GGSRDN
Educational Services Private Limited
9th, 10th, NEET, JEE(Main/Advanced)
अभ्यास ही सबसे बड़ा गुरू है।

CLASS : XII (MATHS)

DPP

DAILY PRACTICE PROBLEM

DPP-21 to 30

MATHEMATICS DPP

DAILY PRACTICE PROBLEMS

DPP No. 21

Total Marks : 31

Max. Time : 38 min.

Topics : Fundamentals of Mathematics, Function, Limits

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1	(3 marks, 3 min.)	[3, 3]
Subjective Questions (no negative marking) Q.2,3,4,5,6,7,8	(4 marks, 5 min.)	[28, 35]

1. Given $x^2 - xy + y^2 = 4(x + y - 4)$, where x, y both are real numbers. The number of pairs (x, y) satisfying the equation is

(A) only one (B) only two (C) three (D) None of these

2. Evaluate

(i) $\lim\limits_{n\to\infty} (2^n + 3^n)^{1/n}$ (ii) $\lim\limits_{x\to -1} \dfrac{\cos 2 - \cos 2x}{x^2 - |x|}$

3. Evaluate

(i) $\lim\limits_{x\to 0} \dfrac{\tan\sqrt[3]{x}\ \ell n(1+3x)}{(\tan^{-1}\sqrt{x})^2(e^{5\sqrt[3]{x}} - 1)}$ (ii) $\operatorname{Lim}\limits_{x\to 1} \dfrac{\sqrt[3]{x} + \sqrt{x} + x\sqrt{x} - 3}{x^3 - 1}$

4. (a) $\lim\limits_{x\to\infty} \left(\dfrac{x-2}{x^2 - 4x + 3}\right)^x$ is equal to

(b) $\lim\limits_{x\to 2} [x]$ (where [.] denotes greatest integer function) is equal to

(c) $\left[\lim\limits_{x\to 2} x\right]$ (where [.] denotes greatest integer function) is equal to

5. Solve $\dfrac{1}{[x]} + \dfrac{1}{[2x]} = \{x\} + \dfrac{1}{3}$, where [.] denotes greatest integral function and {x} denotes fractional part of x.

6. (a) Whether function $f(x) = \begin{cases} x & 0 \le x < 1 \\ 3 - x & 1 \le x \le 2 \end{cases}$ is invertible ? If yes, then find its inverse.

(b) If Domain of f(x) is $[\pi, 3\pi)$ & $g(x) = \pi + x + \sin x$, then find domain of f(g(x)).

7. Evaluate :

(i) $\lim\limits_{x\to\infty} \dfrac{(2+x)^{40}\ (4+x)^5}{(2-x)^{45}}$ (ii) $\operatorname{Lim}\limits_{x\to 0} \dfrac{1 - \cos^3 x}{x \sin x \cos x}$ (iii) $\operatorname{Lim}\limits_{x\to 0} \dfrac{\ell n(1+2x) - 2\ell n(1+x)}{x^2}$

8. Evaluate %

(i) $\lim\limits_{x\to 0} \dfrac{x^2 2^{2x} - x^2.2^{x+1} + x^2}{\cos 2x - 4\cos x + 3}$ (ii) $\lim\limits_{x\to\infty} \dfrac{(x+1)^4 - (x-1)^4}{(x+1)^4 + (x-1)^4}$

MATHEMATICS
DPP
DAILY PRACTICE PROBLEMS

DPP No. 22

Total Marks : 32

Max. Time : 35 min.

Topics : Fundamentals of Mathematics, Function, Limits

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3,4	(3 marks, 3 min.)	[12, 12]
Subjective Questions (no negative marking) Q.5,6,7	(4 marks, 5 min.)	[12, 15]
Match the Following (no negative marking) Q.8	(8 marks, 8 min.)	[8, 8]

1. Range of the function $f(x) = \frac{\ell nx}{\sqrt{x}}$ is

(A) $(-\infty, e)$ (B) $(-\infty, e^2)$ (C) $\left(-\infty, \frac{2}{e}\right]$ (D) $\left(-\infty, \frac{1}{e}\right]$

2. Let $\tan(2\pi|\sin\theta|) = \cot(2\pi|\cos\theta|)$, where $\theta \in R$ and $f(x) = (|\sin\theta| + |\cos\theta|)^x$, $x \ge 1$. Then range of f(x) does not include

(A) 1 (B) 2 (C) 3 (D) 4

3. Range of the function $f(x) = \sqrt{\sin^{-1}|\sin x| - \cos^{-1}|\cos x|}$ is

(A) {0} (B) $\left[0, \sqrt{\pi/2}\right]$ (C) $\left[0, \sqrt{\pi}\right]$ (D) none of these

4. If $f(4) = g(4) = 2$, $f'(4) = 9$, $g'(4) = 6$, then $\lim_{x\to 4} \frac{\sqrt{f(x)} - \sqrt{g(x)}}{\sqrt{x} - 2}$ is equal to

(A) $3\sqrt{2}$ (B) $\frac{3}{\sqrt{2}}$ (C) 0 (D) does not exists

5. Evaluate :

(i) $\lim_{x\to\infty} \frac{\cot^{-1}\left(\sqrt{x+1} - \sqrt{x}\right)}{\sec^{-1}\left(\left(\frac{2x+1}{x-1}\right)^x\right)}$ (ii) $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{r}{1+r^2+r^4}$

6. Let $f(x) = \begin{cases} \frac{\sin ax^2}{x^2}; & x \ne 0 \\ \frac{3}{4} + \frac{1}{4a}; & x = 0 \end{cases}$. For what values of a, f(x) is continuous at x = 0 ?

