DPP No.-29 to 32

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

DPP No.-29

[SINGLE CORRECT CHOICE TYPE] **[8 × 3 = 24]**

Q.1 The value of $\lim\limits_{x\to 0} x^2\left(1+2+3+4+......+\left[\frac{1}{|x|}\right]\right)$ equals

[**Note:** [k] denotes greatest integer function less than or equal to k.]

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) 2

Q.2 If the sum of the coefficients in the expansion of $\left(\frac{1}{3}-15x+15x^3\right)^{2014}$ $(17x - 17x^5 + 3)^{2015}$ is n, then the number of dissimilar terms in the expansion of $(1 + x)^n$ is

(A) 1 (B) 2 (C) 3 (D) 4

Q.3 Number of integral value of x satisfying $\log_4|x|<1$ is equal to

(A) 4 (B) 5 (C) 6 (D) 7

Q.4 The angle which is equivalent to $4\tan^{-1}\left(\frac{1}{5}\right)-\tan^{-1}\left(\frac{1}{239}\right)$ is equal to

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\tan^{-1}\left(\frac{4}{5}\right)$ (D) $\tan^{-1}\left(\frac{3}{5}\right)$

Q.5 In ΔABC with altitude AD, $\angle BAC=\frac{\pi}{4}$, $BD=\frac{3}{2}$ and DC = 1, the length of side AB is

(A) $\frac{3\sqrt{5}}{2}$ (B) $\sqrt{3}$ (C) $3\sqrt{5}$ (D) 3

Q.6 The value of $\sin\frac{\pi}{16}\sin\frac{3\pi}{16}\sin\frac{5\pi}{16}\sin\frac{7\pi}{16}$ is

(A) $\frac{1}{8\sqrt{2}}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4\sqrt{2}}$ (D) $\frac{1}{4}$

Q.7 The range of the function $f(x) = \sec^{-1}x + \tan^{-1}x + \text{cosec}^{-1}x$, is

(A) $\left(0,\frac{\pi}{2}\right)$ (B) $(0, \pi)$ (C) $\left(0,\frac{\pi}{4}\right)\cup\left[\frac{3\pi}{4},\pi\right)$ (D) $\left(0,\frac{\pi}{4}\right]\cup\left[\frac{3\pi}{4},\pi\right)$

Q.8 If $\lim\limits_{x\to 0} \dfrac{\ln \cot\left(\dfrac{\pi}{4} - k_1 x\right)}{\tan k_2 x} = 1$ then

(A) $k_1 = k_2$ (B) $2k_1 = k_2$ (C) $k_1 = 2k_2$ (D) $k_1 = 4k_2$

DPP No.-30

[SINGLE CORRECT CHOICE TYPE] **[5 × 3 = 15]**

Q.1 The value of $\left(\text{cosec}^2(\cot^{-1}x) - \cot^2(\text{cosec}^{-1}x)\right)$ is equal to

(A) $2(1 + x^2)$ (B) 2 (C) 2x (D) $2x^2$

Q.2 If $e^x + e^y = e^{x+y}$ then $\frac{dy}{dx}$ is not equal to

(A) $e^{x-y}\left(\frac{1-e^y}{e^x-1}\right)$ (B) $\frac{1}{1-e^x}$ (C) $e^y - 1$ (D) $\frac{-e^y}{e^x}$

Q.3 Assuming that $f(t) = \frac{t-1}{t+1}$ is an invertible function, then $f^{-1}\left(\frac{1}{c}+1\right)$ is equal to

(A) $-(2c + 1)$ (B) $\frac{1}{(2+c)}$ (C) $2c - 1$ (D) $(2c + 1)$

Q.4 The number of solutions of the equation
$\log(x^{2016} + 1) + \log(1 + x^2 + x^4 + \ldots\ldots + x^{2014}) = \log 2016 + 2015 \log x$
is equal to

(A) 1 (B) 2 (C) 3 (D) infinite

Q.5 If length of the perpendicular from the origin upon the tangent drawn to the curve $x^2 - xy + y^2 + \alpha(x-2) = 4$ at (2, 2) is equal to 2 then α equals

(A) –2 (B) 0 (C) 2 (D) 4

[INTEGER TYPE] **[1 × 5 = 5]**

Q.6 Let $f(x) = \sum_{r=1}^{9}(-1)^{r-1}\cos(rx)\cos((r+1)x)$. If $l = \lim_{x\to 0}\frac{e^{1-f(x)}-1}{12\tan^2 x}$, then $[\,l\,]$ is

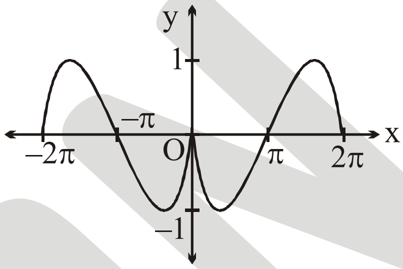
[**Note :** [k] denotes greatest integer value less than or equal to k.]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

DPP No.-31

[SINGLE CORRECT CHOICE TYPE] **[8 × 3 = 24]**

Q.1 Which function corresponds to the given graph ?



