GGSRDN
Educational Services Private Limited
9th, 10th, NEET, JEE(Main/Advanced)
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CLASS : XII (MATHS)

DPP

DAILY PRACTICE PROBLEM

DPP-31 to 40

MATHEMATICS DPP
DAILY PRACTICE PROBLEMS

DPP No. 31

Total Marks : 27

Max. Time : 27 min.

Topics : Sequence & Series, Application of Derivatives

Type of Questions		M.M., Min.
Comprehension (no negative marking) Q.1 to Q.3	(3 marks, 3 min.)	[9, 9]
Single choice Objective (no negative marking) Q.4,5,6	(3 marks, 3 min.)	[9, 9]
Multiple choice objective (no negative marking) Q.7	(5 marks, 4 min.)	[5, 4]
Subjective Questions (no negative marking) Q.8	(4 marks, 5 min.)	[4, 5]

COMPREHENSION (Q. NO. 1 TO 3)

If $S = -1 -1 + 1 + 7 + 19 + 39 + 69 + \dots\dots$, then

1. n^{th} term (t_n) will be

(A) $\frac{-6+(n-1)(n-2)^2}{6}$ (B) $\frac{-3+(n-1)(n-2)^2}{6}$

(C) $\frac{n^3-3n^2+2n-3}{3}$ (D) None of these

2. t_{10} is equal to

(A) 299 (B) 239 (C) 171 (D) 211

3. Sum of first 10 term (S_{10}) is equal to -

(A) 650 (B) 659 (C) 560 (D) 625

4. The gradient of the common tangent to the two curves $y = x^2 - 5x + 6$ and $y = x^2 + x + 1$ is :

(A) – 1/3 (B) – 2/3 (C) – 1 (D) – 3

5. A curve with equation of the form $y = ax^4 + bx^3 + cx + d$ has zero gradient at the point (0, 1) and also touches the x – axis at the point (– 1, 0) then the values of x for which the curve has a negative gradient are :

(A) $x > -1$ (B) $x < 1$ (C) $x < -1$ (D) $-1 \le x \le 1$

6. The equation of the tangent to the curve $y = e^{-|x|}$ at the point where the curve cuts the line x = 1 is

(A) $x + y = e$ (B) $e(x + y) = 1$ (C) $y + ex = 1$ (D) None of these

7. If a line is tangent to one point and normal at another point on the curve $x = 4t^2 + 3$, $y = 8t^3 - 1$, then slope of such a line is

(A) – 1 (B) 1 (C) $-\sqrt{2}$ (D) $\sqrt{2}$

8. Show that the curves $x^3 - 3xy^2 = a$ and $3x^2y - y^3 = b$ cut each other orthogonally where a and b are constants.

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 32

Total Marks : 29

Max. Time : 31 min.

Topics : **Method of Differentiation, Complex Number, Continuity & Derivability, Application of Derivatives, Sequence & Series, Function**

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3,4	(3 marks, 3 min.)	[12, 12]
Multiple choice objective (no negative marking) Q.5	(5 marks, 4 min.)	[5, 4]
Subjective Questions (no negative marking) Q.6,7,8	(4 marks, 5 min.)	[12, 15]

1. Let $y = \tan^{-1}\left(\dfrac{2\cos(3x^2-2)+5\sin(3x^2-2)}{5\cos(3x^2-2)-2\sin(3x^2-2)}\right)$, then $\dfrac{dy}{dx}$ =

(A) $6x - 2$ (B) $6x$ (C) $5x$ (D) $\dfrac{6x}{x^2+1}$

2. If $y = at^2 + 2bt + c$ and $t = ax^2 + 2bx + c$, then $\dfrac{d^3y}{dx^3}$ equals

(A) $24a^2(at + b)$ (B) $24a(ax + b)^2$ (C) $24a(at + b)^2$ (D) $24a^2(ax + b)$

3. The complex number $z = x + iy$ for which $\log_{1/2}|z-2| > \log_{1/2}|z|$, are given by:

(A) $Re(z) \le 1$ (B) $Im(z) \le 1$ (C) $Re(z) > 1$ (D) $Im(z) > 1$

4. If $g(x) = \dfrac{2h(x)+|h(x)|}{2h(x)-|h(x)|}$ where $h(x) = \sin x - \sin^n x$, $n \in R^+$, the set of positive real numbers, and

$$f(x) = \begin{cases} [g(x)], & x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right) \\ 3, & x = \frac{\pi}{2} \end{cases}$$

where [.] denotes greatest integer function. Then

(A) f(x) is continuous and differentiable at $x = \frac{\pi}{2}$, when $0 < n < 1$

(B) f(x) is continuous and differentiable at $x = \frac{\pi}{2}$, when $n > 1$

(C) f(x) is continuous but not differentiable at $x = \frac{\pi}{2}$, when $0 < n < 1$

(D) f(x) is continuous but not differentiable at $x = \frac{\pi}{2}$, when $n > 1$

5. For the series $S = 1 + \dfrac{1}{(1+3)}(1+2)^2 + \dfrac{1}{(1+3+5)}(1+2+3)^2 + \dfrac{1}{(1+3+5+7)}(1+2+3+4)^2 + \ldots\ldots$

(A) 7^{th} term is 16 (B) 7^{th} term is 18

(C) sum of first 10^{th} terms is $\dfrac{505}{4}$ (D) sum of first 10^{th} term is $\dfrac{405}{4}$

6. Let $f(x) = \dfrac{1}{1-x}$, $g(x) = fofofofofofof(x)$ and $h(x) = \tan^{-1}(g(-x^2-x))$, then find $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n} h(r)$.

7. Prove that in the curve $y = a\,\ell n\,(x^2 - a^2)$, sum of the tangent and subtangent varies as the product of the coordinates of the point of contact.

8. If the equation $a_0x^n + a_1x^{n-1} + \text{-------} + a_{n-1}x = 0$, (where n is natural number) has a positive root α, prove that the equation $na_0x^{n-1} + (n-1)a_1x^{n-2} + \text{-------} + a_{n-1} = 0$ also has a positive root smaller than α.

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 33

Total Marks : 27

Max. Time : 30 min.

Topics : **Method of Differentiation, Complex Number, Continuity & Derivability, Application of Derivatives, Sequence & Series, Straight Line**

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.1,2,3,4,5	**(3 marks, 3 min.)**	**[15, 15]**
Subjective Questions (no negative marking) Q.6,7,8	**(4 marks, 5 min.)**	**[12, 15]**

1. Sum to infinite terms of the series $\frac{1}{1.3} + \frac{2}{1.3.5} + \frac{3}{1.3.5.7} + \frac{4}{1.3.5.7.9} + \ldots\ldots\ldots$ is

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) none of these

2. Consider the funciton $f(x) = x - |x - x^2|$, $-1 \le x \le 2$. Then point of discontinuities of f(x) for $x \in [-1, 2]$ are

(A) x = 0, 1 (B) x = 1, 2 (C) x = 0, $\frac{1}{2}$,1 (D) None of these

3. Given that f is a real valued differentiable function such that $f(x)\, f'(x) < 0$ for all real x, it follows that

(A) f(x) is an increasing function (B) f(x) is a decreasing function

(C) |f(x)| is an increasing function (D) |f(x)| is a decreasing function

4. If $f'(1) = -2\sqrt{2}$ and $g'(\sqrt{2}) = 4$, then the derivative of f(tan x) with respect to g(sec x) at $x = \frac{\pi}{4}$, is

(A) 1 (B) – 1 (C) 2 (D) 4

5. If $y = (\sqrt{x})^{x^{x^{\ldots\ldots\text{to}\,\infty}}}$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{y^3}{2x(1-y^2\ell nx)}$ (B) $\frac{y^2}{2x(1+y^2\ell nx)}$ (C) $\frac{y^2}{2x(1-y^2\ell nx)}$ (D) $\frac{y^3}{2x(1+y^2\ell nx)}$

6. If $y = \left(\frac{ax+b}{x^2+c}\right)$, then show that $(2xy' + y)\, y''' = 3\,(xy'' + y')y''$, where a, b, c are constants

7. If the lines $L_1 : 2x - 3y - 6 = 0$, $L_2 : x + y - 4 = 0$ and $L_3 : x + 2 = 0$ taken pair wise in order constitute the angles A, B and C respectively of ΔABC, then find the equation whose roots are tan A, tan B and tan C.

8. Sketch the region given by $|z| \le 4$ & Arg $(z - i - 1) > \pi/4$

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 34

Total Marks : 27

Max. Time : 27 min.