7. Find all values of the parameter 'a' for which the inequality $4^x - a2^x - a + 3 \le 0$ is satisfied by at least one real x.

8.

	Column - I		Column - II
(A)	$\lim_{x\to\pi/2} [\sin^{-1}\sin x] =$	(p)	-2
(B)	$\lim_{x\to-\infty} [\tan^{-1}x] =$	(q)	0
(C)	$\lim_{x\to\frac{\pi}{4}} \frac{\sqrt{1-\sqrt{\sin 2x}}}{\pi - 4x} =$	(r)	1
(D)	$\lim_{x\to 0^+} \left[\frac{\sin\|x\|}{x}\right] =$	(s)	does not exist.

($\because$ where [.] denotes greatest integer function)

MATHEMATICS
DPP
DAILY PRACTICE PRODUCTS

DPP No. 23

Total Marks : 31

Max. Time : 35 min.

Topics : **Continuity & Derivability, Function, Limits, Quadratic Equation, Trigonometric Ratio**

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2	(3 marks, 3 min.)	[6, 6]
Multiple choice objective (no negative marking) Q.3	(5 marks, 4 min.)	[5, 4]
Subjective Questions (no negative marking) Q.4,5,6,7,8	(4 marks, 5 min.)	[20, 25]

1. If $f(x) = \dfrac{a\cos x - \cos bx}{x^2}$, $x \neq 0$ and $f(0) = 4$ is continuous at $x = 0$, then the ordered pair (a, b) is

(A) $(\pm 1, 3)$ (B) $(1, \pm 3)$ (C) $(-1, -3)$ (D) $(-1, 3)$

2. Let A = {9, 10, 11, 12, 13} and $f : A \to N$ be a function defined as f(x) = Highest prime factor of x. Then number of elements in the range of f(x) is :-

(A) 5 (B) 4 (C) 3 (D) None of these

3. Which of the statements(s) is/are INCORRECT ?

(A) If f + g is continuous at x = a, then f and g are continuous at x = a.

(B) If $\lim\limits_{x\to a}$ (f g) exists, then $\lim\limits_{x\to a}$ f and $\lim\limits_{x\to a}$ g both exists.

(C) Discontinuity at x = a $\Rightarrow$ non existences of limit

(D) All functions defined on a closed interval attain maximum or a minimum value in its interval.

4. Evaluate &

(i) $\lim\limits_{x\to 0} \dfrac{\cos(xe^x) - \cos(xe^{-x})}{x^3}$ (ii) $\lim\limits_{x\to 0} (\cos ax)^{\operatorname{cosec}^2 bx}$

5. Evaluate :

(i) $\lim\limits_{x\to 2a^+} \dfrac{\sqrt{x-2a} + \sqrt{x} - \sqrt{2a}}{\sqrt{x^2 - 4a^2}}$ (ii) $\lim\limits_{x\to 0^+} \left(\dfrac{e^{x\ln(2^x-1)} - (2^x-1)^x \sin x}{e^{x\ln x}}\right)^{1/x}$

6. Find the sum of an infinite geometric progression whose first term is the limiting value of the function $f(x) = \dfrac{\sin\left(x - \frac{\pi}{6}\right)}{\sqrt{3} - 2\cos x}$ at $x = \dfrac{\pi}{6}$ and whose common ratio is the limiting value of the function

$g(x) = \dfrac{\sin(x)^{1/3}\, \ell n\,(1+3x)}{(\arctan\sqrt{x})^2\left(e^{5\cdot x^{1/3}} - 1\right)}$ as $x \to 0^+$.

7. Find the exact value of the expression $\dfrac{\tan 70^\circ - \tan 20^\circ - 2\tan 40^\circ}{\tan 10^\circ}$.

8. Find all values of a for which the inequality $(a - 3)x^2 - 2ax + 3a - 6 > 0$ is satisfied for all values of x.

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 24

Total Marks : 30

Max. Time : 30 min.

