DPP No.-25 TO 28

DPP. NO.-25

[SINGLE CORRECT CHOICE TYPE] [5 × 3 = 15]

Q.1 The value of $\lim_{x\to 1}\left(\frac{4}{\pi}\tan^{-1}x\right)^{\frac{1}{(x^2-1)}}$ is equal to

(A) $\frac{1}{\pi}$ (B) $\frac{-1}{\pi}$ (C) $e^{\frac{1}{\pi}}$ (D) $e^{\frac{-1}{\pi}}$

Q.2 If f and g are two functions with $g(x) = x - \frac{1}{x}$ and $fog(x) = x^3 - \frac{1}{x^3}$, then f'(x) is

(A) $3x^2 + 3$ (B) $x^2 - \frac{1}{x^2}$ (C) $1 + \frac{1}{x^2}$ (D) $3x^2 + \frac{3}{x^4}$

Q.3 For $x \in (0, 1)$, let $\alpha = \sin^{-1}x$, $\beta = x$, $\gamma = \tan^{-1}x$, $\delta = \cot^{-1}x - \frac{\pi}{2}$

then identify the correct relation ?

(A) $\alpha > \beta > \delta > \gamma$ (B) $\beta > \alpha > \gamma > \delta$ (C) $\alpha > \beta > \gamma > \delta$ (D) $\beta > \alpha > \delta > \gamma$

Q.4 Let $f'(x) = \sin x^2$ and $y = f(x^2 + 1)$ then $\frac{dy}{dx}$ at $x = 2$ is

(A) 0 (B) 1 (C) 4 sin 5 (D) 4 sin 25

Q.5 Point P lies in a triangle ABC which is right angled at B. If each side subtends an angle of $\frac{2\pi}{3}$ at P and PA = 10, PB = 6, then PC is equal to

(A) 11 (B) 22 (C) 33 (D) 44

[INTEGER TYPE / SUBJECTIVE] [1 × 5 = 5]

Q.6 Let x, y be real number such that $x\cos^2 39° = \tan 26° \cot 39° \tan 86° \cos 78° \tan 34°$ and $y\cos 81° = \cos 36° - \sin 36°$, then find the value of $x + y^2$.

DPP. NO.-26

[SINGLE CORRECT CHOICE TYPE] [4 × 3 = 12]

Q.1 If $y = x + e^x$, then $\frac{d^2x}{dy^2}$ is equal to

(A) e^x (B) $-\frac{e^x}{(1+e^x)^3}$ (C) $-\frac{e^x}{(1+e^x)^2}$ (D) $\frac{1}{(1+e^x)^2}$

Q.2 If $f(x) = 2ax - x^2 - 4$ has a positive integral value at only one real x, then product of possible values of a is

(A) 5 (B) 9 (C) $\frac{-9}{2}$ (D) -5

Q.3 Let $P(n) = \prod_{n=2}^{n}\left(1 - \frac{4}{(2n-1)^2}\right)$ then $\lim_{n\to\infty} P(n)$ equals

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{5}$ (D) $\frac{1}{6}$

Q.4 Let f and g be differentiable function on R such that $g(1) = 1 = g'(1)$ and $f'(0) = 3$.
If $h(x) = f\left(2x\, g(x) + \cos \pi x - 1\right)$, then $h'(1)$ equals

(A) 8 (B) 12 (C) 24 (D) 36

[MULTIPLE CORRECT CHOICE TYPE] [2 × 4 = 8]

Q.5 Let $f(x) = 2|x|^3 + 3x^2 - 12|x| + 1$, where $x \in [-1, 2]$, then the greatest value of $f(x)$ is more than

(A) 7 (B) 6 (C) 3 (D) 4

Q.6 Miss C has written a result that $\sin^{-1}(\sin 7\, \text{sgn}(P)) < 6 - 2\pi$ but she forget to write the condition on P, then

(A) Miss C is right if $P > 0$ (B) Miss C is wrong if $P > 0$
(C) Miss C is right if $P < 0$ (D) Miss C is wrong if $P < 0$

