sqrt{2}}\right) + C \text{ Ans.]}$$

Q.5 In a $\Delta ABC$, sides a, b, c are in G.P. and angles A, B, C are in A.P. If area of $\Delta ABC = 3\sqrt{3}$ then $(R + r)$ is equal to

(A) – 2 (B) 0 (C) 2 (D*) 3

[Sol.$_{522/seq/SC}$ $b^2 = ac$

$\Rightarrow$ $\sin^2 B = \sin A \sin C$

$\sin^2 60° = \sin(60° - \theta) \sin(60° + \theta)$ $\Rightarrow$ $\sin\theta = 0$ $\Rightarrow$ $\theta = 0°$

triangle is equilateral triangle

$\Delta = \frac{\sqrt{3}}{4}a^2 = 3\sqrt{3}$ $\Rightarrow$ $a = 2\sqrt{3}$. ]

**[INTEGER TYPE]** **[1 × 5 = 5]**

Q.6 Let [k] denotes the greatest integer less than or equal to k.

If number of positive integral solutions of the equation $\left[\frac{x}{[\pi^2]}\right] = \left[\frac{x}{\left[11\frac{1}{2}\right]}\right]$ is n,

then find the value of $\sqrt{n-8}$. [Ans. 4]

[Sol.$_{\text{50011/func/OMR}}$ $\left[\frac{x}{9}\right] = \left[\frac{x}{11}\right]$

**Case-I :** $0 \le \frac{x}{9} < 1$ and $0 \le \frac{x}{11} < 1$

$\Rightarrow 0 \le x < 9$ and $0 \le x < 11 \Rightarrow$ common value of x is $\{1, 2, 3, ......8\}$

**Case-II :** $1 \le \frac{x}{9} < 2$ and $1 \le \frac{x}{11} < 2$

$\Rightarrow 9 \le x < 18$ and $11 \le x < 22 \Rightarrow x \in \{11, 12, ......, 17\}$

**Case-III :** $2 \le \frac{x}{9} < 3$ and $2 \le \frac{x}{11} < 3$

$\Rightarrow 18 \le x < 27$ and $22 \le x < 33 \Rightarrow x \in \{22, 23, ......, 26\}$

**Case-IV :** $3 \le \frac{x}{9} < 4$ and $3 \le \frac{x}{11} < 4$

$\Rightarrow 27 \le x < 36$ and $33 \le x < 44 \Rightarrow x \in \{33, 34, 35\}$

**Case-V :** $4 \le \frac{x}{9} < 5$ and $4 \le \frac{x}{11} < 5 \Rightarrow x = 44$

$\therefore$ total positive integer $x = 8 + 7 + 5 + 3 + 1 = 24$

$\therefore$ Answer $= \sqrt{24-8} = 8.$ ]

**Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)**

## DPP. NO.-36

### [SINGLE CORRECT CHOICE TYPE] [2 × 3 = 6]

Q.1 If $x^2 + y^2 = 1$, then
(A) $yy'' - 2(y')^2 + 1 = 0$ (B*) $yy'' + (y')^2 + 1 = 0$
(C) $yy'' - (y')^2 - 1 = 0$ (D) $yy'' + 2(y')^2 + 1 = 0$
[ JEE 2000, Screening, 1 out of 35 ]

[Sol.$_{181/mod/SC}$ **[B]**
$x^2 + y^2 = 1$
$2x + 2yy' = 0 \quad \Rightarrow \quad x + yy' = 0$
$$y' = -\frac{x}{y}$$
$1 + yy'' + (y')^2 = 0$ **Ans.**]

Q.2 Range of the function $f(x) = 4\tan^{-1} x + 3\sin^{-1} x + \sec^{-1} x$ is
(A) $\left\{\frac{-3\pi}{2}, \frac{-5\pi}{2}\right\}$ (B) $\left\{\frac{3\pi}{2}, \frac{5\pi}{2}\right\}$ (C) $\left(\frac{-3\pi}{2}, \frac{5\pi}{2}\right)$ (D*) $\left\{\frac{-3\pi}{2}, \frac{5\pi}{2}\right\}$

[Sol.$_{197/itf/SC}$ Domain of $f(x)$ is $x \in R \cap x \in [-1, 1] \cap |x| \geq 1 \Rightarrow x \in \{-1, 1\}$
$\Rightarrow$ Range of $f(x)$ contains $\{f(-1), f(1)\}$ i.e. $\left\{\frac{-3\pi}{2}, \frac{5\pi}{2}\right\}$. **Ans.**]

### [MULTIPLE CORRECT CHOICE TYPE] [3 × 4 = 12]

Q.3 Consider, $P = \frac{x^2 - 2x}{x^2 + x + 1}$, $Q = \frac{y - 1}{y^2 + y + 1}$ and $R = \frac{2}{z^2 + z + 1}$ where $x, y, z \in R$.
If $k = [P + Q + R] - ([P] + [Q] + [R])$ then the possible value(s) of k is(are)
(A*) 0 (B*) 1 (C*) 2 (D) 3
**[Note :**[λ] denotes the greatest integer less than or equal to λ.]