Topics : Limits, Straight Line, Continuity & Derivability, Function, Sequence & Series

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.1,2,3,4	(3 marks, 3 min.)	[12, 12]
Multiple choice objective (no negative marking) Q.5,6	(5 marks, 4 min.)	[10, 8]
Subjective Questions (no negative marking) Q.7,8	(4 marks, 5 min.)	[8, 10]

1. $\lim\limits_{x\to 0^+} \dfrac{\left(\dfrac{\pi}{2}-\cot^{-1}\{x\}\right)x}{\text{sgn}(x)-\cos x}$ (where {.} and sgn(.) denotes fractional part function and signum function respectively) is equal to :

(A) 2 (B) 1 (C) 0 (D) does not exist

2. Let $\lim\limits_{x\to 0} \dfrac{[x]^2}{x^2} = \ell$ and $\lim\limits_{x\to 0} \dfrac{[x^2]}{x^2} = m$, then

(A) ℓ exists but m does not (B) m exists but ℓ does not

(C) ℓ and m both exist (D) neither ℓ nor m exists

3. Least value of function $f(x) = \dfrac{2\sec^2 x + 2\sec x + 1}{\sec^2 x + \sec x + 5}$ is :

(A) 2 (B) $\dfrac{1}{5}$ (C) $\dfrac{2}{19}$ (D) $\dfrac{5}{7}$

4. Through the centriod of an equilateral triangle a line parallel to the base is drawn. On this line, an arbitrary point P is taken inside the triangle. Let h denote the distance of P from the base of the triangle. Let h_1 and h_2 be the distance of P from the other two sides of the triangle, then

(A) h is the H.M. of h_1, h_2 (B) h is the G.M. of h_1, h_2

(C) h is the A.M. of h_1, h_2 (D) none of these

5. Given two straight lines x – y – 7 = 0 and x – y + 3 = 0. Equation of a line which divides the distance between them in the ratio 3 : 2 (internally) can be :

(A) x – y – 1 = 0 (B) x – y – 3 = 0 (C) y = x (D) x – y + 1 = 0

6. If f(x) = [x], g(x) = $\begin{cases} 0 & , \quad x \in Z \\ x^2 & , \quad x \in (R - Z) \end{cases}$, then (where [.] is greatest integer function)

(A) $\lim_{x \to 1}$ g(x) exists but g(x) is discontinuous at x = 1

(B) $\lim_{x \to 1}$ f(x) does not exist and f(x) is not continuous at x = 1

(C) gof is continuous function

(D) g(x) is discontinuous at all integer points

7. Let f(x) = cosec 2x + cosec 2^2 x + cosec 2^3 x + ...+ cosec 2^n x , x ∈ $\left(0, \frac{\pi}{2}\right)$ and g(x) = f(x) + cot 2^n x.

If H(x) = $\begin{cases} (\cos x)^{g(x)} + (\sec x)^{\operatorname{cosec} x} & \text{if} \quad x > 0 \\ p & \text{if} \quad x = 0 \\ \dfrac{e^x + e^{-x} - 2\cos x}{x \sin x} & \text{if} \quad x < 0 \end{cases}$. Find the value of p, if possible to make the function H(x) continuous at x = 0.

8. (i) If $\lim_{x \to 0} \dfrac{729^x - 243^x - 81^x + 9^x + 3^x - 1}{x^3}$ = K(ℓn3)^3, then find the value of k.

(ii) If $\lim_{x \to 0} \dfrac{(1 + a^3) + 8e^{\frac{1}{x}}}{1 + (2 + b + b^2)e^{\frac{1}{x}}}$ = 2, where a, b ∈ R, then find the possible ordered pair (a, b).

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 25

Total Marks : 27

Max. Time : 28 min.

Topics : Limits, Straight Line, Continuity & Derivability

Type of Questions		M.M.,	Min.
Comprehension (no negative marking) Q.1 to Q.2	(3 marks, 3 min.)	[6,	6]
Single choice Objective (no negative marking) Q.3,4	(3 marks, 3 min.)	[6,	6]
Multiple choice objective (no negative marking) Q.5	(5 marks, 4 min.)	[5,	4]
True or False (no negative marking) Q.6	(2 marks, 2 min.)	[2,	2]
Subjective Questions (no negative marking) Q.7,8	(4 marks, 5 min.)	[8,	10]

COMPREHENSION (FOR Q.NO. 1 TO 2)

If f(x) = maximum $\left(\cos x, \frac{1}{2}, \{\sin x\}\right)$, $0 \le x \le 2\pi$, where { . } represents fractional part function, then

1. Number of points of discontinuity of f(x) is

(A) 1 (B) 2 (C) 3 (D) 4

2. Number of points where f(x) is not differentiable is

(A) 4 (B) 5 (C) 6 (D) 7

3. Consider a function f(x) : R → R and if $\lim_{x \to a}$ [f(x)] does not exist, where [ ] denotes greatest integer function, then

(A) $\lim_{x \to a}$ f(x) will never exist

(B) f(x) may be continuous at x = a

(C) Function will not have a tangent at x = a

(D) None of these

4. The angle between straight lines joining the origin and intersection points of the straight line bx + ay = ab and circle $x^2 + y^2 = ax + by$ is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

5. Two consecutive vertices of a rectangle of area 10 unit^2 are (1,3) and (–2, –1). Other two vertices are

(A) $\left(\frac{-3}{5}, \frac{21}{5}\right), \left(-\frac{18}{5}, \frac{1}{5}\right)$