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

DPP. NO.-27

[SINGLE CORRECT CHOICE TYPE] [4 × 3 = 12]

Q.1 $\lim\limits_{x\to 0^+}\left((\sin x)^{\frac{1}{x}}+(\cos x)^{\frac{1}{x}}+(\operatorname{cosec} x)^{x}\right)$ is equal to

(A) 0 (B) 1 (C) 2 (D) e + 1

Q.2 The sum of series $\tan^{-1}\left(\frac{4}{1+3\times4}\right)+\tan^{-1}\left(\frac{6}{1+8\times9}\right)+\tan^{-1}\left(\frac{8}{1+15\times16}\right)+\ldots\ldots\infty$ is equal to

(A) $\cot^{-1}2$ (B) $\tan^{-1}2$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

Q.3 The graph of the function $y = f(x)$ has a unique tangent at $(e^a, 0)$ through which the graph passes then $\lim\limits_{x\to e^a}\frac{\ln(1+7f(x))-\sin(f(x))}{3f(x)}$, is equal to

(A) 1 (B) 3 (C) 2 (D) 7

Q.4 If f is twice differentiable function on R and $g(x) = f(x)\cdot\sin\left(\frac{x}{2}\right)$ (where $f'(2\pi)=1$), then $g''(2\pi)$ is

(A) 1 (B) –1 (C) 0 (D) 2

[INTEGER TYPE] [2 × 4 = 8]

Q.5 If $x, y, z > 0$ and $x+y+z=1$, then find the least value of $E=\frac{2x}{1-x}+\frac{2y}{1-y}+\frac{2z}{1-z}$.

Q.6 If $f(x)=x^3+px^2+qx+r=0$ has three distinct non negative integral roots and $(x^2-4x+7)^3+p(x^2-4x+7)^2+q(x^2-4x+7)+r=0$ has no real roots, then find the value of $(p+2q+r)$.

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

DPP. NO.-28

[SINGLE CORRECT CHOICE TYPE] [5 × 3 = 15]

Q.1 The number of solutions to the equation $\frac{\pi}{2}|\sin^{-1}(\sin x)| = \left|x^2 - \pi|x|\right|$ is equal to

(A) 3 (B) 4 (C) 5 (D) 6

Q.2 If the domain of the function $f(x) = \sqrt{12 - 3^x - 3^{3-x}} + \sin^{-1}\left(\frac{2x}{3}\right)$ is [a, b], then $3a + 2b$ is equal to

(A) 4 (B) 5 (C) 6 (D) 7

Q.3 $\lim_{x\to\infty}(e^{2x} + e^x + x)^{\frac{1}{x}}$ equals

(A) e^{-2} (B) e (C) e^2 (D) e^4

Q.4 If a function $f: R \to R$ is defined such that $f(x) = \text{sgn}\left((x^2 - ax + 2)(2x^2 + bx + 3)\right)$. If for exactly two distinct values of $\alpha \in R$, $\lim_{x\to\alpha} f(x)$ exists but $f(x)$ is non derivable at $x = \alpha$, then $a^2 + b^2$ is equal to

(A) 8 (B) 16 (C) 32 (D) 44

Q.5 Let h(x) = x g(x) where g is the inverse of f(x). Also the values of f(x) and f'(x) are given as

x	2	3	5
f(x)	4	5	1
f'(x)	−1	2	3

then the value of h'(5) equals

(A) $\frac{11}{2}$ (B) $\frac{9}{2}$ (C) 5 (D) $\frac{5}{2}$

[INTEGER TYPE] [1 × 4 = 4]

Q.6 Find the value of $\lim_{x\to 1}\frac{\left[\sum_{K=1}^{100} x^k\right] - 100}{x - 1}$.