(A) $y = |\sin x|$
(B) $y = \sin|x|$
(C) $y = -|\sin x|$
(D) $y = -\sin|x|$

Q.2 If $(a-2)^2 - 2k(a+1) = (a-1)^2 - 2k(a+2) = -1$, $k \in R$, then the value of $(k^2 + 6k)$ is equal to

(A) $\frac{5}{4}$ (B) $\frac{1}{4}$ (C) $\frac{-3}{4}$ (D) 1

Q.3 If $\lim_{x\to 0} \frac{\ln(3+x)-\ln(3-x)}{x} = k$, the value of k is

(A) $\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) $-\frac{2}{3}$ (D) 0

Q.4 If $\operatorname{cosec}\theta + \cot\theta = 5$, then value of $\cos\theta$ is equal to

(A) $\frac{1}{5}$ (B) $\frac{5}{12}$ (C) $\frac{5}{13}$ (D) $\frac{12}{13}$

Q.5 Let α and β (α > β) are two values of x satisfying the equation $(\tan^{-1}x)^2 = 1 + \frac{\pi^2}{16} - x^2$.
Then the value of $(\sin^{-1}\alpha + \cot^{-1}\beta + \sec^{-1}\beta + \operatorname{cosec}^{-1}\alpha)$, is

(A) 0 (B) 2π (C) $\frac{7\pi}{4}$ (D) $\frac{11\pi}{4}$

Q.6 If $\sin^{-1}\alpha$, $\tan^{-1}\alpha$, $\cos^{-1}\alpha$ are in arithmetic progression then the value of $\lim_{x\to\alpha}\left(\tan\left(\frac{\pi}{4}+\ln x\right)\right)^{\frac{1}{\ln x}}$ equal to

(A) e^{-1} (B) e^{2} (C) e^{-2} (D) e^{3}

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Q.7 The length of the common chord of the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x + 2 = 0$, is

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 1 (D) 2

Q.8 Let $f(x) = \dfrac{\sqrt{\text{sgn}\,(\alpha x^2 + \alpha x + 1)}}{\cot^{-1}(x^2 - \alpha)}$. If $f(x)$ is continuous for all $x \in R$, then number of integers in the range of α, is

(A) 0 (B) 4 (C) 5 (D) 6

[**Note :** sgn k denotes signum function of k.]

DPP No.-32

Q.1 Evaluate $\lim\limits_{x\to\infty}\left(\frac{x}{e}-x\left(\frac{x}{x+1}\right)^x\right)$ **[3]**

[SINGLE CORRECT CHOICE TYPE] [3 × 3 = 9]

Q.2 The number of prime numbers in the range of the function, $f(x)=\frac{96}{7(x^4+3x^2+1)}$ is

(A) 4 (B) 6 (C) 13 (D) 14

Q.3 Two mutually perpendicular straight lines through origin form an isosceles triangle with the line $2x+y=5$, then the area of triangle is

(A) 5 (B) 3 (C) $\frac{5}{2}$ (D) 1

Q.4 Let S_1, S_2, S_3 be the respective sums of first n, 2n and 3n terms of the same arithmetic progression with a as the first term and d as the common difference. If $R=S_3-S_2-S_1$, then R depends on
(A) a and d (B) d and n (C) a and n (D) a, d and n

[MULTIPLE CORRECT CHOICE TYPE] [2 × 4 = 8]

Q.5 If the independent variable x is changed to y then the differential equation $x\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3-\frac{dy}{dx}=0$ is changed to $x\frac{d^2x}{dy^2}+\left(\frac{dx}{dy}\right)^2=k$ where k is equal to

(A) $\lim\limits_{x\to0}\left[\frac{2\tan x}{x}\right]$ (B) $\lim\limits_{x\to0}\left[\frac{2x}{\tan x}\right]$ (C) $\lim\limits_{x\to0}\left[\frac{\tan x}{x}\right]$ (D) 1

[**Note:** [y] denotes greatest integer function less than or equal to y.]

Q.6 Let $f(x)=\begin{cases}\frac{\ln(1+2x)}{x}, & \frac{-1}{2}<x<0\\ 2\cos x, & x=0\\ \frac{e^{2x}-1}{x}, & 0<x<1\\ e^2-1, & x\ge1\end{cases}$

then
(A) f(x) is continuous at $x=0$. (B) f(x) is not differentiable at $x=0$.
(C) f(x) is continuous at $x=1$. (D) $\lim\limits_{x\to0^+}[f(x)]=1$.

[**Note:** [k] denotes greatest integer less than or equal to k.]