Topics : **Method of Differentiation, Continuity & Derivability, Application of Derivatives, Sequence & Series, Straight Line**

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3,4,5,6	(3 marks, 3 min.)	[18, 18]
Multiple choice objective (no negative marking) Q.7	(5 marks, 4 min.)	[5, 4]
Subjective Questions (no negative marking) Q.8	(4 marks, 5 min.)	[4, 5]

1. If $f(x) = \max \{\sin x, \sin^{-1} (\cos x)\}$, then
(A) f is differentiable everywhere
(B) f is continuous but not differentiable everywhere
(C) f is discontinuous at $x = \frac{n\pi}{2}$, $n \in I$
(D) none of these

2. The radius of a right circular cyliner increases at a constant rate. Its altitude is a linear function of the radius and increases three times as fast as radius. When the radius is 1 cm the altitude is 6 cm. When the radius is 6cm, the volume is increasing at the rate of 1 Cu cm/sec. When the radius is 36cm, the volume is increasing at a rate of n cu. cm/sec. The value of 'n' is equal to :
(A) 12
(B) 22
(C) 30
(D) 33

3. If $y = (A + Bx) e^{mx} + (m - 1)^{-2} . e^x$, then $\frac{d^2y}{dx^2} - 2m \frac{dy}{dx} + m^2y$ is equal to
(A) e^{mx}
(B) e^{-mx}
(C) $e^{(1-m)x}$
(D) e^x

4. If $\tan y = \frac{2^x}{1+2^{2x+1}}$, then $\frac{dy}{dx}$ at x = 0 is
(A) $-\frac{3}{10}$
(B) $-\frac{3}{10}\ell n\, 2$
(C) $-\frac{1}{10}$
(D) $-\frac{1}{10}\ell n\, 2$

5. If y = sin x, then $\frac{d^2(\cos^7 x)}{dy^2}$ is equal to
(A) $35 \cos^3x - 42 \cos^5x$
(B) $35 \cos^3x + 42 \cos^5x$
(C) $42 \cos^3x - 35 \cos^5 x$
(D) $-35 \cos^3x - 42 \cos^5x$

6. If 2a + 3b + c = 3 ; a > 0, b > 0, c > 0, then the greatest value of $a^2 b^5 c^2$
(A) $\frac{5^5\ 2^2}{3^{23}}$
(B) $\frac{5^5 . 2^2}{3^{14}}$
(C) $\frac{4.5^5}{9^9}$
(D) $\frac{5^6 . 2^2}{3^4.9^{10}}$

7. The function $f(x) = (\tan^{-1}x)^3 - (\cot^{-1}x)^2 + \tan^{-1}x + 2$ is
(A) decreasing $\forall$ x $\in$ R.
(B) Increasing $\forall$ x $\in$ R.
(C) Bounded
(D) Many one function.

8. The tangent to $y = ax^2 + bx + \frac{7}{2}$ at (1, 2) is parallel to the normal at the point (–2, 2) on the curve $y = x^2 + 6x + 10$. Find the value of a and b.

MATHEMATICS DPP

DAILY PRACTICE PROBLEMS

DPP No. 35

Total Marks : 28

Max. Time : 32 min.

Topics : **Complex Number, Continuity & Derivability, Application of Derivatives, Sequence & Series**

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3,4	(3 marks, 3 min.)	[12, 12]
Subjective Questions (no negative marking) Q,5,6,7,8	(4 marks, 5 min.)	[16, 20]

1. The angle at which the curve $y = 2\,e^{2x}$ intersects the y-axis is

(A) $\tan^{-1}4$ (B) $\cot^{-1}4$ (C) $\tan^{-1}2$ (D) $\cot^{-1}2$

2. The subnormal at any point on the curve $xy^n = a^{n+1}$ is constant for:

(A) n = 0 (B) n = 1 (C) n = – 2 (D) no value of n

3. Let the sequence $a_1, a_2, a_3, ... a_{2n-1}, a_{2n}$ form an A.P. Then the value of,

$$a_1^2 - a_2^2 + a_3^2 - .... + a_{2n-1}^2 - a_{2n}^2 \text{ is :}$$

(A) $\dfrac{2n}{n-1}\left(a_{2n}^2 - a_1^2\right)$ (B) $\dfrac{n}{2n-1}\left(a_1^2 - a_{2n}^2\right)$

(C) $\dfrac{n}{n+1}\left(a_1^2 + a_{2n}^2\right)$ (D) $\dfrac{n}{n-1}\left(a_1^2 + a_{2n}^2\right)$

4. Let f(x) = max. $\left\{|x^2 - 2|x||, |x|\right\}$ and g(x) = min. $\left\{|x^2 - 2|x||, |x|\right\}$, then

(A) both f(x) and g(x) are non differentiable at 5 points.

(B) f(x) is not differentiable at 5 points and g(x) is non differentiable at 7 points.

(C) number of points of non differentiability for f(x) and g(x) are 7 and 5 respectively.

(D) both f(x) and g(x) are non differentiable at 3 and 5 points respectively.

5. If $f(x) = \dfrac{2}{\sqrt{3}}\tan^{-1}\left(\dfrac{2x+1}{\sqrt{3}}\right) - \ell n\,(x^2 + x + 1) + (k^2 - 5k + 3)\,x + 10$ is a decreasing function for all $x \in R$, find the permissible values of k.

6. Using monotonicity find range of the function $f(x) = \sqrt{x-1} + \sqrt{6-x}$.

7. The centre of a square is at the point with complex number $z_0 = 1 + i$ and one of its vertices is at the points $z_1 = 1 - i$. The complex numbers which correspond to the other vertices are ______, ______ & ______.

8. Find the length of arc given by $\text{Arg}\left(\dfrac{z-1}{z+2i}\right) = \pi/3$

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 36

Total Marks : 56

Max. Time : 59 min.

Topic : Complex Number

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q. 1,2,3,4,5,6	(3 marks, 3 min.)	[18, 18]
Multiple choice objective (no negative marking) Q.7, 8	(5 marks, 4 min.)	[10, 8]
Subjective Questions (no negative marking) Q. 9,10,11,12,14	(4 marks, 5 min.)	[20, 25]
Match the Following (no negative marking) Q.13	(8 marks, 8 min.)	[8, 8]

1. The number of complex numbers z such that $|z-1| = |z+1| = |z-i|$ equals

(A) 1 (B) 2 (C) ∞ (D) 0

2. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$

(A) -1 (B) 1 (C) 2 (D) -2

3. If ω be an imaginary cube root of unity, then the number :
$(1-\omega-\omega^2)^3 + (\omega-1-\omega^2)^3 + (\omega^2-\omega-1)^3$ is:

(A) divisible by 3 but not by 8 (B) divisible by 8 but not by 3
(C) divisible by both 3 & 8 (D) none of these

4. If the imaginary part of the expression $\frac{z-1}{e^{i\theta}} + \frac{e^{i\theta}}{z-1}$ be zero, then the locus of z is

(A) a straight line parallel to x-axis (B) a parabola
(C) a circle of radius 1 (D) a straight line passing through (1, 0)

5. The reflection of the complex number $(2-i)$ in the straight line $iz = \bar{z}$ is

(A) $4-3i$ (B) $3+4i$ (C) $2+i$ (D) $1-2i$

6. If z_1, z_2, z_3, z_4 are imaginary 5th roots of unity, then the value of $\sum_{r=1}^{16}(z_1^r + z_2^r + z_3^r + z_4^r)$, is

(A) 0 (B) -1 (C) 20 (D) 19

7. If z_1 and z_2 are two complex numbers satisfying the equation

$\left|\frac{z_1+z_2}{z_1-z_2}\right| = 1$ then z_1/z_2 is a number which is

(A) positive real (B) negative real (C) imaginary (D) purely imaginary

8. The complex number z satisfying $|z+\bar{z}| + |z-\bar{z}| = 2$ and $|iz-1| + |z-i| = 2$ is/are

(A) i (B) $-i$ (C) $\frac{1}{i}$ (D) $\frac{1}{i^3}$

9. Compute the product, $\left[1+\left(\frac{1+i}{2}\right)\right]\left[1+\left(\frac{1+i}{2}\right)^{2}\right]\left[1+\left(\frac{1+i}{2}\right)^{2^2}\right]......\left[1+\left(\frac{1+i}{2}\right)^{2^n}\right]$ where $n \geq 2$

10. Let A and B be two complex numbers such that $\frac{A}{B}+\frac{B}{A}$ = 1, then prove that the origin and the two points represented by A and B form vertices of an equilateral triangle.

11. Find the equation of line joining the points (1 + i) and 2 – i in complex plane.

12. Let z_1 = 10 + 6i and z_2 = 4+ 2i be two complex nubmers and z be a complex number such that $\arg\left(\frac{z-z_1}{z-z_2}\right) = \frac{\pi}{4}$. Find the centre and radius of the locus of complex number z.

13. **Match the column :**

Column- I		Column-II	
(A)	If ω_1, ω_2 be imaginary cube roots of unity, then $\omega_1^4 + \omega_2^4$ is equal to	(p)	$-\frac{1}{\omega_1\omega_2}$
(B)	If $\omega \neq 1$ be nth roots of unity, then $\omega + \omega^2 + \omega^3 + ... + \omega^{n-1}$ is equal to	(q)	–1
(C)	If z_1 and z_2 be two nth roots of unity, then $\arg\left(\frac{z_1}{z_2}\right)$ is a multiple of	(r)	$\frac{2\pi}{n}$
(D)	If $\omega \neq 1$ be nth roots of unity, then value of $(1-\omega)(1-\omega^2)........(1-\omega^{n-1})$ is equal to	(s)	n

14. Draw the locus of z :

(i) $\arg(z-1+i) \leq -\frac{\pi}{3}$

(ii) $|z+1-i| = |z-2|$

(iii) $|z| \leq 1$ and $-\frac{\pi}{4} \leq \arg(z) \leq \frac{\pi}{4}$

(iv) $\arg\left(\frac{z+i}{z-i}\right) = \frac{2\pi}{3}$

MATHEMATICS DPP

DAILY PRACTICE PROBLEMS

DPP No. 37

Total Marks : 27

Max. Time : 30 min.