DPP No.-25 TO 28 (SOLUTIONS)

DPP. NO.-25

[SINGLE CORRECT CHOICE TYPE] [5 × 3 = 15]

Q.1 The value of $\lim\limits_{x\to1}\left(\frac{4}{\pi}\tan^{-1}x\right)^{\frac{1}{(x^2-1)}}$ is equal to

(A) $\frac{1}{\pi}$ (B) $\frac{-1}{\pi}$ (C*) $e^{\frac{1}{\pi}}$ (D) $e^{\frac{-1}{\pi}}$

[Sol.$_{517/lcd(l)/SC}$ $\lim\limits_{x\to1}\left(\frac{4}{\pi}\tan^{-1}x\right)^{\frac{1}{x^2-1}} = e^{\lim\limits_{x\to1}\frac{1}{x^2-1}\left(\frac{4}{\pi}\tan^{-1}x-1\right)} = e^{\frac{1}{\pi}}$,

as $\lim\limits_{x\to1}\frac{\left(\frac{4}{\pi}\tan^{-1}x-1\right)}{x^2-1} = \frac{4}{\pi}\lim\limits_{x\to1}\left(\frac{\tan^{-1}x-\frac{\pi}{4}}{x^2-1}\right) = \frac{1}{\pi}$. **Ans.**]

Q.2 If f and g are two functions with $g(x) = x - \frac{1}{x}$ and $fog(x) = x^3 - \frac{1}{x^3}$, then f'(x) is

(A*) $3x^2 + 3$ (B) $x^2 - \frac{1}{x^2}$ (C) $1 + \frac{1}{x^2}$ (D) $3x^2 + \frac{3}{x^4}$

[Sol.$_{280/mod/SC}$ $t = x - \frac{1}{x}$

$t^3 = x^3 - \frac{1}{x^3} - 3t \Rightarrow f(t) = t^3 + 3t \Rightarrow f'(t) = 3t^2 + 3.$]

Q.3 For $x \in (0, 1)$, let $\alpha = \sin^{-1}x$, $\beta = x$, $\gamma = \tan^{-1}x$, $\delta = \cot^{-1}x - \frac{\pi}{2}$

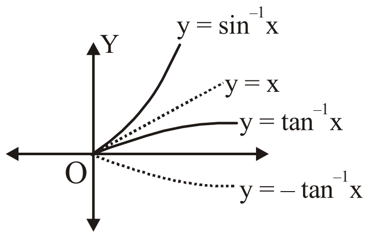
then identify the correct relation ?

(A) $\alpha > \beta > \delta > \gamma$ (B) $\beta > \alpha > \gamma > \delta$ (C*) $\alpha > \beta > \gamma > \delta$ (D) $\beta > \alpha > \delta > \gamma$

[Sol.$_{154/itf/SC}$ For $x \in (0, 1)$

$\sin^{-1}x > x > \tan^{-1}x > \cot^{-1}x - \frac{\pi}{2}$

$\cot^{-1}x - \frac{\pi}{2} = -(\tan^{-1}x)$.]



Q.4 Let $f'(x) = \sin x^2$ and $y = f(x^2 + 1)$ then $\frac{dy}{dx}$ at $x = 2$ is

(A) 0 (B) 1 (C) 4 sin 5 (D*) 4 sin 25

[Sol.$_{202/mod/SC}$ $f'(x) = \sin x^2$

$y = f(x^2 + 1)$

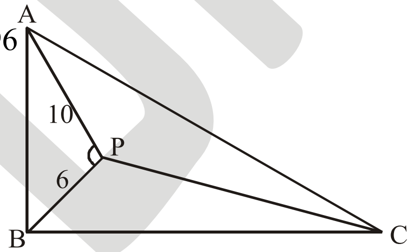
$\frac{dy}{dx} = f'(x^2 + 1) \cdot 2x$

$\left.\frac{dy}{dx}\right|_{x=2} = 4 f'(5) = 4 \sin 25.$]

Q.5 Point P lies in a triangle ABC which is right angled at B. If each side subtends an angle of $\frac{2\pi}{3}$ at P and PA = 10, PB = 6, then PC is equal to

