GGSRDN
NEET, IIT(JEE-Mains/Advanced)
अभ्यास ही सबसे बड़ा गुरू है।

MATHEMATICS
DAILY PRATICE PROBLEM

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-23 TO 24

Dpp. No.-23

[SINGLE CORRECT CHOICE TYPE] **[5 × 3 = 15]**

Q.1 The number of roots of the equation $2^{\tan\left(x-\frac{\pi}{4}\right)}-2(0.25)^{\frac{\sin^2\left(x-\frac{\pi}{4}\right)}{\cos 2x}}+1=0$ is

(A) 0 (B) 1 (C) 2 (D) 3

Q.2 Let $f: R \to R$ be a positive increasing function with $\lim\limits_{x\to\infty}\frac{f(5x)}{f(x)}=1$ then $\lim\limits_{x\to\infty}\frac{f(3x)}{f(x)}$ is equal to

(A) 1 (B) $\frac{5}{3}$ (C) $\frac{3}{5}$ (D) 3

Q.3 If $a, b \in R$ and satisfy $a=b+\cfrac{1}{a+\cfrac{1}{b+\cfrac{1}{a+\ldots\ldots\infty}}}$, $b=a-\cfrac{1}{b+\cfrac{1}{a-\cfrac{1}{b+\ldots\ldots\infty}}}$

then a^2-b^2 is equal to

(A) 0 (B) 1 (C) 2 (D) –1

Q.4 Let $f(x)$ be derivable at $x=2$ and $\lim\limits_{h\to 0}\frac{f(2+h)}{\sin h}=3$ then $\frac{f(2)+f'(2)}{f'(2)-f(2)}$ is equal to

(A) 0 (B) 1 (C) 3 (D) – 1

Q.5 If $[x]^2+[x-2]<0$ and $\{x\}=\frac{1}{2}$, then the number of possible values of x, is

[**Note :** $[x]$ and $\{x\}$ denote greatest integer less than or equal to x and fractional part of x respectively.]

(A) 4 (B) 3 (C) 2 (D) 1

[MATRIX TYPE] **[3+3+3=9]**

Q.6

	Column-I		Column-II
(A)	Let $f(x)=\sqrt{\log(\cos[\{x\}])}$, then f(x) is	(P)	Even and Periodic function
(B)	Let $f:(-1, 1)\to R$ be defined as $f(x)=\sum_{r=1}^{100}[x^{2r}]$, then $f(x)$ is	(Q)	Bounded
		(R)	Domain contains at least one integer and at most 3 integers
(C)	Let $f(x)=\cos^{-1}([e^x]-1)+\sin^{-1}([e^x])$, then $f(x)$ is	(S)	Both many one and odd function

[**Note :** [y] and {y} denote greatest integer and fractional part function of y respectively.]

Dpp. No.-24

[SINGLE CORRECT CHOICE TYPE] **[6 × 3 = 18]**

Q.1 For the equation $\cos^{-1}x + \cos^{-1}2x + \pi = 0$, the number of solutions will be
(A) 0 (B) 1 (C) 2 (D) ∞

Q.2 Let $f(x) = \begin{cases} 7-2x & ; \quad x<2 \\ \dfrac{x^2}{2}-5 & ; \quad 2 \le x \le 4 \\ 7-x & ; \quad x>4 \end{cases}$. The number of points at which $y = |f(x)|$ is not differentiable are
(A) 2 (B) 3 (C) 4 (D) more than four

Q.3 Let $f(x) = \text{sgn}(|x| - |x-2| + k^2 - 2)$. If $f(x)$ is continuous $\forall\, x \in R$, then the least positive integral value of k is
[**Note:** sgn(y) denotes signum function of y.]
(A) 1 (B) 2 (C) 3 (D) 4

Q.4 Two tour guides are leading seven tourists. The guides decide to split up. Each tourist must choose one of the guides, but with the condition that each guide must take atleast one tourist. Number of different groupings of guides and tourist are
(A) 120 (B) 124 (C) 126 (D) 127