Topics : **Sequence & Series, Application of Derivatives, Limits, Continuity & Derivability**

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q. 1,2,3,4,5	**(3 marks, 3 min.)**	**[15, 15]**
Subjective Questions (no negative marking) Q. 6,7,8	**(4 marks, 5 min.)**	**[12, 15]**

1. If a, b,c,d, e are five positive numbers, then

(A) $\left(\frac{a}{b}+\frac{b}{c}\right)\left(\frac{c}{d}+\frac{d}{e}\right) \ge 4\sqrt{\frac{a}{e}}$ (B) $\frac{b}{a}+\frac{c}{b}+\frac{d}{c}+\frac{e}{d}+\frac{a}{e} \ge \frac{1}{5}$

(C) $\frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{d}{e}+\frac{e}{a} < 5$ (D) None of these

2. Set of all possible values of a such that $f(x) = e^{2x} - (a + 1) e^x + 2x$ is monotonically increasing for all $x \in R$, is

(A) (3, 4) (B) $(-\infty, 0)$ (C) $(-\infty, 3]$ (D) $(3, \infty)$

3. If at each point of the curve $y = x^3 - ax^2 + x + 1$, tangent is inclined at an acute angle with the positive direction of the x-axis then

(A) $a > 0$ (B) $a \le \sqrt{3}$ (C) $-\sqrt{3} < a < \sqrt{3}$ (D) none of these

4. If f(x) is differentiable for all $x \in R$ so that $f(2) = 4$ and $f'(x) \ge 5$ for all $x \in [2, 6]$, then f(6)

(A) ≥ 24 (B) ≤ 24 (C) ≥ 9 (D) none of these

5. Let $U_n = \frac{n!}{(n+2)!}$ where $n \in N$. If $S_n = \sum_{n=1}^{n} U_n$, then $\lim_{n\to\infty} S_n$ equals

(A) 2 (B)1 (C) $\frac{1}{2}$ (D) non existent

6. If the equation $x^2 e^x = k$ possess three real roots then the range of values of k is _______

7. Find value of a, b, c such that curves $y = x^2 + ax + b$ and $y = cx - x^2$ will touch each other at the point (1, 0).

8. If f(x) and g(x) are continuous functions in [a, b] and they are differentiable in (a, b) then prove that

$$\begin{vmatrix} f(a) & f(b) \\ g(a) & g(b) \end{vmatrix} = (b - a)\begin{vmatrix} f(a) & f'(c) \\ g(a) & g'(c) \end{vmatrix}$$ where a < c < b.

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 38

Total Marks : 32

Max. Time : 37 min.

Topics : Application of Derivatives, Limits

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q. 1	(3 marks, 3 min.)	[3, 3]
Multiple choice objective (no negative marking) Q.2	(5 marks, 4 min.)	[5, 4]
Subjective Questions (no negative marking) Q. 3,4,5,6,7,8	(4 marks, 5 min.)	[24, 30]

1. At (0, 0), the curve $y^2 = x^3 + x^2$

(A) touches X-axis
(B) bisects the angle between the axes
(C) makes an angle of 60° with OX
(D) none of these

2. Let $f(\theta) = \dfrac{1+\sin\theta}{5+4\cos\theta}$, then

(A) $\dfrac{1}{5} \le f(\theta) \le 1$
(B) $0 \le f(\theta) \le 3$
(C) in $(0, \pi/2)$, $f(\theta)$ is increasing
(D) none of these

3. Find the number of critical points of the following functions.

(i) $f(x) = -\dfrac{3}{4}x^4 - 8x^3 - \dfrac{45}{2}x^2 + 105$; $x \in R$

(ii) $f(x) = |x - 2| + |x + 1|$; $x \in R$

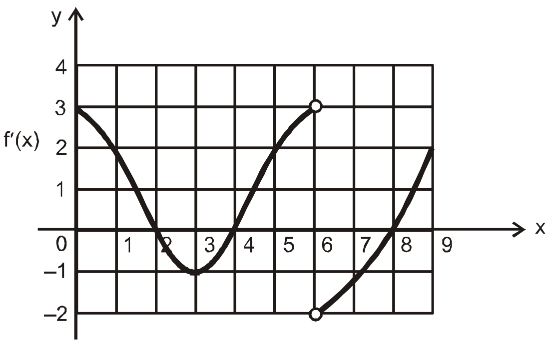
(iii) $f(x) = \min(\tan x, \cot x)$; $x \in (0, \pi)$

4. Discuss monotonocity of the function Q(x), where $Q(x) = 2f\left(\dfrac{x^2}{2}\right) + f(6 - x^2)$, $\forall x \in R$ & $f'' > 0$.

5. The number of distinct tangents to the curve $y^2 - 2x^3 - 4y + 8 = 0$ which pass through the point (1, 2) is

6. If $\lim\limits_{x\to 3}\left(\dfrac{\sqrt{2x+3}-x}{\sqrt{x+1}-x+1}\right)^{\frac{x-1-\sqrt{x^2-5}}{x^2-5x+6}}$ can be expressed in the form $\dfrac{a\sqrt{b}}{c}$ where a, b, c, $\in$ N, then find the least value of $(a^2 + b^2 + c^2)$.

7. The graph of the derivative f' of a continuous function f is shown with f(0) = 0, then for f(x) find



(i) Intervals of monotonicity

(ii) Points of local minima-maxima .

(iii) Intervals of concavity

(iv) Points of inflection

(v) Critical points

8. P(x) is a polynomial function with real coefficients. Let a, b $\in$ R with a < b, are two consecutive roots of the equation P(x) = 0, then show that there exists atleast one 'c' such that $a \le c \le b$ and $P'(c) + 100\,P(c) = 0$.

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MATHEMATICS DPP

DAILY PRACTICE PROBLEMS

DPP No. 39

Total Marks : 25

Max. Time : 26 min.

Topics : Application of Derivatives, Solution of Triangle

Type of Questions			M.M., Min.
Comprehension (no negative marking) Q.1 to Q.3	(3 marks, 3 min.)	[9,	9]
Single choice Objective (no negative marking) Q. 4,5,6,7	(3 marks, 3 min.)	[12,	12]
Subjective Questions (no negative marking) Q.8	(4 marks, 5 min.)	[4,	5]

COMPREHENSION (Q. NO. 1 TO 3)

Let f(x) be a function such that it is thrice differentiable in (a, b). Consider a function

$\phi(x) = f(b) - f(x) - (b-x) f'(x) - \frac{(b-x)^2}{2} f''(x) - (b-x)^3 \lambda$. and $\phi(x)$ follows all conditions of Rolle's theorem on [a, b]

1. If there exist some number $c \in (a, b)$ such that $\phi'(c) = 0$ and $f(b) = f(a) + (b-a) f'(a) + \frac{(b-a)^2}{2} f''(a) + \mu (b-a)^3 f'''(c)$, then μ is

(A) $\frac{1}{2}$ (B) $\frac{1}{6}$ (C) $\frac{1}{8}$ (D) $-\frac{1}{2}$

2. Let $f(x) = x^4 - 6x^3 + 12x^2 - 8x + 3$. If Rolle's theorem is applicable to $\phi(x)$ on [2, 2 + h] and there exist $c \in (2, 2+h)$ such that $\phi'(c) = 0$ and $\frac{f(2+h) - f(2)}{h^3} = g(c)$, then slope of tangent of curve y = g(x) at x = 5 is

(A) 4 (B) 5 (C) 6 (D) 10

3. Let $f(x) = e^{2x}$ and b = a + h. If there exists a real number $\theta \in (0, 1)$ such that $\phi(a + \theta h) = 0$ and $\frac{e^{2h} - 1 - 2h - 2h^2}{h^3} = Ae^{B\theta h}$, then the value of $\frac{2B}{A}$ is equal to

(A) 4 (B) 3 (C) 6 (D) 8

4. The curve $y = x^3 + x^2 - x$ has two horizontal tangents. The distance between these two horizontal lines, is

(A) $\frac{13}{9}$ (B) $\frac{11}{9}$ (C) $\frac{22}{27}$ (D) $\frac{32}{27}$

5. If a, b > 0, then minimum value of $y = \frac{b^2}{a-x} + \frac{a^2}{x}$ in (0, a) is

(A) $\frac{a+b}{a}$ (B) $\frac{ab}{a+b}$ (C) $\frac{1}{a}+\frac{1}{b}$ (D) none of these

6. Find maximum possible area that can be enclosed by a wire of length 20 cm by bending it in form of a circular sector.

(A) 10 (B) 25 (C) 30 (D) 20

7. If the sides a, b, c of a triangle ABC are the roots of the equation $x^3 - 13x^2 + 54x - 72 = 0$, then the value of

$\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$ is equal to (with usual notation in ΔABC)

(A) $\frac{169}{144}$ (B) $\frac{61}{72}$ (C) $\frac{61}{144}$ (D) $\frac{169}{72}$

8. If $x = e^t \sin t$, $y = e^t \cos t$, show that $\frac{d^2y}{dx^2} = \frac{-2(x^2+y^2)}{(x+y)^3}$

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 40

Total Marks : 30

Max. Time : 30 min.