(B) $\left(-\frac{3}{5}, \frac{21}{5}\right), \left(-\frac{2}{5}, -\frac{11}{5}\right)$

(C) $\left(-\frac{2}{5}, -\frac{11}{5}\right), \left(\frac{13}{5}, \frac{9}{5}\right)$

(D) $\left(\frac{13}{5}, \frac{9}{5}\right), \left(-\frac{18}{5}, \frac{1}{5}\right)$

6. True / False

(A) $\lim\limits_{x\to\infty} \dfrac{\ell nx}{[x]} = \lim\limits_{x\to\infty} \dfrac{\{x\}}{\ell n x}$

where [.] is G.I.F. & { . } denotes fractional part function

(B) If $\lim\limits_{x\to\infty} \left(\sqrt{x^4+ax^3+3x^2+bx+2} - \sqrt{x^4+2x^3-cx^2+3x-d}\right) = 4,$

then absolute value of a – c is 3.

(C) $\lim\limits_{x\to 0} \left[\dfrac{\sin(\text{sgn}(x))}{\text{sgn}(x)}\right] = 1$ where [.] is greatest integer function

(D) $\lim\limits_{x\to\infty} \sec^{-1}\left(\dfrac{x}{\sin x}\right) = \lim\limits_{x\to\infty} \sec^{-1}\left(\dfrac{\sin x}{x}\right)$

7. Consider the function $g(x) = \begin{cases} \dfrac{1-a^x+xa^x\ell na}{a^x x^2} & ; x<0 \\ \dfrac{2^x a^x - x\ell n2 - x\ell na - 1}{x^2} & ; x>0 \end{cases}$ where a > 0. Find the value of a and g(0) so that the function g(x) is continuous at x = 0.

8. Consider the function $f(x) = \begin{cases} x^2\left|\cos\dfrac{\pi}{2x}\right| & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0 \end{cases}$. Find LHD and RHD at $x = \dfrac{1}{3}$

MATHEMATICS
DPP
DAILY PRACTICE PROBLEMS

DPP No. 26

Total Marks : 27

Max. Time : 30 min.

Topics : **Method of Differentiation, Straight Line, Continuity & Derivability, Circle**

Type of Questions		M.M., Min.
Comprehension (no negative marking) Q.1 to Q.2	(3 marks, 3 min.)	[6, 6]
Single choice Objective (no negative marking) Q.3,4,5	(3 marks, 3 min.)	[9, 9]
Subjective Questions (no negative marking) Q.6,7,8	(4 marks, 5 min.)	[12, 15]

COMPREHENSION (1 - 2)

In calculus the derivative of any function y = f(x) is defined as

$$D\,f(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Now instead of this usual definition of derivative Df(x), define a new kind of derivative D*f(x), which can be calculated by the formula

$$D^*\,f(x) = \lim_{h\to 0}\frac{f^2(x+h)-f^2(x)}{h}$$

where $f^2(x) = (f(x))^2$.

1. If $f(x) = \frac{x}{\ell nx}$, then D* f(x) is

(A) $\frac{\ell nx-1}{(\ell nx)^2}$ (B) $\frac{2x(\ell nx-1)}{(\ell nx)}$ (C) $\frac{2x(\ell nx-1)}{(\ell nx)^2}$ (D) $\frac{2x(\ell nx-1)}{(\ell nx)^3}$

2. If function $g(x) = x^x$, then $D^*\,g(x)\,|_{x=1}$ is

(A) 1 (B) $2e^e$ (C) 2 (D) not defined

3. The point ([P + 1], [P]) lies inside the circle $x^2 + y^2 - 2x - 15 = 0$, then set of all values of P is (where [.] represents greatest integer function)

(A) [–2, 3) (B) (–2, 3) (C) [–2, 0) ∪ (0, 3) (D) [0, 3)

4. The line L given by $\frac{x}{5}+\frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3} = 1$. Then the distance between L and K is

(A) $\sqrt{17}$ (B) $\frac{17}{\sqrt{15}}$ (C) $\frac{23}{\sqrt{17}}$ (D) $\frac{23}{\sqrt{15}}$

5. If the normal to differentiable curve y = f(x) at x = 0 be given by the equation 3x – y + 3 = 0, then the value of

$$\lim_{x\to 0}\frac{x^2}{f(x^2)-5f(4x^2)+4f(7x^2)}$$ is

(A) 1/3

(B) – 1/3

(C) – 1/5

(D) 1/4

6. A triangle has two of its sides along the lines $y = m_1 x$ & $y = m_2 x$, where m_1, m_2 are the roots of the equation $3x^2 + 10x + 1 = 0$. If H (6 , 2) be the orthocentre of the triangle, find the equation of the third side of the triangle.