Q.5 Let $y = f(x)$ be an even continuous function on R whose graph is passing through the point (1, 1). If $\lim\limits_{x\to 0} f(x) = 2$ and g(x) be a function defined on R such that $\lim\limits_{x\to 0} g(x) = 3$, then which one of the following statement is incorrect?
(A) $\lim\limits_{x\to 0} f(x-1) = 1$
(B) $\lim\limits_{x\to 0} g(-x) = 3$
(C) $\lim\limits_{x\to 0} \left(f(2x) + g(-x) \right) = 5$
(D) $\lim\limits_{x\to 0} \left(5f(x-1) - 2g(-x) \right) = 1$

Q.6 Let $f(x) = \begin{cases} |1-x^2|, & 0 \le x \le 2 \\ \{\alpha x\} + \beta, & 2 < x < p \end{cases}$ where α, β and $p \in R$.
If $f(x)$ is non-derivable at exactly one point in $[0, p)$ then maximum value of $(\alpha + \beta + 4p)$ is equal to
[**Note:** $\{k\}$ denotes the fractional part function of k.]
(A) 8 (B) 9 (C) 12 (D) 16

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-23 TO 24
(SOLUTIONS)

Dpp. No.-23

[SINGLE CORRECT CHOICE TYPE] **[5 × 3 = 15]**

Q.1 The number of roots of the equation $2^{\tan\left(x-\frac{\pi}{4}\right)} - 2(0.25)^{\frac{\sin^2\left(x-\frac{\pi}{4}\right)}{\cos 2x}} + 1 = 0$ is

(A*) 0 (B) 1 (C) 2 (D) 3

[Sol.$_{138/ph-2/SC}$ Simplify $\frac{\sin^2\left(x-\frac{\pi}{4}\right)}{\cos 2x} = \frac{-1}{2}\tan\left(x-\frac{\pi}{4}\right)$

Now, equation

$$2^{\tan\left(x-\frac{\pi}{4}\right)} - 2(0.25)^{\frac{-1}{2}\tan\left(x-\frac{\pi}{4}\right)} + 1 = 0$$

$$2^{\tan\left(x-\frac{\pi}{4}\right)} - 2(2)^{\tan\left(x-\frac{\pi}{4}\right)} + 1 = 0$$

$$2^{\tan\left(x-\frac{\pi}{4}\right)} = 1$$

$$\tan\left(x-\frac{\pi}{4}\right) = 0$$

$$x = \frac{\pi}{4}$$

Not possible value because $\cos 2x = 0$.]

Q.2 Let $f: R \to R$ be a positive increasing function with $\lim_{x\to\infty} \frac{f(5x)}{f(x)} = 1$ then $\lim_{x\to\infty} \frac{f(3x)}{f(x)}$ is equal to

(A*) 1 (B) $\frac{5}{3}$ (C) $\frac{3}{5}$ (D) 3

[Sol.$_{750/lcd(I)/SC}$ $\because$ $f(5x) > f(3x) > f(x)$ as $x \to \infty$

$$\Rightarrow \frac{f(5x)}{f(x)} > \frac{f(3x)}{f(x)} > 1$$

$$\because \lim_{x\to\infty} \frac{f(5x)}{f(x)} = 1$$

Q.3 If $a, b \in R$ and satisfy $a = b + \cfrac{1}{a + \cfrac{1}{b + \cfrac{1}{a + \ldots\ldots\infty}}}$, $b = a - \cfrac{1}{b + \cfrac{1}{a - \cfrac{1}{b + \ldots\ldots\infty}}}$

then $a^2 - b^2$ is equal to

(A) 0 (B*) 1 (C) 2 (D) –1

[Sol.$_{426/qe/SC}$ We have $a = b + \cfrac{1}{a + \cfrac{1}{a}} \Rightarrow (a - b) = \cfrac{1}{a + \cfrac{1}{a}}$ ......(1)

Also, $b = a - \cfrac{1}{b + \cfrac{1}{b}} \Rightarrow (a - b) = \cfrac{1}{b + \cfrac{1}{b}}$ ......(2)

$\therefore$ From (1) and (2), we get

$$a + \frac{1}{a} = b + \frac{1}{b} \Rightarrow (a - b) + \left(\frac{1}{a} - \frac{1}{b}\right) = 0 \Rightarrow (a - b)\left(1 - \frac{1}{ab}\right) = 0$$

But reject $a = b$ (Think ?).