DPP No.-29 to 32
SOLUTION

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

DPP. NO.-29

[SINGLE CORRECT CHOICE TYPE] **[8 × 3 = 24]**

Q.1 The value of $\lim\limits_{x\to 0} x^2\left(1+2+3+4+\ldots\ldots+\left[\frac{1}{|x|}\right]\right)$ equals

[**Note:** [k] denotes greatest integer function less than or equal to k.]

(A) 1 (B) 0 (C*) $\frac{1}{2}$ (D) 2

[Sol.$_{705/lcd/SC}$ $x^2\left(1+2+3+\ldots\ldots+\left[\frac{1}{|x|}\right]\right)=\frac{x^2}{2}\left[\frac{1}{|x|}\right]\left(\left[\frac{1}{|x|}\right]+1\right)$

$$\frac{1-|x|}{2}=\frac{x^2}{2}\left(\frac{1}{|x|}-1\right)\left(\frac{1}{|x|}\right)<x^2\left(1+2+\ldots\ldots+\left[\frac{1}{|x|}\right]\right)\le\frac{x^2}{2}\frac{1}{|x|}\frac{|x|+1}{|x|}=\frac{|x|+1}{2}$$

By sandwich theorem

$$\lim_{x\to 0} x^2\left(1+2+\ldots\ldots+\left[\frac{1}{|x|}\right]\right)=\frac{1}{2}$$ **Ans.**]

Q.2 If the sum of the coefficients in the expansion of $\left(\frac{1}{3}-15x+15x^3\right)^{2014}(17x-17x^5+3)^{2015}$ is n, then the number of dissimilar terms in the expansion of $(1+x)^n$ is

(A) 1 (B) 2 (C) 3 (D*) 4

[Sol.$_{159/bin/SC}$ put $x = 1$
$n = 3$
no. of dissimilar terms is 4]

Q.3 Number of integral value of x satisfying $\log_4|x|<1$ is equal to

(A) 4 (B) 5 (C*) 6 (D) 7

[Sol.$_{340/qe/SC}$ We have $\log_4|x|<1 \Rightarrow |x|<4 \Rightarrow -4<x<4-\{0\}$
$\Rightarrow x\in\{-3,-2,-1,1,2,3\} \Rightarrow$ 6 integral values. **Ans.**]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Q.4 The angle which is equivalent to $4\tan^{-1}\left(\frac{1}{5}\right)-\tan^{-1}\left(\frac{1}{239}\right)$ is equal to

(A) $\frac{\pi}{6}$ (B*) $\frac{\pi}{4}$ (C) $\tan^{-1}\left(\frac{4}{5}\right)$ (D) $\tan^{-1}\left(\frac{3}{5}\right)$

[Sol.$_{158/itf/SC}$

$\alpha = 2\tan^{-1}\left(\frac{2/5}{1-1/25}\right)-\tan^{-1}\left(\frac{1}{239}\right)$ $\left[2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right),\ -1<x<1\right]$

$= 2\tan^{-1}\left(\frac{5}{12}\right)-\tan^{-1}\left(\frac{1}{239}\right) = \tan^{-1}\left(\frac{5/6}{1-25/144}\right)-\tan^{-1}\left(\frac{1}{239}\right)$

$\alpha = \tan^{-1}\left(\frac{120}{119}\right)-\tan^{-1}\left(\frac{1}{239}\right)$ ....(1)

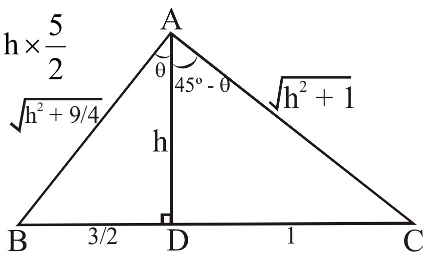
But $\frac{\pi}{4}+\tan^{-1}\left(\frac{1}{239}\right) = \tan^{-1}1 + \tan^{-1}\left(\frac{1}{239}\right) = \tan^{-1}\left(\frac{240/239}{1-1/239}\right) = \tan^{-1}\left(\frac{120}{119}\right)$

$\therefore\ \tan^{-1}\left(\frac{120}{119}\right)-\tan^{-1}\left(\frac{1}{239}\right) = \frac{\pi}{4}$. **Ans.**]

Q.5 In ΔABC with altitude AD, $\angle BAC = \frac{\pi}{4}$, $BD = \frac{3}{2}$ and $DC = 1$, the length of side AB is

(A*) $\frac{3\sqrt{5}}{2}$ (B) $\sqrt{3}$ (C) $3\sqrt{5}$ (D) 3

[Sol.$_{217/sot/SC}$ Equating the area of $\Delta = \frac{1}{2\sqrt{2}}\sqrt{h^2+\frac{9}{4}}\cdot\sqrt{h^2+1} = \frac{1}{2}\times h\times\frac{5}{2}$