7. f(x) is defined as under : $f(x) = \begin{cases} ax(x-1)+b & ; \quad x<1 \\ x-1 & ; \quad 1\le x\le 3 \\ cx^2+dx+2 & ; \quad x>3 \end{cases}$

Determine the constants a, b, c and d, given that

(i) f(x) is continuous for all x

(ii) f '(1) does not exist

(iii) f '(x) is continuous at x = 3

8. Let f(x) be a function of x defined as $f(x) = \begin{cases} \dfrac{x^2-1}{x^2-2|x-1|-1} & , \quad x\ne 1 \\ \dfrac{1}{2} & , \quad x=1 \end{cases}$

Discuss the continuity of function at x = 1.

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 27

Total Marks : 30

Max. Time : 33 min.

Topics : Method of Differentiation, Continuity & Derivability, Limits, Solution of Triangle

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3	(3 marks, 3 min.)	[9, 9]
Multiple choice objective (no negative marking) Q.4	(5 marks, 4 min.)	[5, 4]
Subjective Questions (no negative marking) Q.5,6,7,8	(4 marks, 5 min.)	[16, 20]

1. The number of points where $f(x) = [\sin x + \cos x]$, where [.] denotes the greatest integer function, $x \in (0, 2\pi)$ is not continuous is :

(A) 3 (B) 4 (C) 5 (D) 6

2. $\lim\limits_{x \to 0} \dfrac{\sin[\cos x]}{1+[\cos x]}$ ([.] denotes the greatest integer function) is equal to

(A) equal to 1 (B) equal to 0 (C) does not exist (D) none of these

3. If $x = \cos\theta$, $y = \sin^3\theta$, then $\left.\left(\dfrac{dy}{dx}\right)^2 + y\dfrac{d^2y}{dx^2} + 3\right|_{\theta=\pi/3}$ is equal to

(A) 0 (B) 1 (C) $\dfrac{16}{57}$ (D) $\dfrac{57}{16}$

4. If $4a^2 + c^2 = b^2 - 4ac$, then the variable line $ax + by + c = 0$ always passes through two fixed points. The coordinates of the fixed points can be

(A) (–2, –1) (B) (2, –1) (C) (–2, 1) (D) (2, 1)

5. Let $f(x) = x^3 - 9x^2 + 15x + 6$ and $g(x) = \begin{cases} \min f(t); & 0 \le t \le x, 0 \le x \le 6 \\ x - 18\ ; & x > 6 \end{cases}$

Draw the graph of g(x) and discuss the continuity and differentiability of g(x).

6. If $f(x) = \begin{cases} -x\ , & x \le 1 \\ 3 + x\ , & x > 1 \end{cases}$ and $g(x) = \begin{cases} 3x\ , & x \le 1 \\ 2 + x\ , & x > 1 \end{cases}$, then define f(g(x)) and also examine its continuity.

7. If $\cos^{-1}(y/a) = \log(x/n)^n$ satisfies the equation $x^2\dfrac{d^4y}{dx^4} + 5x\dfrac{d^3y}{dx^3} + 8\dfrac{d^2y}{dx^2} = 0$, then find the value of n.

8. The distance between the two parallel lines is 1 unit . A point 'A' is chosen to lie between the lines at a distance 'd' from one of them . Triangle ABC is equilateral with B on one line and C on the other parallel line. Find the length of the side of the equilateral triangle

MATHEMATICS DPP

DAILY PRACTICE PROBLEMS

DPP No. 28

Total Marks : 38

Max. Time : 38 min.

Topic : Straight Line

Type of Questions		M.M., Min.
Single choice Objective ('–1' negative marking) Q.2, 3, 5, 6, 7, 9	(3 marks, 3 min.)	[18, 18]
Multiple choice objective ('–1' negative marking) Q.8	(5 marks, 4 min.)	[5, 4]
Subjective Questions ('–1' negative marking) Q.4,	(4 marks, 5 min.)	[4, 5]
Assertion and Reason (no negative marking) Q.1,	(3 marks, 3 min.)	[3, 3]
Match the Following (no negative marking) (2 × 4) Q.10	(8 marks, 8 min.)	[8, 8]

1. The line $L_1 : y – x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement-1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$

Statement-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(A) Statement-1 is true, Statement-2 is true ; Statement-2 is correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true ; Statement-2 is **not** a correct explanation for Statement-1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true

2. If the straight lines joining the origin and the points of intersection of the curve
$5x^2 + 12xy – 6y^2 + 4x – 2y + 3 = 0$ and $x + ky – 1 = 0$
are equally inclined to the co-ordinate axes then the value of k :
(A) is equal to 1
(B) is equal to –1
(C) is equal to 2
(D) does not exist in the set of real numbers .

3. Consider points A(3, 4) and B(7, 13). If P be a point on the line y = x such that PA + PB is minimum, then coordinates of P are

(A) $\left(\frac{12}{7}, \frac{12}{7}\right)$ (B) $\left(\frac{13}{7}, \frac{13}{7}\right)$ (C) $\left(\frac{31}{7}, \frac{31}{7}\right)$ (D) (0, 0)

4. Let the algebraic sum of the perpendicular distance from the points (2, 0), (0, 2) and (1, 1) to a variable straight line be zero, then the line passes through a fixed point whose coordinates are ....