Topics : Set, Relation & Binary Operation

Type of Questions **M.M., Min.**

Single choice Objective (no negative marking) Q. 1,2,3,4,5,6,7,8,9,10 (3 marks, 3 min.) [30, 30]

1. The number of proper subsets of the set {1,2,3} is -
(A) 8 (B) 7 (C) 6 (D) 5

2. If N_a = {an ; n∈N}, then the set $N_5 \cap N_7$ =
(A) N_7 (B) N_5 (C) N_{35} (D) N_{12}

3. A class has 175 students. The following data shows the number of students offering one or more subjects : Mathematics 100, Physics 70, Chemistry 40, Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23, Mathematics, Physics and Chemistry 18. How many student have offered Mathematics alone ?
(A) 35 (B) 48 (C) 60 (D) 22

4. Let A = {1, 2, 3} and B = {2, 3, 4}, then which of the following relation is a function from A to B.
(A) {(1, 2), (2, 3), (3, 4), (2, 2)} (B) {(1, 2), (2, 3), (1, 3)}
(C) {(1, 3), (2, 3), (3, 3)} (D) {(1, 1), (2, 3), (3, 4)}

5. Let R be a relation on the set of integers given by aRb ⇒ $a = 2^K.b$ for some integer k. then R is
(A) An equivalence relation (B) Reflexive but not symmetric
(C) Reflexive and transitive (D) Reflexive and symmetric but not transitive

6. If A is the set of even natural numbers less than 8 and B is the set of prime numbers less than 7, then the number of relations from A to B is
(A) 2^9 (B) 9^2 (C) 3^2 (D) $2^9 - 1$

7. Let S be the set of all real numbers. Then the relation R = {(a, b) : 1 + ab > 0} on S is
(A) An equivalence relations (B) Reflexive but not symmetric
(C) Reflexive and transitive (D) Reflexive and symmetric but not transitive

8. Which of the following binary operations is commutative :
(A) * on R, given by $a * b = ab^2$
(B) * on R, given by $a * b = a^b$
(C) * on P(S), the power set of a set S given by A * B = A Δ B
(D) None of these

9. A binary operation * is defined on the set of real number by a * b = 1 + ab. then the operation * is
(A) Commutative but not associative (B) Associative but not commutative
(C) Both commutative and associative (D) Neither commutative nor associative

10. Let z be the set of integers and * be a binary operation on z defined by a * b = a + b – ab for all a, b ∈ z. The inverse of an element a (≠ 1) ∈ z is
(A) $\frac{a}{a-1}$ (B) $\frac{a}{1-a}$ (C) $\frac{1-a}{a}$ (D) None of these

DPP 31 TO 40 (ANSWER KEY)

DPP NO. - 31

1. (C) **2.** (B) **3.** (A) **4.** (A)
5. (C) **6.** (D) **7.** (C)(D)

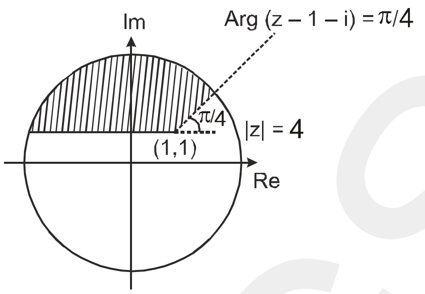
DPP NO. - 32

1. (B) **2.** (D) **3.** (C) **4.** (B)
5. (A)(C) **6.** $\frac{\pi}{4}$

DPP NO. - 33

1. (B) **2.** (D) **3.** (D) **4.** (B)
5. (A) **7.** $2x^3 - 15x^2 + 28x - 15 = 0$

8.



DPP NO. - 34

1. (B) **2.** (D) **3.** (D) **4.** (D)
5. (A) **6.** (B) **7.** (B)(C) **8.** $a = 1,\ b = -\frac{5}{2}$

DPP NO. - 35

1. (B) **2.** (C) **3.** (B) **4.** (B)
5. $k \in \left[\frac{5-\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right]$ **6.** $\left[\sqrt{5}, \sqrt{10}\right]$
7. $-1+i,\ 1+3i,\ 3+i$ **8.** $\frac{\sqrt{5}}{\sqrt{3}}, \frac{4\pi}{3}$

DPP NO. - 36

1. (A) **2.** (B) **3.** (C) **4.** (C)
5. (D) **6.** (B) **7.** (C)(D)
8. (A)(B)(C)(D) **9.** $\left(1 - \frac{1}{2^{2^n}}\right)(1 + i)$
11. $z(1+2i) - \bar{z}(1-2i) - 6i = 0$
12. centre: 9 + i, radius = $\sqrt{26}$
13. (A) → (p,q), (B) → (p,q), (C) → (r), (D) → (s)

DPP NO. - 37

1. (A) **2.** (C) **3.** (C) **4.** (A)
5. (C) **6.** $k \in (0, 4e^{-2})$ **7.** a = – 3, b = 2, c = 1

DPP NO. - 38

1. (B) **2.** (B)(C)
3. (i) 3 points, x = 0, –3, – 5
(ii) ∞ points, $x \in [-1, 2]$
(iii) 2 points, $x = \frac{\pi}{4}, \frac{3\pi}{4}$
4. M.I. in $[-2, 0] \cup [2, \infty)$ & M.D. in $(-\infty, -2] \cup [0, 2)$
5. 2 **6.** 29
7. (i) MI $x \in [0, 2] \cup [4, 6) \cup [8, 9]$, MD $[2, 4] \cup (6, 8]$
(ii) Local minima x = 0, 4, 8, Local maxima x = 2, 6, 9
(iii) Concaveup $x \in [3, 6) \cup (6, 9]$,
Concavedown $x \in [0, 3)$
(iv) Inflection point x = 3
(v) Critical points 2, 4, 6, 8

DPP NO. - 39

1. (B) **2.** (A) **3.** (B) **4.** (D)
5. (D) **6.** (B) **7.** (C)

DPP NO. - 40

1. (B) **2.** (C) **3.** (C) **4.** (C)
5. (A) **6.** (A) **7.** (D) **8.** (C)
9. (A) **10.** (A)

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CLASS : XII (MATHS)

DPP

DAILY PRACTICE PROBLEM

Solutions

DPP-31 to 40

DPP NO. - 31

1. –1, –1, 1, 7, 19, 39, 69
first difference : 0, 2, 6, 12, 20, 30
IInd difference
2, 4, 6, 8, 10 are in A.P.
there for n^{th} term

$t_n = (-1) + 0.{}^{n-1}C_1 + 2.{}^{n-1}C_2 + 2.{}^{n-1}C_3$

$= -1 + \frac{2(n-1)(n-2)}{2} + \frac{2(n-1)(n-2)(n-3)}{3.2.1}$

$t_{10} = \frac{n^3 - 3n^2 + 2n - 3}{3}$

2. $t_{10} = \frac{10^3 - 3(10)^2 + 2(10) - 3}{3} = \frac{717}{3} = 239$

3. $S_{10} = \frac{\sum n^3 - 3\sum n^2 + 2\sum n - 30}{3}$

$= \frac{1}{3}\left(\left(\frac{10(11)}{2}\right)^2 - 3\frac{(10)(11)(21)}{6} + \frac{2(10)(11)}{2} - 30\right)$

$= \frac{1}{3}\left(\frac{(10)(11)}{2}\left(\frac{110}{2} - 21 + 2\right) - 30\right)$

$= \frac{1}{3}\left(\left(\frac{110}{2}\right)\left(\frac{110-38}{2}\right) - 30\right) = \frac{1}{3}\left(\frac{110\times 72}{4} - 30\right)$

= (110×18 – 30) = 110 × 6 – 10 = 660 – 10 = 650

4. y = ax + b on solving with both curves and putting D = 0 gives
$a^2 + 10a + 4b + 1 = 0$
and $a^2 - 2a + 4b - 3 = 0 \Rightarrow a = -1/3$ & $b = 5/9$
$\Rightarrow$ 3x + 9y = 5;
point of contact (7/3, – 2/9) & (– 2/3, 7/9)]

5. $\left.\frac{dy}{dx}\right|_{(0,1)} = 0 \Rightarrow c = 0$

(0, 1) $\Rightarrow$ d = 1 $\Rightarrow$ $y = ax^4 + bx^3 + 1$
(–1, 0) $\Rightarrow$ 0 = a – b + 1
$y = ax^4 + (a + 1)x^3 + 1$

$\left.\frac{dy}{dx}\right|_{(-1,0)} = 0 \Rightarrow -4a + 3(a + 1) = 0 \Rightarrow a = 3$

$y = 3x^4 + 4x^3 + 1$

$\frac{dy}{dx} = 12x^2(x + 1) < 0 \Rightarrow x < -1$

6. $y = e^{-|x|}$, at x = 1 curve is $y = e^{-x}$

$\frac{dy}{dx} = -e^{-x}$

$\left(\frac{dy}{dx}\right)_{x=1} = -e^{-1} = \frac{-1}{e}$

equation of tangent is $\left(y - \frac{1}{e}\right) = \frac{-1}{e}(x-1)$

ey – 1 = –x + 1 $\Rightarrow$ x + ey = 2.