$\left(h^2+\frac{9}{4}\right)(h^2+1) = \frac{25h^2}{2}$

$(4h^2+9)(h^2+1) = 50h^2$

$4h^4 + 13h^2 + 9 = 50h^2$

$4h^4 - 37h^2 + 9 = 0$

$4h^4 - 36h^2 - h^2 + 9 = 0$

$4h^2(h^2-9) - 1(h^2-9) = 0$

$h = \frac{1}{2}$ or $h = 3$

$AB = \sqrt{\frac{10}{4}} = \frac{\sqrt{10}}{2}$ or $\frac{3\sqrt{5}}{2}$]

Q.6 The value of $\sin\frac{\pi}{16}\sin\frac{3\pi}{16}\sin\frac{5\pi}{16}\sin\frac{7\pi}{16}$ is

(A*) $\frac{1}{8\sqrt{2}}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4\sqrt{2}}$ (D) $\frac{1}{4}$

[Sol.$_{334/ph-1/SC}$ Let $\frac{\pi}{16}=\theta$

$\therefore y=\sin\theta\ \sin3\theta\sin5\theta\sin7\theta$

$4y=(2\sin\theta.\sin7\theta).(2\sin3\theta\sin5\theta)$ $\qquad \frac{\pi}{2}=8\theta$

$=(2\sin\theta\cos\theta).(2\sin3\theta\cos3\theta)$ $\qquad \therefore 7\theta=\frac{\pi}{2}-\theta$ and $5\theta=\frac{\pi}{2}-3\theta$

$=\sin2\theta.\sin6\theta$ $\qquad$ and $6\theta=\frac{\pi}{2}-2\theta$

$8y=2\sin2\theta.\sin6\theta\ =\ 2\sin2\theta.\cos2\theta\ =\ \sin4\theta\ =\ \sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$

$\therefore y=\frac{1}{8\sqrt{2}}$.]

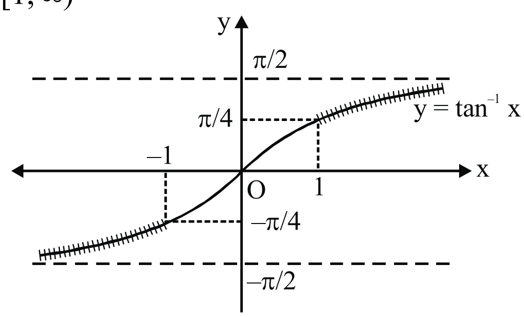
Q.7 The range of the function $f(x)=\sec^{-1}x+\tan^{-1}x+\text{cosec}^{-1}x$, is

(A) $\left(0,\frac{\pi}{2}\right)$ (B) $(0,\pi)$ (C) $\left(0,\frac{\pi}{4}\right)\cup\left[\frac{3\pi}{4},\pi\right)$ (D*) $\left(0,\frac{\pi}{4}\right]\cup\left[\frac{3\pi}{4},\pi\right)$

[Sol.$_{227/itf/SC}$ The domain of the function f(x) is $(-\infty,-1]\cup[1,\infty)$

$f(x)=\frac{\pi}{2}+\tan^{-1}x$

$\tan^{-1}x\in\left(\frac{-\pi}{2},\frac{-\pi}{4}\right]\cup\left[\frac{\pi}{4},\frac{\pi}{2}\right)$



Hence, range of f(x) is $\left(0,\frac{\pi}{4}\right]\cup\left[\frac{3\pi}{4},\pi\right)$.]

Q.8 If $\lim\limits_{x\to0}\frac{\ln\cot\left(\frac{\pi}{4}-k_1x\right)}{\tan k_2x}=1$ then

(A) $k_1=k_2$ (B*) $2k_1=k_2$ (C) $k_1=2k_2$ (D) $k_1=4k_2$

[Sol.$_{609/lcd(l)/SC}$ $\lim\limits_{x\to0}\frac{\left(\frac{\ln(1\cdot\cot k_1\ x+1)}{\cot k_1x-1}\right)}{\frac{\tan k_2}{k_2x}\cdot k_2x}=\lim\limits_{x\to0}\frac{\ln\left(\frac{1+\tan k_1x}{1-\tan k_1x}\right)}{k_2x}=1$

$$\Rightarrow \lim_{x\to 0} \frac{\ln\left(1+\frac{2\tan k_1x}{1-\tan k_1x}\right)}{\frac{2\tan k_1x}{1-\tan k_1x}} \cdot \frac{\left(\frac{2\tan k_1x}{1-\tan k_1x}\right)}{k_2x} = 1 \Rightarrow \frac{2k_1}{k_2} = 1 \Rightarrow 2k_1 = k_2 \textbf{ Ans. }]$$

DPP. NO.-30

[SINGLE CORRECT CHOICE TYPE] [5 × 3 = 15]