5. The straight line x – y – 2 = 0 cuts the axis of x at A. It is rotated about A in such a manner that it is perpendicular to ax + by + c = 0. Its equation is :
(A) bx – ay – 2b = 0
(B) ax – by – 2a = 0
(C) bx + ay – 2b = 0
(D) ax + by + 2a = 0

6. Chords of the curve $4x^2 + y^2 – x + 4y = 0$ which subtend a right angle at the origin pass through a fixed point whose co-ordinates are :

(A) $\left(\frac{1}{5}, -\frac{4}{5}\right)$ (B) $\left(-\frac{1}{5}, \frac{4}{5}\right)$ (C) $\left(\frac{1}{5}, \frac{4}{5}\right)$ (D) $\left(-\frac{1}{5}, -\frac{4}{5}\right)$

7. The interior angle bisector of angle A for the triangle ABC whose coordinates of the vertices are A(–8, 5) ; B(–15, –19) and C(1, –7) has the equation ax + 2y + c = 0, then (a, c) =

(A) (10, 77) (B) (11, 78) (C) (12, 78) (D) (9, 67)

8. The graph of y = f(x) is symmetrical about the line x = 1, then

(A) f(–x) = f(x)
(B) f(1 + x) = f(1 – x)
(C) f(x + 1) = f(x – 1)
(D) f(x) = f(2 – x)

9. The straight line, ax + by = 1 makes with the curve $px^2 + 2axy + qy^2 = r$ a chord which subtends a right angle at the origin . Then :

(A) $r(a^2 + b^2) = p + q$
(B) $r(a^2 + p^2) = q + b$
(C) $r(b^2 + q^2) = p + a$
(D) none of these

10. Consider the general equation of second degree $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$. If this represents a pair of straight lines, match the two columns in the most accurate sense.

Match the column

Column – I		**Column – II**	
(A)	If (x_1, y_1) is the point of intersection of the two lines, then $(ax_1 + hy_1)(hx_1 + by_1)$ =	(p)	$\dfrac{c}{\sqrt{(a-b)^2 + 4h^2}}$
(B)	$af^2 + bg^2 + ch^2$ =	(q)	ab
(C)	The lines are parallel if h^2 =	(r)	gf
(D)	Product of perpendiculars from the origin	(s)	abc + 2fgh

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 29

Total Marks : 33

Max. Time : 33 min.

Topics : Continuity & Derivability, Straight Line, Application of Derivatives, Method of Differentiation

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2	(3 marks, 3 min.)	[6, 6]
Multiple choice objective (no negative marking) Q.3,4,5	(5 marks, 4 min.)	[15, 12]
Subjective Questions (no negative marking) Q.6,7,8	(4 marks, 5 min.)	[12, 15]

1. Let f(x) be defined as follows :

$$f(x) = \begin{cases} (\cos x - \sin x)^{\operatorname{cosec} x} & , \ -\frac{\pi}{2} < x < 0 \\ a & , \ x = 0 \\ \dfrac{e^{1/x} + e^{2/x} + e^{3/x}}{ae^{2/x} + be^{3/x}} & , \ 0 < x < \frac{\pi}{2} \end{cases}$$

If f(x) is continuous at x = 0, then (a, b) =

(A) $\left(e, \frac{1}{e}\right)$ (B) $\left(\frac{1}{e}, e\right)$ (C) (e, e) (D) (e^{-1}, e^{-1})

2. If $ax^2 + bx + c = 0$ has imaginary roots and $a - b + c > 0$, then the set of points (x, y) satisfying the equation

$$\left| a\left(x^2 + \frac{y}{a}\right) + (b+1)x + c \right| = |ax^2 + bx + c| + |x + y|$$

consists of the region in the xy-plane which is

(A) on or above the bisector of I and III quadrant (B) on or above the bisector of II and IV quadrant
(C) on or below the bisector of I and III quadrant (D) on or below the bisector of II and IV quadrant

3. Equation of a tangent to the curve $y \cot x = y^3 \tan x$ at the point where the abscissa is $\pi/4$, is:

(A) $4x + 2y = \pi + 2$ (B) $4x - 2y = \pi + 2$
(C) $x = 0$ (D) $y = 0$

4. If the tangent to the curve $2y^3 = ax^2 + x^3$ at the point (a, a) cuts off intercepts α, β on co-ordinate axes, where $\alpha^2 + \beta^2 = 61$, then the value of 'a' is equal to :

(A) 20 (B) 25 (C) 30 (D) – 30

5. The equation of tangents to the curve $y = \cos(x + y)$, $-2\pi \le x \le 2\pi$, that are parallel to the line $x + 2y = 0$ is/are :

(A) $x + 2y = \pi/2$ (B) $x + 2y = -3\pi/2$ (C) $x - 2y = \pi/2$ (D) $x - 2y = -3\pi/2$

6. If $\left(\frac{x+b}{2}\right) = a \tan^{-1}(a \ln y)$, a > 0, then prove that $yy'' - yy' \ln y = (y')^2$

7. Find the equation of the normal to the curve $y = (1 + x)^y + \sin^{-1}(\sin^2 x)$ at x = 0

8. If x = a(t + sin t), y = a(1 – cost), then find

(i) $\frac{dy}{dx}$ (ii) $\frac{d^2y}{dx^2}$ (iii) $\frac{d^3y}{dx^3}$

MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 30

Total Marks : 31

Max. Time : 38 min.