$\therefore$ $ab = 1$. So, from (1), we get $(a - b)(a + b) = 1 \Rightarrow a^2 - b^2 = 1$ **Ans.**]

Q.4 Let $f(x)$ be derivable at $x = 2$ and $\lim\limits_{h \to 0} \dfrac{f(2+h)}{\sin h} = 3$ then $\dfrac{f(2) + f'(2)}{f'(2) - f(2)}$ is equal to

(A) 0 (B*) 1 (C) 3 (D) – 1

[Sol.$_{751/lcd(d)/SC}$ $\because \lim\limits_{h \to 0} \dfrac{f(2+h)}{\sin h} = 3$

$\because D^r \to 0 \quad \therefore N^r \to 0 \Rightarrow f(2) = 0$

$\therefore$ $f'(2) = 3$

$\therefore$ Given expression = 1.]

Q.5 If $[x]^2 + [x - 2] < 0$ and $\{x\} = \dfrac{1}{2}$, then the number of possible values of x, is

[**Note :** $[x]$ and $\{x\}$ denote greatest integer less than or equal to x and fractional part of x respectively.]

(A) 4 (B) 3 (C*) 2 (D) 1

[Sol.$_{170/func/S}$ $[x]^2 + [x] - 2 < 0 \Rightarrow ([x]+2)([x]-1) < 0 \Rightarrow -2 < [x] < 1 \Rightarrow x \in [-1, 1)$

As, $\{x\} = \frac{1}{2} \Rightarrow x = -\frac{1}{2}, \frac{1}{2}$ **Ans.**]

[MATRIX TYPE]

[3+3+3=9]

Q.6

Column-I		Column-II	
(A)	Let $f(x) = \sqrt{\log(\cos[\{x\}])}$, then f(x) is	(P)	Even and Periodic function
(B)	Let $f : (-1, 1) \to R$ be defined as $f(x) = \sum_{r=1}^{100} [x^{2r}]$, then $f(x)$ is	(Q)	Bounded
		(R)	Domain contains at least one integer and at most 3 integers
(C)	Let $f(x) = \cos^{-1}([e^x] - 1) + \sin^{-1}([e^x])$, then $f(x)$ is	(S)	Both many one and odd function

[**Note :** [y] and {y} denote greatest integer and fractional part function of y respectively.]

[Ans. (A) P, Q, S; (B) Q, R, S; (C) Q]

[Sol.$_{92011/itf/MTC}$

(A) $\left.\begin{array}{l}[\{x\}] = 0 \\ \cos 0 = 1 \\ \log 1 = 0\end{array}\right\} x \in R$

$\therefore f(x) = 0$ for $x \in R$. **Ans**

(B) $x \in (-1, 1)$

$x^2 \in (0, 1)$

$[x^2] = [x^4] \;\ldots\ldots = 0$

$\therefore f(x) = 0$. **Ans.**

(C) $f(x) = \cos^{-1}([e^x] - 1) + \sin^{-1}([e^x])$

For Domain $[e^x] \in [0, 1]$

or $x \in (-\infty, \ln 2)$

$[e^x] = 0$ or $[e^x] = 1$

$\therefore f(x) = \pi$. **Ans.**]

Dpp. No.-24

[SINGLE CORRECT CHOICE TYPE] **[6 × 3 = 18]**

Q.1 For the equation $\cos^{-1}x + \cos^{-1}2x + \pi = 0$, the number of solutions will be

(A*) 0 (B) 1 (C) 2 (D) ∞

[Sol.$_{435/itf/SC}$ $\because$ $0 \le \cos^{-1}x \le \pi$

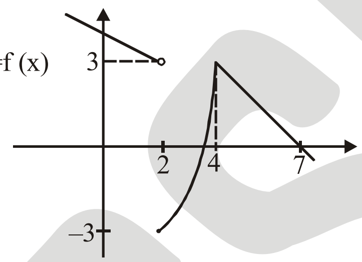
$\therefore$ L.H.S. $\ge \pi$

$\therefore$ no solution.]