Q.1 The value of $\left(\text{cosec}^2(\cot^{-1}x) - \cot^2(\text{cosec}^{-1}x)\right)$ is equal to

(A) $2(1 + x^2)$ (B*) 2 (C) 2x (D) $2x^2$

[Sol.$_{157/itf/SC}$ As, $1 + \cot^2\theta = \text{cosec}^2\theta$

so, $\text{cosec}^2(\cot^{-1}x) = 1 + \cot^2(\cot^{-1}x)$

and $\cot^2(\text{cosec}^{-1}x) = \text{cosec}^2(\text{cosec}^{-1}x) - 1 = (x^2 - 1)$

so, $\text{cosec}^2(\cot^{-1}x) - \cot^2(\text{cosec}^{-1}x) = (1 + x^2)$

so, $\text{cosec}^2(\cot^{-1}x) - \cot^2(\text{cosec}^{-1}x) = (1 + x^2) - (x^2 - 1) = 1 + 1 = 2$ **Ans.**]

Q.2 If $e^x + e^y = e^{x+y}$ then $\frac{dy}{dx}$ is not equal to

(A) $e^{x-y}\left(\frac{1-e^y}{e^x-1}\right)$ (B) $\frac{1}{1-e^x}$ (C*) $e^y - 1$ (D) $\frac{-e^y}{e^x}$

[Sol.$_{341/mod/SC}$ $e^x + e^y = e^{x+y}$

$\Rightarrow e^x + e^y y' = e^xe^y + e^xe^y y'$

$$\Rightarrow y' = \frac{e^x(e^y-1)}{e^y(1-e^x)} = \frac{(e^y-1)(e^xe^y-e^y)}{e^y(1-e^x)} \Rightarrow y' = 1 - e^y$$

$$e^y = \frac{e^x}{e^x-1} \quad \Rightarrow \quad y' = \frac{1}{1-e^x} = \frac{-e^y}{e^x}. \quad]$$

Q.3 Assuming that $f(t) = \frac{t-1}{t+1}$ is an invertible function, then $f^{-1}\left(\frac{1}{c}+1\right)$ is equal to

(A*) $-(2c + 1)$ (B) $\frac{1}{(2+c)}$ (C) $2c - 1$ (D) $(2c + 1)$

[Sol.$_{156/func/SC}$ Let $f(t) = \frac{t-1}{t+1}$

$$ty + y = t - 1 \Rightarrow 1 + y = t(1 - y) \Rightarrow t = \frac{1+y}{1-y}$$

$$f^{-1}(y) = \frac{1+y}{1-y}$$

$$f^{-1}\left(\frac{1}{c}+1\right) = \frac{1+1+\frac{1}{c}}{1-1-\frac{1}{c}} = \frac{2+\frac{1}{c}}{\frac{-1}{c}} \Rightarrow f^{-1}\left(\frac{1}{c}+1\right) = -(2c+1). \textbf{ Ans.}]$$

Q.4 The number of solutions of the equation
$\log(x^{2016} + 1) + \log(1 + x^2 + x^4 + \ldots\ldots + x^{2014}) = \log 2016 + 2015 \log x$
is equal to
(A*) 1 (B) 2 (C) 3 (D) infinite

[Sol.$_{312/log/SC}$ Given equation can be written as
$(x^{2016} + 1)(1 + x^2 + x^4 + \ldots\ldots + x^{2014}) = 2016\, x^{2015}$

$$\Rightarrow \left(x + \frac{1}{x^{2015}}\right)(1 + x^2 + x^4 + \ldots\ldots + x^{2014}) = 2016$$

$$\Rightarrow x + x^3 + x^5 + \ldots\ldots + x^{2015} + \frac{1}{x^{2015}} + \frac{1}{x^{2013}} + \ldots\ldots + \frac{1}{x} = 2016$$

$$\therefore \left(x + \frac{1}{x}\right) + \left(x^3 + \frac{1}{x^3}\right) + \ldots\ldots \left(x^{2015} + \frac{1}{x^{2015}}\right) \geq 2 \times 1008$$
$$\geq 2016$$

$$\therefore \left(x + \frac{1}{x}\right) + \left(x^3 + \frac{1}{x^3}\right) + \ldots\ldots \left(x^{2015} + \frac{1}{x^{2015}}\right) = 2016$$

For $x = 1$]

Q.5 If length of the perpendicular from the origin upon the tangent drawn to the curve $x^2 - xy + y^2 + \alpha(x - 2) = 4$ at $(2, 2)$ is equal to 2 then α equals
(A*) –2 (B) 0 (C) 2 (D) 4

[Sol.$_{264/st.line/SC}$ Tangent at (2, 2) to the curve is $x\left(1 + \frac{\alpha}{2}\right) + y - \alpha - 4 = 0$

$$p = \left|\frac{\alpha + 4}{\sqrt{\left(1 + \frac{\alpha}{2}\right)^2 + 1}}\right| = 2$$

$$\Rightarrow (\alpha + 4)^2 = 4\left[\left(1 + \frac{\alpha}{2}\right)^2 + 1\right] \Rightarrow (\alpha + 4)^2 = 4\left[1 + \frac{\alpha}{2} + \alpha + 1\right]$$