7. $x = 4t^2 + 3$, $y = 8t^3 - 1$

$\frac{dx}{dt} = 8t$, $\frac{dy}{dt} = 24t^2$

$\frac{dy}{dx} = \frac{24t^2}{8t} = 3t$

$y - (8t^3 - 1) = 3t(x - (4t^2 + 3))$
Pass $(8t_1^3 - 1) - (8t^3 - 1) = 3t((4t_1^2 + 3) - (4t^2 + 3))$
$8(t_1^3 - t^3) = 3t\,4(t_1^2 - t^2)$
$2(t_1 - t)(t_1^2 + t^2 + t_1t) = 3(t_1 - t)(t_1 + t)$
$\Rightarrow 2t_1^2 + 2t^2 + 2tt_1 = 3tt_1 + 3t^2$
$\Rightarrow 2t_1^2 - tt_1 - t^2 = 0 \Rightarrow 2t_1^2 - 2tt_1 + tt_1 - t^2 = 0$
$\Rightarrow 2t_1(t_1 - t) + t(t_1 - t) = 0$

$\Rightarrow (t_1 - t)(2t_1 + t) = 0 \Rightarrow t_1 = -\frac{t}{2}$ ...(1)

$(3t) = \frac{-1}{3t_1}$

$(9t\,t_1) = -1$

$\Rightarrow 9t\left(\frac{-t}{2}\right) = -1 \Rightarrow (3t)^2 = 2$

Slope of tangent = 3t = $\pm\sqrt{2}$

8. $x^3 - 3xy^2 = a$

$\Rightarrow x^3 - 3(y^2 + x.2y\frac{dy}{dx}) = 0$

$\Rightarrow x^2 - y^2 - 2xy\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$

and $3x^2y - y^3 = b$

$\Rightarrow 3(2xy + x^2\frac{dy}{dx}) - 3y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2xy}{x^2 - y^2}$

Let two curve interset at (α, β)

$\therefore m_1 m_2 = \left(\frac{\alpha^2 - \beta^2}{2\alpha\beta}\right)\left(\frac{-2\alpha\beta}{\alpha^2 - \beta^2}\right) = -1$

$\Rightarrow$ cut orthogonally.

DPP NO. - 32

1. $y = \tan^{-1}\left(\frac{5\cos(3x^2-2)\left(\frac{2}{5}+\tan(3x^2-2)\right)}{5\cos(3x^2-2)\left(1-\frac{2}{5}\tan(3x^2-2)\right)}\right)$

$y = \tan^{-1}(2/5)+\tan^{-1}(\tan(3x^2-2))$

$\frac{dy}{dx} = 6x$

2. $\frac{dy}{dt} = 2at + 2b, \quad \frac{dt}{dx} = 2ax + 2b$

$\frac{dy}{dx} = (2at + 2b).(2ax + 2b)$

$\frac{d^2y}{dx^2} = (2at + 2b).(2a) + (2ax + 2b).2a\frac{dt}{dx}$

$\frac{d^2y}{dx^2} = (2at + 2b)2a + 2a(2ax + 2b)^2$

$\frac{d^3y}{dx^3} = 2a\ 2a\frac{dt}{dx} + 2a.2\ (2ax + 2b).2a$

$\frac{d^3y}{dx^3} = (2ax + 2b)\ [4a^2 + 8a^2]$

$\frac{d^3y}{dx^3} = 24a^2(ax + b)$

3. $\log_{1/2}|z-2| > \log_{1/2}|z|$

$\Rightarrow |z-2| < |z|$

$\Rightarrow |(x-2) + iy| < |x + iy|$

$\Rightarrow (x-2)^2 + y^2 < x^2 + y^2$

$\Rightarrow x > 1$

4. (B)

$g(x) = \frac{2(\sin x - \sin^n x) + |\sin x - \sin^n x|}{2(\sin x - \sin^n x - |\sin x - \sin^n x|}$

for $0 < n < 1$, $\sin x < \sin^n x$, $g(x) = \frac{1}{3}$ and for $n > 1$, $\sin x > \sin^n x$, $g(x) = 3$

$\therefore$ for $n > 1$, $f(x) = 3$, $x \in (0, \pi)$

$\therefore$ f(x) is continuous and differentiable at $x = \frac{\pi}{2}$

and

for $0 < n < 1$

$$f(x) = \begin{cases} \left[\frac{1}{3}\right] = 0, & x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right) \\ 3, & = \frac{\pi}{2} \end{cases}$$

$\therefore$ f(x) is not continuous at $x = \frac{\pi}{2}$. Hence f(x) is also not differentiable at $x = \frac{\pi}{2}$.

5. $T_r = \frac{\left(\frac{r(r+1)}{2}\right)^2}{r^2} = \frac{1}{4}(r+1)^2 \quad \Rightarrow T_7 = 16$

$S = \sum \frac{1}{4}(r+1)^2$

$S_{10} = \frac{1}{4}(2^2 + 3^2 + \ldots + 11^2)$

$= \frac{1}{4}\left(\frac{11(12)\,23}{6} - 1\right) = \frac{505}{4}$

6. $fof(x) = \frac{1}{1-f(x)} = \frac{1}{1-\frac{1}{1-x}} = \frac{1-x}{-x} = \frac{x-1}{x}$

$fofof(x) = \frac{f-1}{f}\ \frac{\frac{1}{1-x}-1}{\frac{1}{1-x}} = x$

$\therefore g(x) = fofofofofofof(x) = fofofof(x) = f(x) = \frac{1}{1-x}$

Now $h(x) = \tan^{-1}\left(\frac{1}{1+x^2+x}\right) = \tan^{-1}\left(\frac{1+x-x}{1+x(x+1)}\right) = \tan^{-1}(x+1) - \tan^{-1}x$

Now $\lim_{n\to\infty}\sum_{r=1}^{n}\left(\tan^{-1}(r+1) - \tan^{-1}(r)\right)$

$= \lim_{n\to\infty} (\tan^{-1}(2) - \tan^{-1}1) + (\tan^{-1}(3) - \tan^{-1}2) + \ldots\ldots + \tan^{-1}(n+1) - \tan^{-1}n$

$= \lim_{n\to\infty} (\tan^{-1}(n+1) - \tan^{-1}1)$

$= \tan^{-1}(\infty) - \tan^{-1}(1)$

$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

7. We have to prove that $\frac{T+S.T.}{xy}$ = constant

or $\frac{1}{xy}\left[\frac{y}{y'}\sqrt{1+y'^2} + \frac{y}{y'}\right]$ = constant.

or $\frac{1}{xy'}\left[\sqrt{1+y'^2} + 1\right]$ = constant ...(1)

Differentiating, $y' = \frac{2ax}{x^2 - a^2}$

$\therefore \sqrt{1+y'^2} = \frac{x^2+a^2}{x^2-a^2}$

Putting in (1), we

L.H.S. $= \frac{x^2-a^2}{2ax^2}\left[\frac{x^2+a^2}{x^2-a^2} + 1\right] = \frac{1}{a}$ = constant

8. $a_0\alpha^n + a_1\alpha^{n-1} + ... + a_{n-1}\alpha = 0$ ......(1)

Let $f'(x) = na_0x^{n-1} + (n-1)a_1x^{n-2} + ... + a_{n-1}$

$f(x) = a_0x^n + a_1x^{n-1} + ... + a_{n-1}x + c$

$f(0) = c = f(\alpha)$ (using (1))

f(x) satisfies conditions in Rolle's theorem.

$\Rightarrow f'(c) = 0$ for at least one $c \in (0, \alpha)$

DPP NO. - 33

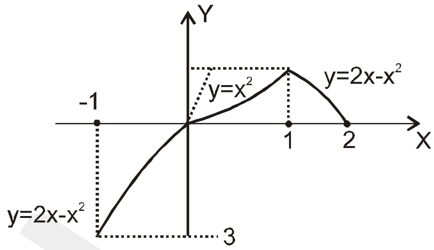
1. $\frac{1}{2}\left(\frac{3-1}{1.3} + \frac{5-1}{1.3.5} + \frac{7-1}{1.3.5.7} + \frac{9-1}{1.3.5.7.9} + ...\right)$

$= \frac{1}{2}\left(1 - \frac{1}{1.3} + \frac{1}{1.3} - \frac{1}{1.3.5} + \frac{1}{3.5.7} - \frac{1}{1.3.5.7} + ...\right)$

$= \frac{1}{2}$

2. $f(x) = x - |x - x^2| = \begin{cases} x + (x - x^2) & \text{if } x - x^2 < 0 \\ x - (x - x^2) & \text{if } x - x^2 \geq 0 \end{cases}$ o r

$f(x) = \begin{cases} 2x - x^2 & \text{if } x < 0 \\ x^2 & \text{if } 0 \leq x < 1 \\ 2x - x^2 & \text{if } x > 1 \end{cases}$



3. Given that f is a real valued function s.t

$f(x) f'(x) < 0 \ \forall \ x \in R$

Now, $\frac{d}{dx}|f(x)| = \frac{f(x)}{|f(x)|} f'(x)$

since $f(x) f'(x) < 0$

$\Rightarrow \frac{d}{dx}|f(x)| < 0$

$\Rightarrow$ |f(x)| is a decreasing function

4. $\frac{f'(\tan x)\sec^2 x}{g'(\sec x)\sec x \tan x}$

at $x = \frac{\pi}{4}$

$\frac{f'(1)\left(\sqrt{2}\right)^2}{g'\left(\sqrt{2}\right)\sqrt{2}.1} \Rightarrow \frac{-2\sqrt{2}\,2}{4.\sqrt{2}} = -1$

5. $y^2 = x^{x^{x^{.........\infty}}}$

$y^2 = x^{y^2}$

$\ln(y^2) = y^2 \ln x$

$2\ln y = y^2 \ln x$

$\frac{2}{y}\frac{dy}{dx} = \frac{y^2}{x} + \ln x.\, 2y\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{y^3}{2x(1 - y^2\ln x)}$