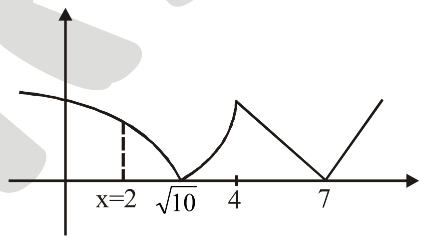
Q.2 Let $f(x) = \begin{cases} 7-2x & ; & x<2 \\ \dfrac{x^2}{2}-5 & ; & 2 \le x \le 4 \\ 7-x & ; & x>4 \end{cases}$. The number of points at which $y = |f(x)|$ is not differentiable are

(A) 2 (B*) 3 (C) 4 (D) more than four

[Sol.$_{756/lcd(d)/SC}$



$\Rightarrow$



at $x = 2$ differentiable and at $x = \sqrt{10}$, 4 , 7 not differentiable.]

Q.3 Let $f(x) = \text{sgn}(|x| - |x-2| + k^2 - 2)$. If $f(x)$ is continuous $\forall\, x \in R$, then the least positive integral value of k is

[**Note:** sgn(y) denotes signum function of y.]

(A) 1 (B) 2 (C*) 3 (D) 4

[Sol.$_{765/lcd/SC}$ $f(x) = \text{sgn}(|x| - |x-2| + k^2 - 2)$

$|x| - |x-2| \ge -2$

$\Rightarrow |x| - |x-2| - 2 + k^2 - 4 = 0\ \forall\, k \in R$

$\Rightarrow k > 2$

$\therefore$ Least value of k = 3.]

Q.4 Two tour guides are leading seven tourists. The guides decide to split up. Each tourist must choose one of the guides, but with the condition that each guide must take atleast one tourist. Number of different groupings of guides and tourist are

(A) 120 (B) 124 (C*) 126 (D) 127

[Sol.$_{193/perm/SC}$ $2({}^7C_1 + {}^7C_2 + {}^7C_3) = 2(7 + 21 + 35) = 126$. **Ans.**]

Q.5 Let $y = f(x)$ be an even continuous function on R whose graph is passing through the point (1, 1). If $\lim\limits_{x\to 0} f(x) = 2$ and g(x) be a function defined on R such that $\lim\limits_{x\to 0} g(x) = 3$, then which one of the following statement is incorrect?

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(C) $\lim\limits_{x\to 0} \left(f(2x)+g(-x)\right) = 5$
(D*) $\lim\limits_{x\to 0} \left(5f(x-1)-2g(-x)\right) = 1$

[Sol.$_{59/lcd/SC}$ As f is continuous $\Rightarrow \lim\limits_{x\to 0} f(x-1) = f(-1) = f(1) = 1$

Also, $\lim\limits_{x\to 0} g(-x) = \lim\limits_{x\to 0} g(x) = 3$

Now, verify alternatives.]

Q.6 Let $f(x) = \begin{cases} |1-x^2|, & 0 \le x \le 2 \\ \{\alpha x\}+\beta, & 2 < x < p \end{cases}$ where α, β and p ∈ R.

If f(x) is non-derivable at exactly one point in [0, p) then maximum value of $(\alpha + \beta + 4p)$ is equal to
[**Note:** {k} denotes the fractional part function of k.]

(A) 8 (B) 9 (C) 12 (D*) 16

[Sol.$_{767/lcd/SC}$ $f(x) = \begin{cases} |1-x^2|, & 0 \le x \le 2 \\ \{\alpha x\}+\beta, & 2 < x < p \end{cases}$

f(x) is non-derivable at x = 1

∴ It should be continuous and derivable at x = 2.

$$f(x) = \begin{cases} 1-x^2, & 0 \le x < 1 \\ x^2-1, & 1 \le x \le 2 \\ \alpha x - [\alpha x] + \beta, & 2 < x < p \end{cases}$$

for the continuity at x = 2
$2\alpha - [2\alpha] + \beta = 3$
for the derivability at x = 2
$\alpha = 4$ and $\beta = 3$
Now, $f(x) = \{4x\} + 3$ in $x \in (2, p)$
for the function to be continuous

$$8 < 4x < 9 \quad \Rightarrow \quad 2 < x < \frac{9}{4}$$

∴ largest value of p must be $\frac{9}{4}$

∴ $(\alpha + \beta + 4p)|_{max.} = 16$ Ans.]