$\Rightarrow \alpha^2 + 16 + 8\alpha = 8 + \alpha^2 + 4\alpha \Rightarrow 4\alpha = -8 \Rightarrow \alpha = -2$. **Ans.**]

[INTEGER TYPE]

[1 × 5 = 5]

Q.6 Let $f(x) = \sum_{r=1}^{9} (-1)^{r-1} \cos(rx)\cos((r+1)x)$. If $l = \lim_{x \to 0} \frac{e^{1-f(x)} - 1}{12\tan^2 x}$, then $[\,l\,]$ is
[**Note :** [k] denotes greatest integer value less than or equal to k.] [Ans. 4]

[Sol.$_{50076/lcd(l)/OMR}$ $f(x) = \cos x \cos 2x - \cos 2x \cos 3x + \cos 3x \cos 4x + \ldots\ldots + \cos 9x \cos 10x$

$f(0) = 1$

$$l = \lim_{x\to 0}\frac{e^{1-f(x)}-1}{12x^2} = \lim_{x\to 0}\frac{1-f(x)}{12x^2}$$

$$= \lim_{x\to 0}\frac{1-(\cos x\cdot\cos 2x - \cos 2x\cos 3x + \ldots\ldots + \cos 9x\cos 10x)}{12x^2}$$

$$= \frac{1^2+2^2-(2^2+3^2)+(3^2+4^2)+\ldots\ldots+(9^2+10^2)}{24}$$

$$= \frac{101}{24}$$

$\therefore\ [l] = 4$ **Ans.**]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

DPP. NO.-31

[SINGLE CORRECT CHOICE TYPE] **[8 × 3 = 24]**

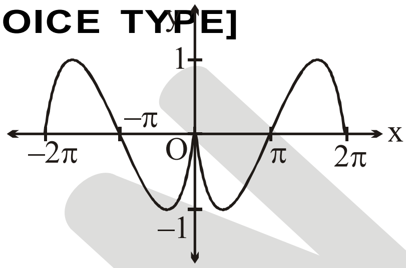
Q.1 Which function corresponds to the given graph ?

(A) $y = |\sin x|$

(B) $y = \sin|x|$

(C) $y = -|\sin x|$

(D*) $y = -\sin|x|$



[Sol.$_{155/func/SC}$]

Q.2 If $(a-2)^2 - 2k(a+1) = (a-1)^2 - 2k(a+2) = -1$, $k \in R$, then the value of $(k^2 + 6k)$ is equal to

(A*) $\frac{5}{4}$ (B) $\frac{1}{4}$ (C) $\frac{-3}{4}$ (D) 1

[Sol.$_{660/qe/SC}$ $(a-2)^2 - 2k(a+1) = -1$ ……(i)

$(a-1)^2 - 2k(a+1) = -1$ ……(ii)

Subtract (i) – (ii)

$\Rightarrow \quad 2k + 3 - 2a = 0$

$\Rightarrow \quad a = \left(\frac{3+2k}{2}\right)$

Substitute in equation (i)

$\Rightarrow \quad k^2 + 6k = \frac{5}{4}$ **Ans.**]

Q.3 If $\lim\limits_{x\to 0} \frac{\ln(3+x) - \ln(3-x)}{x} = k$, the value of k is

(A*) $\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) $-\frac{2}{3}$ (D) 0

[Sol.$_{47/lcd/SC}$ $\text{Limit}_{x\to 0} \frac{\ln(3+x) - \ln(3-x)}{x} = \text{Limit}_{x\to 0} \frac{\ln 3\left(1+\frac{x}{3}\right) - \ln 3\left(1-\frac{x}{3}\right)}{x} = \frac{\ln\left(1+\frac{x}{3}\right) - \ln\left(1-\frac{x}{3}\right)}{x}$

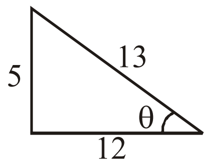
$= \text{Limit}_{x\to 0} \ln\left(1+\frac{x}{3}\right)^{1/x} - \ln\left(1-\frac{x}{3}\right)^{1/x} = \frac{1}{x}\cdot\frac{x}{3} - \frac{1}{x}\left(-\frac{x}{3}\right) = \frac{1}{3}+\frac{1}{3} = \frac{2}{3}$.]