Topics : Application of Derivatives, Circle

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1	(3 marks, 3 min.)	[3, 3]
Subjective Questions (no negative marking) Q.2,3,4,5,6,7,8	(4 marks, 5 min.)	[28, 35]

1. The slope of the normal at the point with abscissa $x = -2$ of the graph of the function $f(x) = |x^2 - |x||$ is

(A) – 1/6 (B) – 1/3 (C) 1/6 (D) 1/3

2. Find the equation of the straight line which is tangent at one point and normal at another point of the curve $x = 3t^2$, $y = 2t^3$.

3. Let P be a point on the curve $x^2 - y^2 = a^2$, where a is a parameter, such that P is nearest to the line $y = 2x$. Find the locus of P.

4. Find the acute angle between the curves $y = |x^2 - 1|$ and $y = |x^2 - 3|$ at their points of intersection.

5. If $x = a \sin 2\theta (1 + \cos 2\theta)$, $y = a \cos 2\theta (1 - \cos 2\theta)$, prove that $\dfrac{\left\{1+\left(\dfrac{dy}{dx}\right)^2\right\}^{3/2}}{(d^2y/dx^2)} = 4a \cos 3\theta$.

6. For the curve $y = 4x^3 - 2x^5$, find points at which tangent passes through the origin.

7. A line meets the x and y axes at A and B respectively. A circle is circumscribed about the triangle OAB. If the distance of the points A and B from the tangent at O, the origin, to the circle are m and n respectively , find the equation of the circle.

8. From a point, common tangents are drawn to the circle $x^2 + y^2 = 8$ and parabola $y^2 = 16x$. Find the area of the quadrilateral formed by the common tangents, the chord of contact of the circle and the chord of contact of the parabola.

DPP - 21 TO 30 (ANSWER KEY)

DPP NO. - 21

1. (A) **2.** (i) 3 (ii) 2 sin 2 **3.** (i) $\frac{3}{5}$ (ii) $\frac{7}{9}$

4. (a) 0 (b) does not exist (c) 2

5. $\frac{29}{12}, \frac{19}{6}, \frac{97}{24}$

6. (a) Yes, $f^{-1}(x) = \begin{cases} x, & 0 \le x < 1 \\ 3 - x, & 1 \le x \le 2 \end{cases}$ (b) $[0, 2\pi)$

7. (i) – 1 (ii) $\frac{3}{2}$ (iii) – 1

8. (i) $2(\ell n\, 2)^2$ (ii) 0

DPP NO. - 22

1. (C) **2.** (A) **3.** (A) **4.** (A)

5. (i) 1 (ii) $\frac{1}{2}$ **6.** $a = 1, a = \frac{-1}{4}$

7. $a \in [2, \infty)$ **8.** (A)→r, (B)→p, (C)→s, (D)→q

DPP NO. - 23

1. (B) **2.** (B) **3.** (A)(B)(C)(D)

4. (i) – 2 (ii) $e^{-\frac{a^2}{2b^2}}$

5. (i) $\frac{1}{2\sqrt{a}}$ (ii) $\frac{1}{e}\ell n\, 2$ **6.** $a = 1, r = \frac{3}{5}, S_\infty = \frac{5}{2}$

7. 4 **8.** $a \in (6, \infty)$

DPP NO. - 24

1. (A) **2.** (B) **3.** (B) **4.** (C)

5. (A)(B) **6.** (A)(B)(C) **7.** p = 2

8. (i) 6 (ii) (1, 1) and (1, –2)

DPP NO. - 25

1. (B) **2.** (C) **3.** (B) **4.** (D)

5. (A)(C)

6. (A) True (B) True (C) False (D) False

7. $a = \frac{1}{\sqrt{2}}$, $g(0) = \frac{1}{8}(\ell n 2)^2$ **8.** LHD=$-\frac{\pi}{2}$ and RHD= $\frac{\pi}{2}$

DPP NO. - 26

1. (D) **2.** (C) **3.** (A) **4.** (C)

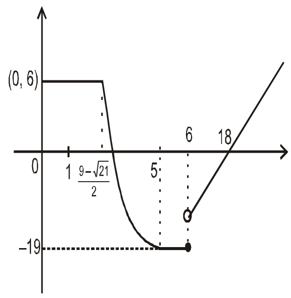
5. (B) **6.** 3x + y + 1 = 0

7. $a \ne 1$, b = 0, c = 1/3, d = –1

DPP NO. - 27

1. (C) **2.** (B) **3.** (D) **4.** (B)(D)

5. f(x) is discontinuous at x = 6 and non-differentiable at x $= \frac{9-\sqrt{21}}{2}, 6$