(A) 11 (B) 22 (C*) 33 (D) 44

[Sol.$_{262/sot/SC}$ In ΔAPB, $\cos 120° = \frac{10^2 + 6^2 - AB^2}{2 \cdot 10 \cdot 6}$ $\Rightarrow$ $AB^2 = 196$



In ΔBPC, $\cos 120° = \frac{6^2 + PC^2 - BC^2}{2 \cdot 6 \cdot PC}$

$\Rightarrow$ $BC^2 = 36 + PC^2 + 6PC$

Similarly, $AC^2 = 100 + PC^2 + 10PC$

$AC^2 - BC^2 = 64 + 4PC = AB^2 = 196$

$4PC = 132$

$PC = 33$ **Ans.**]

[INTEGER TYPE / SUBJECTIVE] [1 × 5 = 5]

Q.6 Let x, y be real number such that $x \cos^2 39° = \tan 26° \cot 39° \tan 86° \cos 78° \tan 34°$ and $y \cos 81° = \cos 36° - \sin 36°$, then find the value of $x + y^2$.

[Ans. 4]

[Sol.$_{50090/ph-1/OMR}$ $x = (\tan 26° \tan 86° \tan 34°) \frac{\cos 39° \cos 78°}{\sin 39° \cos^2 39°}$

$= \frac{2 \tan 78° \cos 78°}{2 \sin 39° \cos 39°} = \frac{2 \tan 78°}{\tan 78°} = 2$

$\frac{\sqrt{2}}{\sqrt{2}} (\cos 36° - \sin 36°) = \sqrt{2} \sin (45° - 36°)$

$= \sqrt{2} \sin 9° = \sqrt{2} \cos 81°$

$\Rightarrow$ $y = \sqrt{2}$

$\Rightarrow$ $x + y^2 = 4$ **Ans.**]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

DPP. NO.-26

[SINGLE CORRECT CHOICE TYPE] **[4 × 3 = 12]**

Q.1 If $y = x + e^x$, then $\dfrac{d^2x}{dy^2}$ is equal to

(A) e^x (B*) $-\dfrac{e^x}{(1+e^x)^3}$ (C) $-\dfrac{e^x}{(1+e^x)^2}$ (D) $\dfrac{1}{(1+e^x)^2}$

[Sol.$_{273/mod/SC}$ $\dfrac{d^2x}{dy^2} = \dfrac{-\dfrac{d^2y}{dx^2}}{\left(\dfrac{dy}{dx}\right)^3} = -\dfrac{e^x}{(1+e^x)^3}$ **Ans.**]

Q.2 If $f(x) = 2ax - x^2 - 4$ has a positive integral value at only one real x, then product of possible values of a is

(A) 5 (B) 9 (C) $\dfrac{-9}{2}$ (D*) -5

[Sol.$_{620/qe/SC}$ $f(x) = 2ax - x^2 - 4$

$\dfrac{-D}{4a} = 1$

$\dfrac{-(4a^2 - 4\cdot 4)}{-4} = 1 \quad \Rightarrow \quad 4a^2 - 16 = 4 \quad \Rightarrow \quad 4a^2 = 20 \quad \Rightarrow \quad a^2 = 5 \Rightarrow a = \pm\sqrt{5}$

$\therefore$ product = –5 **Ans.**]

Q.3 Let $P(n) = \prod\limits_{n=2}^{n}\left(1 - \dfrac{4}{(2n-1)^2}\right)$ then $\lim\limits_{n\to\infty} P(n)$ equals

(A) $\dfrac{1}{4}$ (B*) $\dfrac{1}{3}$ (C) $\dfrac{1}{5}$ (D) $\dfrac{1}{6}$

[Sol.$_{58/lcd/SC}$ Given, $P(n) = \prod\limits_{n=2}^{n}\left(1 - \dfrac{4}{(2n-1)^2}\right)$