Q.4 If $\operatorname{cosec}\theta + \cot\theta = 5$, then value of $\cos\theta$ is equal to

(A) $\frac{1}{5}$ (B) $\frac{5}{12}$ (C) $\frac{5}{13}$ (D*) $\frac{12}{13}$

[Sol.$_{341/ph-1/SC}$ $\operatorname{cosec}\theta + \cot\theta = 5$

$\therefore\ \text{cosec}\theta - \cot\theta = \dfrac{1}{5}$



$2\text{cosec}\theta = \dfrac{26}{5}$

$\therefore\ \text{cosec}\theta = \dfrac{13}{5}$

$\therefore\ \cos\theta = \dfrac{12}{13}$ Ans.]

Q.5 Let α and β $(\alpha > \beta)$ are two values of x satisfying the equation $(\tan^{-1}x)^2 = 1 + \dfrac{\pi^2}{16} - x^2$. Then the value of $(\sin^{-1}\alpha + \cot^{-1}\beta + \sec^{-1}\beta + \text{cosec}^{-1}\alpha)$, is

(A) 0 (B) 2π (C) $\dfrac{7\pi}{4}$ (D*) $\dfrac{11\pi}{4}$

[Sol.$_{159/itf/SC}$ $x^2 + (\tan^{-1}x)^2 = 1 + \dfrac{\pi^2}{16}$

$\therefore$ $x = \pm 1$

$\Rightarrow$ $\alpha = 1$ and $\beta = -1$

$\therefore$ value is $\dfrac{\pi}{2} + \dfrac{3\pi}{4} + \pi + \dfrac{\pi}{2} = \dfrac{11\pi}{4}$ **Ans.**]

Q.6 If $\sin^{-1}\alpha, \tan^{-1}\alpha, \cos^{-1}\alpha$ are in arithmetic progression then the value of $\lim\limits_{x\to\alpha}\left(\tan\left(\dfrac{\pi}{4} + \ln x\right)\right)^{\frac{1}{\ln x}}$ equal to

(A) e^{-1} (B*) e^2 (C) e^{-2} (D) e^3

[Sol.$_{573/lcd/SC}$ $2\tan^{-1}\alpha = \sin^{-1}\alpha + \cos^{-1}\alpha$

$\tan^{-1}\alpha = \dfrac{\pi}{4} \Rightarrow \alpha = 1$

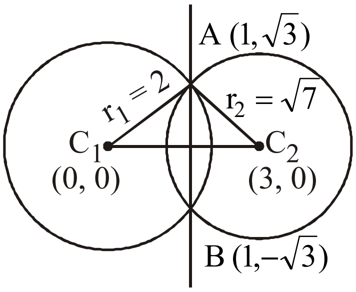
$\lim\limits_{x\to 1}\left(\dfrac{1+\tan(\ln x)}{1-\tan(\ln x)}\right)^{\frac{1}{\ln x}}$ (1^∞ form)

$e^{\lim\limits_{x\to 1}\frac{2\tan(\ln x)}{\ln x}} = e^2$. **Ans.**]

Q.7 The length of the common chord of the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x + 2 = 0$, is

(A) $\sqrt{3}$ (B*) $2\sqrt{3}$ (C) 1 (D) 2

[Sol.$_{60/cir/SC}$



]

Q.8 Let $f(x) = \dfrac{\sqrt{\text{sgn}\,(\alpha x^2 + \alpha x + 1)}}{\cot^{-1}(x^2 - \alpha)}$. If $f(x)$ is continuous for all $x \in R$, then number of integers in the range of α, is

(A) 0 (B*) 4 (C) 5 (D) 6

[**Note :** sgn k denotes signum function of k.]

[Sol.$_{301/lcd/SC}$ $\alpha x^2 + \alpha x + 1 \geq 0 \ \forall \ x \in R$

$D \leq 0$

$\alpha^2 - 4\alpha \leq 0 \Rightarrow \alpha(\alpha - 4) \leq 0 \Rightarrow \alpha \in [0, 4]$ also $\alpha \neq 4$ (think!).]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

DPP. NO.-32

Q.1 Evaluate $\lim\limits_{x\to\infty}\left(\frac{x}{e}-x\left(\frac{x}{x+1}\right)^x\right)$ [Ans. $\frac{-1}{2e}$] **[3]**

[Sol.$_{80005/lcd/SUB}$ $L = -\lim\limits_{x\to\infty} x\left(\left(\frac{x}{x+1}\right)^x-\frac{1}{e}\right) = -\lim\limits_{x\to\infty} x\left(e^{x\ln\left(\frac{x}{x+1}\right)}-e^{-1}\right)$

Put $x=\frac{1}{y}$

$$= \frac{-1}{e}\lim_{y\to0}\frac{1}{y}\left[e^{1+\frac{\ln\left(\frac{1}{1+y}\right)}{y}}-1\right] = \frac{-1}{e}\lim_{y\to0}\frac{1}{y}\left[e^{1-\frac{\ln(1+y)}{y}}-1\right] = -\frac{1}{e}\lim_{y\to0}\frac{1}{y}\left[e^{\frac{y-\ln(1+y)}{y}}-1\right]$$

$$= -\frac{1}{e}\lim_{y\to0}\frac{y-\ln(1+y)}{y^2} = \frac{-1}{2e}$$ **Ans.**]