6. We have $(x^2 + c) = ax + b$. Differentiating w.r.t. x, we get

$y'(x^2 + c) + y \,.\, 2x = a$ ...(1)

Differentiating (1) again, we get

$y''(x^2 + c) + y' \,.\, 2x + y' \,.\, 2x + y \,.\, 2 = 0$

or $y''(x^2 + c) + 4xy' + 2y = 0$ ...(2)

or $x^2 + c = -2(2xy' + y)/y''$ ...(3)

Diferentiating both sides of (2) w.r.t. x, we get

$y'''(x^2 + c) + y'' \,.\, 2x + 4[xy'' + y'] + 2y' = 0$

or $y''' = -\dfrac{6(xy''+y')}{x^2+c} = \dfrac{3y''(xy''+y')}{2xy'+y}$, by (3)

or $y'''(2xy' + y) = 3y''(xy'' + y')$

7. $m_1 = \dfrac{2}{3}$, $m_2 = -1$, $m_3 = 90 - \tan^{-1}(2/3)$

$\tan A = \dfrac{m_1 - m_2}{1 + m_1 m_2} = \dfrac{\frac{5}{3}}{1 - \frac{2}{3}} = 5$

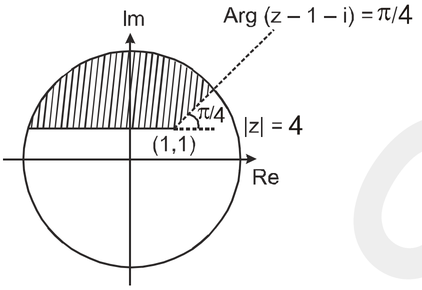
$\tan B = \tan(90 - \tan^{-1} 2/3) = \cot(\tan^{-1} 2/3) = 3/2$

$\tan C = \tan 45° = 1$

equation $x^3 - \left(\dfrac{15}{2}\right)x^2 + \left(5 + \dfrac{15}{2} + \dfrac{3}{2}\right)x - \left(\dfrac{15}{2}\right) = 0$

$2x^3 - 15x^2 + 28x - 15 = 0$

8.

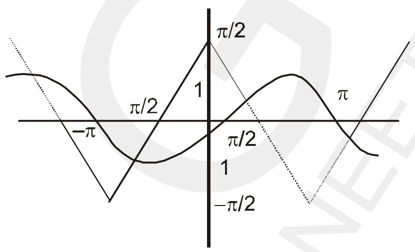


DPP NO. - 34

1. $y = f(x) = \max\{\sin x, \sin^{-1}(\cos x)\}$

$y = f(x)$ is continuous

but not differentiable



2. $\dfrac{dr}{dt} = k$, $\quad h = ar + b$, $\quad \dfrac{dh}{dt} = 3.\dfrac{dr}{dt}$

$\dfrac{dh}{dt} = a.\dfrac{dr}{dt} \Rightarrow a = 3$

also $6 = 3.1 + b$

$\Rightarrow b = 3$

so $h = 3r + 3$

$v = \pi r^2 h$

$\dfrac{dv}{dt} = (\pi.2r(3r+3) + \pi r^2.3)\dfrac{dr}{dt}$

$r = 6$, $\dfrac{dv}{dt} = 1$, find $\dfrac{dr}{dt}$

Now find $\dfrac{dv}{dt}$ at $r = 36$

3. let $\dfrac{e^x}{(m-1)^2} = \alpha$

$\dfrac{dy}{dx} = (A + Bx)e^{mx}m + e^{mx}B + \alpha$

$y' = m(y - \alpha) + Be^{mx} + \alpha$

$y'' = my' - m\alpha + \alpha + Bme^{mx}$

$y'' = my' - m\alpha + \alpha + m(y' - my + m\alpha - \alpha)$

$\Rightarrow y'' - 2my' + m^2y = e^x$

4. $\sec^2 y \dfrac{dy}{dx} = \dfrac{(1+2^{2x+1})(2^x \ln 2) - 2^x(2^{2x+1}.2\ln 2)}{(1+2^{2x+1})^2}$

at $x = 0$

$\Rightarrow \left(\dfrac{\sqrt{10}}{3}\right)^2 \dfrac{dy}{dx} = \dfrac{3\ln 2 - 4\ln 2}{3^2}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-\ln 2}{10}$

5. $\dfrac{dy}{dx} = \cos x$

$\dfrac{d}{dy}\left(\dfrac{d}{dy}\cos^7 x\right)$

$= \dfrac{d}{dy}\left(7.\cos^6 x.(-\sin x).\dfrac{dx}{dy}\right)$

$= \dfrac{d}{dy}\left(-7.\sin x.\cos^5 x\right)$

$= -7[\sin x.\, 5\cos^4 x\,(-\sin x) + \cos^5 x \cos x]\dfrac{dx}{dy}$

$= -7[\cos^3 x.\, 5\sin^2 x + \cos^5 x]$

$= 35\cos^3 x.(1 - \cos^2 x) - 7\cos^5 x$

$= 35\cos^3 x - 42\cos^5 x$

6. $2a + 3b + c = 3$

$\dfrac{2a + 3b + c}{9} \geq \left[a^2.\left(\dfrac{3b}{5}\right)^5.\left(\dfrac{c}{2}\right)^2\right]^{1/9}$

$\frac{1}{3^9} \geq \frac{a^2.b^5.c^2.3^5}{5^5.2^2}$

$a^2\,b^5\,c^2 \leq \frac{5^5\,.\,2^2}{3^{14}}$

7. $f(x) = (\tan^{-1}x)^3 - (\cot^{-1}x)^2 + \tan^{-1}x + 2$

$f'(x) = \frac{3(\tan^{-1}x)^2}{1+x^2} + \frac{2\cot^{-1}x}{1+x^2} + \frac{1}{1+x^2} > 0$

$\Rightarrow$ f(x) is increasing function.

8. $2 = a + b + \frac{7}{2}$ ..... (i)

$\frac{dy}{dx} = 2x + 6$ at $-2, 2$

$\frac{dy}{dx} = 2ax + b$

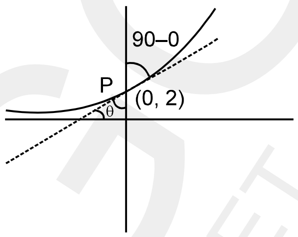
$\Rightarrow$ $m = 2a + b$

$2a + b = -\frac{1}{2}$ .....(ii)

from (i) and (ii) $a = 1$, $b = -5/2$

DPP NO. - 35

1. $y = 2e^{2x}$ ....(i)



$\therefore$ $\frac{dy}{dx} = 2\,.\,e^{2x}\,.\,2 = 4\,.e^{2x}$

$\therefore$ $\left(\frac{dy}{dx}\right)$ at P = 4

$\therefore$ $\tan\theta = 4$ $\Rightarrow$ $q = \tan^{-1}4$

$\therefore$ required angle = $90 - \theta$

$= \frac{\pi}{2} - \tan^{-1}4 = \cot^{-1}4$

2. $xy^n = a^{n+1}$, $a \neq 0$

$y^n + nxy^{n-1}\frac{dy}{dx} = 0$ i.e. $\frac{dy}{dx} = -\frac{y}{nx}$

$\therefore$ length of subnormal = $\left|y.\frac{dy}{dx}\right| = \left|\frac{y^2}{nx}\right|$

$= \left|\frac{y^2.y^n}{na^{n+1}}\right| = \left|\frac{y^{n+2}}{na^{n+1}}\right|$ is constant only if $n = -2$.

3. $a_1^2 - a_2^2 + a_3^2 - \ldots\ldots + a_{2n-1}^2 - a_{2n}^2$

$(a_1 - a_2)(a_1 + a_2) + (a_3 + a_4)(a_3 + a_4) +$
$\ldots\ldots(a_{2n-1} - a_{2n})(a_{2n-1} + a_{2n})$

$-d\,[a_1 + a_2 + a_3 + a_4 + \ldots\ldots + a_{2n-1} + a_{2n}]$

$-d.\frac{2n}{2}[a_1 + a_{2n}]$

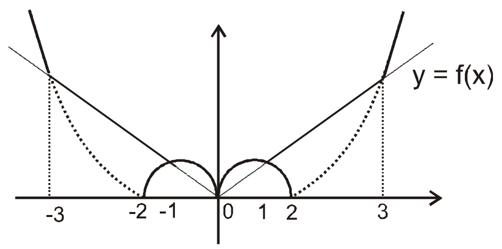
Now $a_{2n} = a_1 + (2n-1)\,d$

$\frac{a_{2n} - a_1}{(2n-1)} = d$

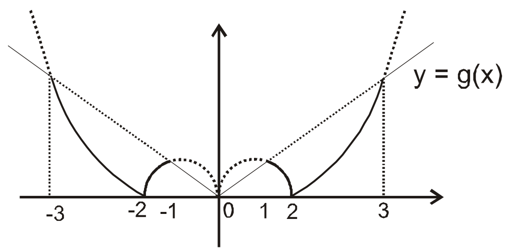
$-\frac{(a_{2n} - a_1)(a_{2n} + a_1).n}{2n-1}$

$\frac{n}{2n-1}\,(a_1^{\,2} - a_{2n}^{\,2})$

4.