So, $P(n) = \prod\limits_{n=2}^{n}\dfrac{(2n-1)^2 - 4}{(2n-1)^2} = \prod\limits_{n=2}^{n}\dfrac{(2n-3)(2n+1)}{(2n-1)^2}$

Now, $P(n) = \prod\limits_{n=2}^{n}\left(\dfrac{2n+1}{2n-1}\right)\cdot\prod\limits_{n=2}^{n}\left(\dfrac{2n-3}{2n-1}\right) = \dfrac{5\cdot 7\cdot 9.....(2n-1)\,(2n+1)}{3\cdot 5\cdot 7........(2n-1)}\cdot\dfrac{1\cdot 3\cdot 5.....(2n-3)}{3\cdot 5\cdot 7........(2n-1)}$

$\Rightarrow \quad P(n) = \dfrac{(2n+1)}{3}\cdot\dfrac{1}{2n-1}$

Hence, $\lim\limits_{n\to\infty} P(n) = \dfrac{1}{3}$ **Ans.**]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Q.4 Let f and g be differentiable function on R such that $g(1) = 1 = g'(1)$ and $f'(0) = 3$.
If $h(x) = f\left(2x\,g(x) + \cos\pi x - 1\right)$, then $h'(1)$ equals
(A) 8 (B*) 12 (C) 24 (D) 36

[Sol.$_{324/mod/SC}$ $h'(x) = f'\left(2x\,g(x) + \cos\pi x - 1\right)[2xg'(x) + 2g(x) - \sin\pi x \cdot \pi]$
$h'(1) = f'(2 - 1 - 1) \cdot \left(2g'(1) + 2g(1)\right)$
$= 3 \cdot (2 + 2) = 12$ **Ans.**]

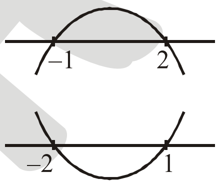
[MULTIPLE CORRECT CHOICE TYPE] **[2 × 4 = 8]**

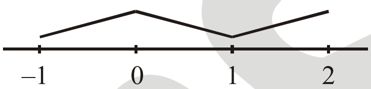
Q.5 Let $f(x) = 2|x|^3 + 3x^2 - 12|x| + 1$, where $x \in [-1, 2]$, then the greatest value of $f(x)$ is more than
(A) 7 (B) 6 (C*) 3 (D*) 4

[Sol.$_{40039/func/MORE}$ $f(x) = \begin{cases} -2x^3 + 3x^2 + 12x + 1; & -1 \le x < 0 \\ 2x^3 + 3x^2 - 12x + 1; & 0 \le x < 2 \end{cases}$

$f'(x) = \begin{cases} -6(x^2 - x - 2); & -1 \le x < 0 \\ 6(x^2 + x - 2); & 0 \le x \le 2 \end{cases}$

$f'(x) = \begin{cases} -6(x-2)(x+1); & -1 \le x < 0 \\ 6(x+2)(x-1); & 0 < x \le 2 \end{cases}$





$f(0) = 1 \Rightarrow f_{max} = 5$
$f(2) = 5$. **Ans.**]

Q.6 Miss C has written a result that $\sin^{-1}(\sin 7\,\text{sgn}\,(P)) < 6 - 2\pi$ but she forget to write the condition on P, then
(A) Miss C is right if $P > 0$ (B*) Miss C is wrong if $P > 0$
(C*) Miss C is right if $P < 0$ (D) Miss C is wrong if $P < 0$

[Sol.$_{40040/itf/MORE}$ (i) If $P > 0$ $\sin^{-1}(\sin 7) < 6 - 2\pi \Rightarrow 7 - 2\pi < 6 - 2\pi$ which is wrong
(ii) If $P < 0$ $\sin^{-1}(\sin(-7)) < 6 - 2\pi \Rightarrow -7 + 2\pi < 6 - 2\pi \Rightarrow 4\pi < 13$ which is right]

Class:XII **DPP. NO.-27**

[SINGLE CORRECT CHOICE TYPE] [4 × 3 = 12]