[SINGLE CORRECT CHOICE TYPE] [3 × 3 = 9]

Q.2 The number of prime numbers in the range of the function, $f(x)=\frac{96}{7(x^4+3x^2+1)}$ is

(A) 4 (B*) 6 (C) 13 (D) 14

[Sol.$_{195/func/SC}$ $f(x)=\frac{96}{7(x^4+3x^2+1)}$

Range of the function is $\left(0, \frac{96}{7}\right]$

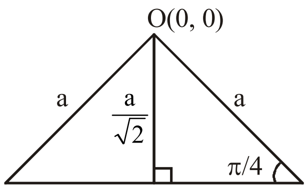
Integers in the range are $1, 2, 3, \ldots\ldots 13$]

Q.3 Two mutually perpendicular straight lines through origin form an isosceles triangle with the line $2x+y=5$, then the area of triangle is

(A*) 5 (B) 3 (C) $\frac{5}{2}$ (D) 1

[Sol.$_{302/st.line/SC}$ $\because \frac{a}{\sqrt{2}}=\sqrt{5} \Rightarrow a=\sqrt{10}$

Area $=\frac{1}{2}a^2=5$



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Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Q.4 Let S_1, S_2, S_3 be the respective sums of first n, 2n and 3n terms of the same arithmetic progression with a as the first term and d as the common difference. If $R = S_3 - S_2 - S_1$, then R depends on

(A) a and d (B*) d and n (C) a and n (D) a, d and n

[Sol.$_{239/seq/SC}$ $R = S_3 - S_2 - S_1$

$$= \frac{3n}{2}\left(2a + (3n-1)d\right) - \frac{2n}{2}\left(2a + (2n-1)d\right) - \frac{n}{2}\left(2a + (n-1)d\right)$$

$$= 2a\left(\frac{3n}{2} - \frac{2n}{2} - \frac{n}{2}\right) + \frac{nd}{2}\left(3(3n-1) - 2(2n-1) - 1(n-1)\right)$$

$$= \frac{nd}{2}(4n) = 2n^2 d$$]

[MULTIPLE CORRECT CHOICE TYPE] [2 × 4 = 8]

Q.5 If the independent variable x is changed to y then the differential equation $x\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 - \frac{dy}{dx} = 0$ is changed to $x\frac{d^2x}{dy^2} + \left(\frac{dx}{dy}\right)^2 = k$ where k is equal to

(A) $\lim_{x\to 0}\left[\frac{2\tan x}{x}\right]$ (B*) $\lim_{x\to 0}\left[\frac{2x}{\tan x}\right]$ (C*) $\lim_{x\to 0}\left[\frac{\tan x}{x}\right]$ (D*) 1

[**Note:** [y] denotes greatest integer function less than or equal to y.]

[Sol.$_{40520/mod/MORE}$ $\frac{dy}{dx} = \frac{1}{dx/dy}$; $\frac{d^2y}{dx^2} = \frac{d}{dy}\left(\frac{1}{dx/dy}\right)\cdot\frac{dy}{dx} = -\frac{1}{(dx/dy)^3}\frac{d^2x}{dy^2}$

hence $x\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 - \frac{dy}{dx} = 0$

becomes $-x\cdot\frac{1}{(dx/dy)^3}\frac{d^2x}{dy^2} + \frac{1}{(dx/dy)^3} - \frac{1}{(dx/dy)} = 0$

$$x\frac{d^2x}{dy^2} - 1 + \left(\frac{dx}{dy}\right)^2 = 0 \quad\Rightarrow\quad x\frac{d^2x}{dy^2} + \left(\frac{dx}{dy}\right)^2 = 1$$

$\therefore$ k = 1 **Ans.**]

Q.6 Let $f(x) = \begin{cases} \frac{\ln(1+2x)}{x}, & \frac{-1}{2} < x < 0 \\ 2\cos x, & x = 0 \\ \frac{e^{2x}-1}{x}, & 0 < x < 1 \\ e^2 - 1, & x \ge 1 \end{cases}$

then

(A*) f(x) is continuous at x = 0. (B*) f(x) is not differentiable at x = 0.

(C*) f(x) is continuous at x = 1. (D) $\lim_{x\to 0^+} [f(x)] = 1$.

[**Note:** [k] denotes greatest integer less than or equal to k.]

[Sol.$_{40001/lcd/MORE}$ $f(0^-) = 2$, $f(0^+) = 2$, $f(0) = 2$; $f'(0^+) = 2$, $f'(0^-) = -2$

$f(1^+) = f(1) = f(1^-) = e^2 - 1$ (each)

Also, $\lim_{x\to 0^+} [f(x)] = 2$.]