f(x) is not differentiable function at $x = -3, -1, 0, 1, 3$



g(x) is not differentialble at $x = -3, -2, -1, 0, 1, 2, 3$

5. $f'(x) = \frac{2}{\sqrt{3}}\,\frac{1}{1+\left(\frac{2x+1}{\sqrt{3}}\right)^2}\cdot\frac{2}{\sqrt{3}} - \frac{2x+1}{x^2+x+1} + (k^2 - 5k + 3)$

$$f'(x) = \frac{4}{3}\,\frac{3}{3+(2x+1)^2} - \frac{2x+1}{x^2+x+1} + k^2 - 5k + 3$$

also $f'(x) \le 0$

$$\frac{4}{4x^2+4x+4} - \frac{2x+1}{x^2+x+1} + (k^2 - 5k + 3) \le 0$$

$\Rightarrow \dfrac{-2x}{x^2+x+1} + k^2 - 5k + 3 \le 0$ (maximum value of $\dfrac{-2x}{x^2+x+1}$ is 2)

$\Rightarrow\ 2 + k^2 - 5k + 3 \le 0 \qquad \Rightarrow\ k^2 - 5k + 5 \le 0$

$$k \in \left[\frac{5-\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right]$$

6. $f(x) = \sqrt{x-1} + \sqrt{6-x}$

domain of f(x) is [1, 6]

Now $f'(x) = \dfrac{1}{2}\left[\dfrac{1}{\sqrt{x-1}} - \dfrac{1}{\sqrt{6-x}}\right]$

$f'(x) > 0 \quad \Rightarrow \quad \dfrac{1}{\sqrt{x-1}} > \dfrac{1}{\sqrt{6-x}}$

$\Rightarrow \sqrt{6-x} > \sqrt{x-1} \quad \Rightarrow \quad 6 - x > x - 1$

$7 > 2x \quad \Rightarrow \quad x < \dfrac{7}{2}$

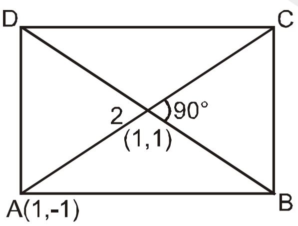
$\Rightarrow$ f′(x) is increasing in $\left[1, \dfrac{7}{2}\right]$ and decreasing in $\left[\dfrac{7}{2}, 6\right]$

$f(1) = \sqrt{5}$, $f(6) = \sqrt{5}$

$f\left(\dfrac{7}{2}\right) = \sqrt{\dfrac{5}{2}} + \sqrt{\dfrac{5}{2}} = \sqrt{10}$

Hence range $[\sqrt{5}, \sqrt{10}]$

7.

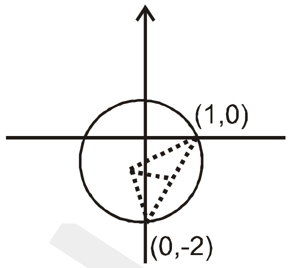


AC is parallel to y-axis

$\Rightarrow$ BD is parallel to x-axis

therefore points B(–1, 1) and D(3, 1)

8.



$\dfrac{\sqrt{5}}{2r} = \sin\dfrac{\pi}{3} \Rightarrow r = \dfrac{\sqrt{5}}{\sqrt{3}}$

length of arc = $\theta\, r = \dfrac{4\pi}{3} \cdot \dfrac{\sqrt{5}}{\sqrt{3}}$

DPP NO. - 36

1. $|z - 1|^2 = |z + 1|^2 \quad \Rightarrow \quad x = 0$

$|z - 1|^2 = |z - i|^2$

$\Rightarrow\ (x - 1)^2 + y^2 = x^2 + (y - 1)^2 \quad \Rightarrow \quad 1 + y^2 = (y - 1)^2$ ($\because$ x = 0)

$\therefore\ y = 0 \quad \Rightarrow \quad (0, 0)$ satisfies

Hence correct option is (1)

2. $x^2 - x + 1 = 0$

$\Rightarrow\ x = -\omega, -\omega^2$

$\therefore\ \alpha^{2009} + \beta^{2009} = -\omega^{2009} - \omega^{4018} = -\omega^2 - \omega = 1$

Hence correct option is (2)

3. $(1 - \omega - \omega^2)^3 + (\omega - 1 - \omega^2)^3 + (\omega^2 - \omega - 1)^3$

$= 2^3 + (2\omega)^3 + (2\omega^2)^3$

$= 8(1 + \omega^3 + \omega^6) = 8(1 + 1 + 1) = 24$

5. $iz = \bar{z} \quad \Rightarrow \quad i(x + iy) = x - iy$

$\Rightarrow\ i(x + y) - (x + y) = 0 \quad \Rightarrow \quad (i - 1)(x + y) = 0$

which represents the line $y = -x$

So, reflection of the point (2, –1) in the line $y = -x$ gives the point (1, –2)

9. Assume $\dfrac{1+i}{2} = z$; multiply numerator and denominator by $(1 - z)$ which simplifies to

$$\frac{\left(1 - z^{2^n}\right)\left(1 - z^{2^n}\right)}{1 - z} = \frac{1 - \left(z^2\right)^{2^n}}{1 - z} \quad ;$$

Now $\dfrac{1}{1-z} = \dfrac{2}{1-i} = (1 + i)$

$$\left(z^{2^n}\right)^2 = \left(z^2\right)^{2^n} = \left[\left(\frac{1+i}{2}\right)^2\right]^{2^n} = \left(\frac{i}{2}\right)^{2^n} \quad ;$$

for $n \geq 2$ $(i)^{2^n} = 1 \Rightarrow \left(z^{2^n}\right)^2 = \frac{1}{2^{2^n}}$

$\Rightarrow$ Given expression $= \left(1 - \frac{1}{2^{2^n}}\right)(1 + i)$

10. $z = \frac{A}{B}$

$z + \frac{1}{z} = 1 \Rightarrow z^2 - z + 1 = 0 \Rightarrow z = \frac{1 \pm \sqrt{3}i}{2}$

$z = \frac{1+\sqrt{3}i}{2}, \frac{1-\sqrt{3}i}{2}$

11. $z_1 = 1 + i$ $\quad z_2 = 2 - i$

equation of line

$$\frac{z - z_1}{\bar{z} - \bar{z}_1} = \frac{z_2 - z_1}{\bar{z}_2 - \bar{z}_1}$$

$$\Rightarrow \frac{z-(1+i)}{\bar{z}-(1-i)} = \frac{(2-i)-(1+i)}{(2+i)-(1-i)}$$

$$\Rightarrow \frac{z-(1+i)}{\bar{z}-(1-i)} = \frac{1-2i}{1+2i}$$

$\Rightarrow z(1+2i) - (1+i)(1+2i)$

$= \bar{z}(1-2i) - (1-i)(1-2i)$

$\Rightarrow z(1+2i) - \bar{z}(1-2i) + 1 - 3i - 3i - 1 = 0$

$\Rightarrow z(1+2i) - \bar{z}(1-2i) - 6i = 0$

DPP NO. - 37

1. $\frac{a}{b} + \frac{b}{c} \geq 2\sqrt{\frac{a}{c}}$

$\frac{c}{d} + \frac{d}{e} \geq 2\sqrt{\frac{c}{e}}$

multiply both these

$$\left(\frac{a}{b} + \frac{b}{c}\right)\left(\frac{c}{d} + \frac{d}{e}\right) \geq 4\sqrt{\frac{a}{c}.\frac{c}{e}} \geq 4\sqrt{\frac{a}{e}}$$

2. $f(x) = e^{2x} - (a + 1)e^x + 2x$

$f'(x) = 2e^{2x} - (a + 1)e^x + 2$

Now, $\quad 2e^{2x} - (a + 1)e^x + 2 \geq 0$ for all $x \in R$

i.e. $2\left(e^x + \frac{1}{e^x}\right) - (a + 1) \geq 0$ for all $x \in R$

i.e. $4 - (a + 1) \geq 0$

i.e. $a \leq 3$

3. $\frac{dy}{dx} = 3x^2 - 2ax + 1 > 0$

$\Rightarrow D < 0$

$4a^2 - 4(3)(1) < 0$

$\Rightarrow a^2 < 3$

$\Rightarrow -\sqrt{3} < a < \sqrt{3}$

4. Using mean value theorem,

$f(6) - f(2) = (6 - 2) f'(c)$, where $c \in (2, 6)$

$\Rightarrow f(6) = f(2) + 4f'(c) = 4 + 4f'(c) > 4 + 4(5)$ $\quad [\because f(x) \geq 5]$

$\Rightarrow f(6) \geq 24$

5. $U_n = \frac{1}{(n+2)(n+1)}$

$S_n = \sum_{n=1}^{n} U_n = \frac{n}{2(n+2)}$

Hence $\lim_{n\to\infty} S_n = \frac{1}{2}$

6. $x^2 e^x = k$

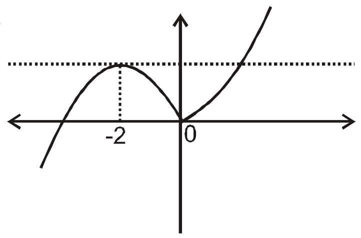
let $f(x) = x^2.e^x$

$f'(x) = 2xe^x + x^2e^x \Rightarrow f'(x) = e^x(x^2 + 2x)$

$x(x + 2) = 0 \Rightarrow x = 0, x = -2$

at $x = 0$, $k = 0$

at $x = -2$, $k = 4e^{-2}$



$k \in (0, 4e^{-2}]$

7. $y = ax^2 + ax + b$, $\quad y = cx - x^2$

touches (1, 0)

$\Rightarrow 0 = 1 + a + b$

$a + b = -1$ $\quad$ and $\quad 0 = c - 1 = c = 1$

$\frac{dy}{dx} = 2x + a = c - 2x$

at (1, 0) $\quad 2 + a = c - 2$

$a = -3$

$\therefore$ $b = 2$

DPP NO. - 38

1. $y^2 = x^2(x + 1)$, $\quad x + 1 \geq 0$

$y = \pm x\sqrt{x+1}$

$\therefore \frac{dy}{dx} = \pm\left(\sqrt{x+1} + \frac{x}{2\sqrt{x+1}}\right)$

$= \pm \dfrac{2(x+1)+x}{2\sqrt{x+1}} = \pm \dfrac{3x+2}{2\sqrt{x+1}}$

$\therefore \left.\dfrac{dy}{dx}\right|_{x=0} = \pm 1$

$\therefore$ at (0, 0) the curve bisects the angle between the axes.