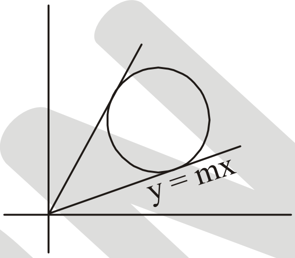
[Sol.$_{40009/func/MORE}$ $P = [P] + f_1$, $Q = [Q] + f_2$, $R = [R] + f_3$
$\therefore \ k = [f_1 + f_2 + f_3] \quad 0 \leq f_1 + f_2 + f_3 < 3$
$\Rightarrow$ possible value(s) of k are 0, 1 & 2. ]

Q.4 Given that $(x - 2)^2 + (y - 2)^2 = 1$ then maximum value of
(A*) $x + y$ is $4 + \sqrt{2}$ (B*) $x - y$ is $\sqrt{2}$ (C*) $\frac{x}{y}$ is $\frac{4+\sqrt{7}}{3}$ (D*) $\frac{y}{x}$ is $\frac{4+\sqrt{7}}{3}$

[Sol.$_{40573/cir/MORE}$ Let $x = 2 + \cos\theta$ and $y = 2 + \sin\theta$
$\therefore \quad x + y = 4 + \cos\theta + \sin\theta$
$\therefore \quad$ maximum of $x + y = 4 + \sqrt{2}$
and $\quad x - y = \cos\theta - \sin\theta$
$\therefore \quad$ maximum of $x - y = \sqrt{2}$
Let $y = mx$ be tangent to circle

$\therefore \quad \dfrac{|2m-2|}{\sqrt{1+m^2}} = 1 \;\Rightarrow\; 4m^2 - 8m + 4 = m^2 + 1 \Rightarrow 3m^2 - 8m + 3 = 0$

$\Rightarrow \quad m = \dfrac{8 \pm \sqrt{64-36}}{6} = \dfrac{4 \pm \sqrt{7}}{3}$




$\therefore \quad$ maximum of $\dfrac{y}{x}$ is $\dfrac{4+\sqrt{7}}{3}$ and $\dfrac{x}{y} = \dfrac{1}{\frac{y}{x}}$

$\therefore \quad$ maximum of $\dfrac{x}{y} = \dfrac{1}{\text{min .of } \frac{y}{x}} = \dfrac{3}{4-\sqrt{7}} = \dfrac{3\left(4+\sqrt{7}\right)}{9} = \dfrac{4+\sqrt{7}}{3}$. ]

Q.5 The value(s) of p for which the inequality $\log_{(p^2+1)}(3x^2 - 2x - p + 6) \geq \cos^2\theta\,(1 - 4\sin^2\theta)^2 - \cos^2 3\theta$ is satisfied for all real values of x and $\theta$ is(are)

(A) 0 (B*) 1 (C*) 4 (D) 5

[Sol.$_{\text{40072/qe/MORE}}$ $\log_{(p^2+1)}(3x^2 - 2x + 6 - p) \geq 0$

$\Rightarrow \quad 3x^2 - 2x + 6 - p \geq 1$

$\Rightarrow \quad 3x^2 - 2x + 5 - p \geq 0 \quad \forall \; x \in R$

$\Rightarrow \quad D \leq 0$

$\Rightarrow \quad 4 - 4 \cdot 3 \cdot (5 - p) \leq 0$

$\Rightarrow \quad 1 - 15 + 3p \leq 0$

$\Rightarrow \quad p \leq \dfrac{14}{3}$

$\Rightarrow \quad p \in \left(-\infty, \dfrac{14}{3}\right] - \{0\}$ ] **[11th, 03-09-2017, P-2]**

## [MATRIX TYPE]

**[2+2+2+2=8]**

Q.6

| Column–I | Column-II |
|---|---|
| (A) $\displaystyle\lim_{x\to 1} \frac{x^3-1}{\ln x}$ is | (P) 2 |
| (B) $\displaystyle\lim_{x\to 0} \frac{x(\cos x - \cos 2x)}{2\sin x - \sin 2x}$ is | (Q) 3 |
| (C) $\displaystyle\lim_{x\to 0} \frac{\tan x\sqrt{\tan x} - \sin x\sqrt{\sin x}}{x^3 \cdot \sqrt{x}}$ is | (R) $\dfrac{3}{2}$ |
| (D) If $f(x) = \cos(x \cos \frac{1}{x})$ and $g(x) = \dfrac{\ln(\sec^2 x)}{x \sin x}$ are | (S) $\dfrac{3}{4}$ |

both continuous at $x = 0$ then $f(0) + g(0)$ equals

[Ans. (A) Q; (B) R; (C) S ; (D) P]

[Sol.$_{92020/lcd/MTC}$

**(A)** $x = 1 + h$, $\qquad \lim\limits_{h\to 0} \dfrac{(1+h)^3 - 1}{h\cdot\dfrac{ln(1+h)}{h}} = \lim\limits_{h\to 0} \dfrac{(1+h)^3-1}{h} = 3$ **Ans.**

**(B)** $\lim\limits_{x\to 0} \dfrac{x2\sin\left(\dfrac{3x}{2}\right)\sin\dfrac{x}{2}}{2\sin x(1-\cos x)} = \lim\limits_{x\to 0} \dfrac{\sin\dfrac{3x}{2}\ \sin\dfrac{x}{2}\cdot x^2}{(1-\cos x)x^2} = \lim\limits_{x\to 0} \dfrac{2\cdot 3\cdot\sin\dfrac{3x}{2}\ \sin\dfrac{x}{2}}{4\cdot\dfrac{3x}{2}\cdot\dfrac{x}{2}} = \dfrac{3}{2}$ **Ans.**

**(C)** $\lim\limits_{x\to 0} \dfrac{(\tan x)^{3/2}[1-(\cos x)^{3/2}]}{x^{3/2}\cdot x^2} = 1\cdot\lim\limits_{x\to 0} \dfrac{1-\cos^3 x}{x^2}\cdot\dfrac{1}{1+(\cos x)^{3/2}}$

$= \dfrac{1}{2}\cdot\dfrac{1}{2}\left(1+\cos x+\cos^2 x\right) = \dfrac{3}{4}$ **Ans.** $\Rightarrow$ **(P)**

**(D)** $f(0) + g(0) = 1 + 1 = 2$ **Ans.** $\Rightarrow$ **(Q)** ]