Q.1 $\lim\limits_{x\to 0^+}\left((\sin x)^{\frac{1}{x}}+(\cos x)^{\frac{1}{x}}+(\operatorname{cosec} x)^{x}\right)$ is equal to

(A) 0 (B) 1 (C*) 2 (D) e + 1

[Sol.$_{299/mod/SC}$ $\lim\limits_{x\to 0^+}(\sin x)^{\frac{1}{x}}=0 \;\; (0^{\infty}=0)$

$$\lim_{x\to 0^+}(\cos x)^{\frac{1}{x}} = e^{\lim\limits_{x\to 0}\frac{1}{x}(\cos x-1)} = e^0 = 1$$

Let $y=\lim\limits_{x\to 0^+}(\operatorname{cosec} x)^{x} \Rightarrow \ln y=\lim\limits_{x\to 0^+} x\ln\operatorname{cosec} x = \lim\limits_{x\to 0^+}\dfrac{-\ln\sin x}{\frac{1}{x}}$

$$\Rightarrow \ln y=\lim_{x\to 0^+}\frac{-\cot x}{\frac{-1}{x^2}}=\lim_{x\to 0}\frac{x^2}{\tan x}=0 \Rightarrow y=1$$

$\therefore$ Given limit = 0 + 1 + 1 = 2 **Ans.**]

Q.2 The sum of series $\tan^{-1}\left(\frac{4}{1+3\times 4}\right)+\tan^{-1}\left(\frac{6}{1+8\times 9}\right)+\tan^{-1}\left(\frac{8}{1+15\times 16}\right)+\ldots\ldots\infty$ is equal to

(A*) $\cot^{-1}2$ (B) $\tan^{-1}2$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

[Sol.$_{192/itf/SC}$ $T_n=\tan^{-1}\left(\dfrac{2n+2}{1+(n^2+2n)\cdot(n+1)^2}\right)=\tan^{-1}\left(\dfrac{(n+1)(n+2)-n(n+1)}{1+(n+1)(n+2)\cdot n(n+1)}\right)$

$$T_n=\left(\tan^{-1}((n+1)(n+2))-\tan^{-1}(n(n+1))\right)$$

$$S_n=\left(\tan^{-1}((n+1)(n+2))-\tan^{-1}2\right)$$

$$\therefore \sum_{n=1}^{\infty}T_n=\left(\frac{\pi}{2}-\tan^{-1}2\right)=\cot^{-1}2.$$]

Q.3 The graph of the function $y=f(x)$ has a unique tangent at $(e^a, 0)$ through which the graph passes then $\lim\limits_{x\to e^a}\dfrac{\ln(1+7f(x))-\sin(f(x))}{3f(x)}$, is equal to

(A) 1 (B) 3 (C*) 2 (D) 7

[Sol.$_{788/lcd/SC}$ Given $f(e^a)=0$

$$\lim_{x\to e^a}\frac{ln(1+7f(x))-\sin(f(x))}{3f(x)}=\frac{1}{3}\lim_{x\to e^a}\left(7\cdot\frac{ln(1+7f(x))}{7f(x)}-\frac{\sin(f(x))}{f(x)}\right)=\frac{1}{3}(7-1)=2\ \textbf{Ans.}]$$

Q.4 If f is twice differentiable function on R and $g(x)=f(x)\cdot\sin\left(\frac{x}{2}\right)$ $(\text{where } f'(2\pi)=1)$, then $g''(2\pi)$ is

(A) 1 (B*) –1 (C) 0 (D) 2

[Sol.$_{315/mod/SC}$ $g'(x)=f'(x)\cdot\sin\left(\frac{x}{2}\right)+f(x)\cdot\frac{1}{2}\cos\left(\frac{x}{2}\right)$

$$g''(x)=f''(x)\cdot\sin\left(\frac{x}{2}\right)+f'(x)\cdot\frac{1}{2}\cos\left(\frac{x}{2}\right)+\frac{1}{2}f'(x)\cdot\cos\left(\frac{x}{2}\right)-\frac{1}{2}\cdot f(x)\cdot\frac{1}{2}\sin\left(\frac{x}{2}\right)$$

$g''(2\pi)=-1$ **Ans.**] **[13th, 25-09-2016, P-2]**

[INTEGER TYPE] **[2 × 4 = 8]**

Q.5 If $x, y, z > 0$ and $x+y+z=1$, then find the least value of $E=\frac{2x}{1-x}+\frac{2y}{1-y}+\frac{2z}{1-z}$.