2. $f(\theta) = \dfrac{1+\sin\theta}{5+4\cos\theta}$

$\Rightarrow f'(\theta) = \dfrac{(5+4\cos\theta)\cos\theta+(1+\sin\theta)(4\sin\theta)}{(5+4\cos\theta)^2}$

$\Rightarrow f'(\theta) = \dfrac{5\cos\theta+4\cos^2\theta+4\sin\theta+4\sin^2\theta}{(5+4\cos\theta)^2}$

$\Rightarrow f'(\theta) = \dfrac{5\cos\theta+4\sin\theta+4}{(5+4\cos\theta)^2}$

In $(0, \pi/2)$

$f'(\theta) > 0$

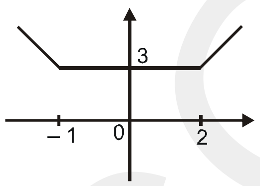
$\Rightarrow f(\theta)$ is increasing

3. (i) $f'(x) = -3x^3 - 24x^2 - 45x$

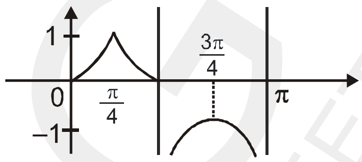
$= -3x(x^2 + 8x + 15) = -3x(x + 5)(x + 3)$

$f'(x) = 0$ at $x = 0$, $x = -5$ and $x = -3$

(ii)



(iii)



5. $2y\dfrac{dy}{dx} - 6x^2 - 4\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = \dfrac{6x^2}{2(y-2)}$

Equation of tanget at (x_1y_1)

$y - y_1 = \dfrac{3x_1^2}{(y_1-2)}(x - x_1)$ pars (1,2)

$(2 - y_1)(y_1 - 2) = 3x_1^2(1 - x_1)$

$y_1^2 - 49_1 + 4 = 3x_1^3 - 3x_1^2$

also point (x_1, y_1) lies on curve

$2x_1^3 - 8 + 4 = 3x_1^3 - 3x_1^2$

$x_1^3 - 3x_1^2 + 4 = 0 \quad \Rightarrow \quad (x_1 + 1)(x_1 - 2)^2 = 0$

$x_1 = -1$

$x_1 = 2$

so two tangents

6. a = 3, b = 2, c = 4

8. Let $f(x) = e^{100x} P(x)$, $x \in [a, b]$

$f'(x) = e^{100x} p'(x) + e^{100x} p(x)(100)$

(i) f(x) is continuous & differentiable(i)

(ii) $f(a) = e^{100a} p(a)$

$f(b) = e^{100b} p(b)$

$\Rightarrow$ By rolles theorem

$f'(x) = 0$

$\Rightarrow e^{100c} p'(c) + e^{100c} P(c)(100) = 0$

$\Rightarrow p'(c) + p(c)(100) = 0$ **Ans.**

DPP NO. - 39

1. Since $\phi(x)$ satisfy Rolle's theorem

$\Rightarrow \phi(a) = \phi(b) = 0$

$\Rightarrow \phi(a) = 0 \because \phi(b) = 0$

$\Rightarrow f(b) - f(a) - (b-a) f'(a)$

$- \dfrac{(b-a)^2}{2} f''(a) - (b-a)^3 \lambda = 0$ ......(i)

also $\phi'(c) = 0$

$- f'(c) - (b - c) f''(c) + f'(c) + (b - c) f''(c)$

$- \dfrac{(b-c)^2}{2} f'''(c) + 3(b - c)^2 \lambda = 0$

Comparing it with equation (i)

$\Rightarrow \lambda = \dfrac{f'''(c)}{6}$

therefore $\mu = \dfrac{1}{6}$

2. In previous problem we get

$f(b) = f(a) + (b - a) f'(a) + \dfrac{(b-a)^2}{2} f''(a)$

$+ \dfrac{1}{6}(b - a)^3 f'''(c)$

put b = 2 + h a = 2 ....(ii)

$f(2 + h) = f(2) + h f'(2) + \dfrac{h^2}{2} f''(2) + \dfrac{1}{6} h^3 f'''(c)$

$f(2 + h) - f(2) = h^3(4c - 6)$

$\Rightarrow \dfrac{f(2+h)-f(2)}{h^3} = 4c - 6$

g(x) = 4x – 6 has slope 4 at all points

3. put b = a + h and c = a + θh in equation (ii) obtained in previous question.

$$e^{2a+2h} = e^{2a} + 2h\,e^{2a} + 2h^2 e^{2a} + \frac{8}{6} h^3 e^{2(a+\theta h)}$$

$$e^{2a}(e^{2h} - 1 - 2h - 2h^2) = \frac{4}{3} h^3 e^{2a+2\theta h}$$

$$\Rightarrow \frac{e^{2h} - 1 - 2h - 2h^2}{h^3} = \frac{4}{3} e^{2\theta h}$$

$$\Rightarrow A = \frac{4}{3},\ B = 2 \quad \Rightarrow \frac{2B}{A} = 3$$

4. $\frac{dy}{dx} = 3x^2 + 2x - 1 = 0$

$\Rightarrow 3x^2 + 3x - x - 1 = 0$

$(3x - 1)(x + 1) = 0$

$\Rightarrow x = -1,\ 1/3$

$y(-1) = -1 + 1 + 1 = 1 \quad \Rightarrow y = 1$

$$y(1/3) = \frac{1}{27} + \frac{1}{9} - \frac{1}{3}$$

$$\Rightarrow \frac{1+3-9}{27} = \frac{-5}{27} \qquad \Rightarrow y = \frac{-5}{27}$$

$$\text{dis} = \left|1 + \frac{5}{27}\right| = \frac{32}{27}$$

5. $$\frac{dy}{dx} = \frac{-b^2}{(a-x)^2}.(-1) - \frac{a^2}{x^2}$$

$$\text{for min.,}\ \frac{dy}{dx} = 0 \Rightarrow x = \frac{a^2}{a \pm b}$$

$$\text{In } (0, a),\ x = \frac{a^2}{a \pm b} \quad \therefore\ x \neq \frac{a^2}{a-b}\ (> a)$$

$$\frac{d^2y}{dx^2} = \frac{2a^2}{x^3} + \frac{2b^2}{(a-x)^3} > 0,\ \text{if } x = \frac{a^2}{a+b}$$

$$\therefore\ y \text{ is min. at } x = \frac{a^2}{a+b} \text{ and}$$

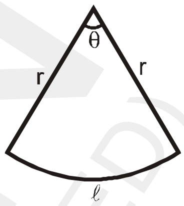
$$y_{min} = \frac{b^2}{\left(a - \frac{a^2}{a+b}\right)} + \frac{a^2}{\left(\frac{a^2}{a+b}\right)} = \frac{(a+b)^2}{a}$$

6. Given $2r + \ell = 20$

$$\text{Area} = \frac{\pi r^2}{360}.\theta$$

$$A = \frac{\pi r^2}{360}.\frac{\ell}{r} = \frac{\pi}{360}\ell r$$

$$A = \frac{\pi r(20 - 2r)}{360} = \frac{\pi}{180}(10\,r - r^2)$$



$$\frac{dA}{dr} = \frac{\pi}{180}(10 - 2r) = 0$$

$\Rightarrow r = 5 \quad \Rightarrow \ell = 20 - 10 = 10$

$$\text{Area} = \frac{\pi}{360}(10/5) = 25.$$

7. $a + b + c = 13$

$\sum ab = 54$

$abc = 72$

$$\text{Now} \quad \frac{b^2 + c^2 - a^2}{2abc} + \frac{a^2 + c^2 - b^2}{2abc} + \frac{b^2 + a^2 - c^2}{2abc}$$

$$= \frac{a^2 + b^2 + c^2}{2abc} = \frac{(a+b+c)^2 - 2\sum ab}{2ab}$$

$$= \frac{169 - 2(54)}{2 \times 72} = \frac{61}{144}$$

8. $$\frac{dx}{dt} = e^t.\cos t + e^t \sin t$$

$$\frac{dy}{dt} = e^t.(-\sin t) + e^t \cos t$$

$$\frac{dy}{dx} = \frac{\cos t - \sin t}{\cos t + \sin t} = \frac{1 - \tan t}{1 + \tan t}$$

$$\frac{dy}{dx} = \frac{1 - x/y}{1 + x/y} = \frac{y - x}{y + x}$$

$$\frac{d^2y}{dx^2} = \frac{-2(x^2 + y^2)}{(x+y)^3}$$