[Ans. 3]

[Sol.$_{50093/seq/OMR}$ $E=2\left[\frac{x}{1-x}+\frac{y}{1-y}+\frac{z}{1-z}\right]=2\left[\frac{1}{1-x}+\frac{1}{1-y}+\frac{1}{1-z}-3\right]$

Now, $$\frac{(1-x)+(1-y)+(1-z)}{3}\ge\frac{3}{\frac{1}{1-x}+\frac{1}{1-y}+\frac{1}{1-z}}$$

$$\frac{2}{3}\ge\frac{3}{\frac{1}{1-x}+\frac{1}{1-y}+\frac{1}{1-z}}$$

$$\Rightarrow\quad\frac{1}{1-x}+\frac{1}{1-y}+\frac{1}{1-z}\ge\frac{9}{2}$$

$$\Rightarrow\quad E\ge 2\left[\frac{9}{2}-3\right]=3\ \Rightarrow\ E_{min}=3.\ \textbf{Ans.}]$$

Q.6 If $f(x)=x^3+px^2+qx+r=0$ has three distinct non negative integral roots and
$(x^2-4x+7)^3+p(x^2-4x+7)^2+q(x^2-4x+7)+r=0$
has no real roots, then find the value of $(p+2q+r)$. [Ans. 1]

[Sol.$_{50039/func/OMR}$ Clearly $x^2-4x+7=(x-2)^2+3\ge 3$
$\Rightarrow$ $x^2-4x+7\ne 0, 1, 2\ \Rightarrow\ f(x)$ has roots $0, 1, 2$
$\therefore$ $f(x)=x(x-1)(x-2)=x^3-3x^2+2x$
$\therefore$ $p+2q+r=-3+4+0=1$ **Ans.**]

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

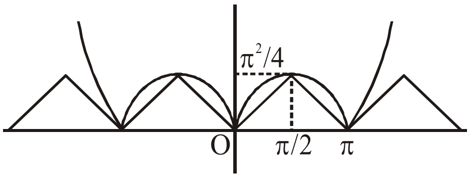
DPP. NO.-28

[SINGLE CORRECT CHOICE TYPE] **[5 × 3 = 15]**

Q.1 The number of solutions to the equation $\frac{\pi}{2}\,|\sin^{-1}(\sin x)| = \left|x^2 - \pi|x|\right|$ is equal to

(A) 3 (B) 4 (C*) 5 (D) 6

[Sol.$_{380/itf/SC}$



Clearly number of solutions = 5 **Ans.**]

Q.2 If the domain of the function $f(x) = \sqrt{12 - 3^x - 3^{3-x}} + \sin^{-1}\left(\frac{2x}{3}\right)$ is [a, b], then $3a + 2b$ is equal to

(A) 4 (B) 5 (C*) 6 (D) 7

[Sol.$_{401/func/SC}$ $\because$ $12 - 3^x - 3^{3-x} \ge 0$ $\Rightarrow$ $3^x + \frac{27}{3^x} - 12 \le 0$

Let $3^x = t$

$\therefore$ $t^2 - 12t + 27 \le 0$

$\Rightarrow$ $(t - 3)(t - 9) \le 0$

$\Rightarrow$ $3 \le t \le 9$

$\Rightarrow$ $1 \le x \le 2$

For $\sin^{-1}\frac{2x}{3}$ to be defined

$-1 \le \frac{2x}{3} \le 1 \Rightarrow \frac{-3}{2} \le x \le \frac{3}{2}$

$\therefore$ Domain of $f(x) = \left[1, \frac{3}{2}\right]$

$\therefore$ $3a + 2b = 3 + 3 = 6$ **Ans.**]

Q.3 $\lim_{x\to\infty} (e^{2x} + e^x + x)^{\frac{1}{x}}$ equals

(A) e^{-2} (B) e (C*) e^2 (D) e^4

[Sol.$_{264/mod/SC}$ $l = e^{\lim_{x\to\infty} \frac{\ln(e^{2x}+e^x+x)}{x}}$

using L'Hospital's rule

$l = e^{\lim_{x\to\infty} \frac{2e^{2x}+e^x+1}{e^{2x}+e^x+1}} = e^{\lim_{x\to\infty} \frac{2+e^{-x}+e^{-2x}}{1+e^{-x}+e^{-2x}}} = e^2$ **Ans.**]

Q.4 If a function $f: R \to R$ is defined such that $f(x) = \text{sgn}\left((x^2 - ax + 2)(2x^2 + bx + 3)\right)$. If for exactly two distinct values of $\alpha \in R$, $\lim_{x\to\alpha} f(x)$ exists but $f(x)$ is non derivable at $x = \alpha$, then $a^2 + b^2$ is equal to

(A) 8 (B) 16 (C*) 32 (D) 44

[Sol.$_{686/lcd(d)/SC}$ Clearly, both roots of $x^2 - ax + 2 = 0$ and $2x^2 + bx + 3 = 0$ can not be same

Both roots of $x^2 - ax + 2 = 0$ should be equal

$\therefore \quad a^2 - 8 = 0 \Rightarrow a^2 = 8$

and both roots of $2x^2 + bx + 3 = 0$ should be same

$\therefore \quad b^2 - 4 \cdot 2 \cdot 3 = 0 \quad \Rightarrow \quad b^2 = 24$

$\therefore \quad a^2 + b^2 = 32$ **Ans.**]

Q.5 Let h(x) = x g(x) where g is the inverse of f(x). Also the values of f(x) and f'(x) are given as

x	2	3	5
f(x)	4	5	1
f'(x)	−1	2	3

then the value of h'(5) equals

(A*) $\frac{11}{2}$ (B) $\frac{9}{2}$ (C) 5 (D) $\frac{5}{2}$

[Sol.$_{201/mod/SC}$ $h'(5) = 5\, g'(5) + g(5) = 5 \cdot \frac{1}{f'(3)} + g(5) = \frac{5}{2} + 3 = \frac{11}{2}$. **Ans.**]

[INTEGER TYPE] [1 × 4 = 4]

Q.6 Find the value of $\lim_{x\to1} \frac{\left[\sum_{K=1}^{100} x^k\right] - 100}{x-1}$. [Ans. 5050]

[Sol.$_{50783/lcd/OMR}$ $\lim_{x\to1} \frac{x + x^2 + x^3 + \ldots.. + x^{100} - 100}{x-1}$

$= \lim_{h\to0} \frac{(1+h) + (1+h)^2 + (1+h)^3 + \ldots.. + (1+h)^{100} - 100}{h}$

$= \lim_{h\to0} \frac{(1+h) + (1+2h) + (1+3h) + \ldots.. + (1+100h) - 100}{h}$

$= \lim_{h\to0} \frac{100 - 100 + h(1+2+3+\ldots\ldots+100)}{h}$

$= \lim_{h\to0} \frac{100\times101}{2} = 101 \times 50 = 5050$ **Ans.**

Alternatively: $\left(\frac{x-1}{x-1}\right) + \left(\frac{x^2-1}{x-1}\right) + \left(\frac{x^3-1}{x-1}\right) + \ldots.. + \left(\frac{x^{100}-1}{x-1}\right) = 1 + 2 + 3 + \ldots.. + 100 = 5050$ Ans.]