6. $\begin{cases} -3x, & x \le \frac{1}{3} \\ 3 + 3x, & \frac{1}{3} < x \le 1 \\ 5 + x, & x > 1 \end{cases}$, discontinuous at x = $\frac{1}{3}$

7. n = ±2 **8.** $2\sqrt{\frac{d^2 - d + 1}{3}}$

DPP NO. - 28

1. (C) **2.** (B) **3.** (C) **4.** (1, 1)

5. (A) **6.** (A) **7.** (B)

8. (B)(D) **9.** (A)

10. (A) → (r), (B) → (s), (C) → (q), (D) → (p)

DPP NO. - 29

1. (B) **2.** (B) **3.** (A)(B)(D) **4.** (C)(D)

5. (A)(B) **7.** $y + x - 1 = 0$

8. If(i) $\tan \frac{t}{2}$ (ii) $\frac{1}{2a} \sec^4\left(\frac{t}{2}\right)$

(iii) $\frac{1}{a^2} \sec^6\left(\frac{t}{2}\right) \tan\left(\frac{t}{2}\right)$

DPP NO. - 30

1. (D) **2.** $y = \pm\sqrt{2}\,(x - 2)$ **3.** $x = 2y$

4. $\theta = \tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)$ **6.** (0, 0), (1, 2), (– 1, – 2)

7. $x^2 + y^2 \pm \sqrt{m(m+n)}\,x \pm \sqrt{n(n+m)}\,y = 0$

8. 60

GGSRDN
Educational Services Private Limited
NEET, JEE(Main/Advanced)
अभ्यास ही सबसे बड़ा गुरू है।

GGSRDN

Educational Services Private Limited

9th, 10th, NEET, JEE(Main/Advanced)

अभ्यास ही सबसे बड़ा गुरू है।

CLASS : XII (MATHS)

DPP

DAILY PRACTICE PROBLEM

Solutions

DPP-21 to 30

DPP NO. - 21

1. $2x^2 - 2xy + 2y^2 = 8(x + y - 4)$

$(x - y)^2 + (x^2 - 8x + 16) + (y^2 - 8y + 16) = 0$

$(x - y)^2 + (x - 4)^2 + (y - 4)^2 = 0$

possible when x = y = 4

pair (4, 4)

2. (i) $\lim\limits_{n\to\infty} (2^n + 3^n)^{1/n} = \lim\limits_{n\to\infty} 3\left(\left(\frac{2}{3}\right)^n + 1\right)^{1/n} = 3$

(ii) as $x \to -1$, $\quad |x| \to -x$

$\lim\limits_{x\to-1} \frac{\cos 2 - \cos 2x}{x^2 + x}$

$= \lim\limits_{x\to-1} \frac{0 + 2\sin 2x}{2x + 1} = \frac{-2\sin 2}{-1} = 2\sin 2$

3. (i) $r = \lim\limits_{x\to 0} \frac{\tan(x)^{1/3}\,\ell n(1+3x)}{(\tan^{-1}\sqrt{x})^2\left(e^{5x^{1/3}} - 1\right)}$

$r = \lim\limits_{x\to 0} \frac{x^{1/3}\frac{\tan(x)^{1/3}}{x^{1/3}}.\frac{\ln(1+3x)}{3x}.3x}{x\left(\frac{\tan^{-1}\sqrt{x}}{\sqrt{x}}\right)^2\left(\frac{e^{5.x^{1/3}} - 1}{5.x^{1/3}}\right).5x^{1/3}}$

$r = \frac{3}{5}$

(ii) $\lim\limits_{x\to 1} \frac{\frac{1}{3}x^{-2/3} + \frac{1}{2\sqrt{x}} + \frac{3}{2}\sqrt{x}}{3x^2}$

$= \frac{\frac{1}{3} + \frac{1}{2} + \frac{3}{2}}{3} = \frac{\frac{1}{3} + 2}{3} = \frac{7}{9}$

4. (a) $(0^\infty \text{form}) = 0$

(b) R.H.L. = $\lim\limits_{x\to 2^+} [x] = 2$

L.H.L. = $\lim\limits_{x\to 2^-} [x] = 1$

so $\lim\limits_{x\to 2} [x]$ = does not exist.

(c) $\left[\lim\limits_{x\to 2} x\right] = [2] = 2$

5. **Case : I** If $0 \le f < \frac{1}{2}$, then [x] = n, [2x] = 2n

$\Rightarrow \frac{1}{n} + \frac{1}{2n} = f + \frac{1}{3}$

$\Rightarrow \frac{3}{2n} = f + \frac{1}{3} \qquad \left\{\text{as } 0 \le f < \frac{1}{2}\right\}$

$\Rightarrow \frac{1}{3} \le \frac{3}{2n} < \frac{5}{6} \quad \Rightarrow \quad 3 \ge \frac{2n}{3} > \frac{6}{5}$

$\Rightarrow \frac{9}{5} < n \le \frac{9}{2} \quad \Rightarrow \quad n = 2, 3, 4$

$n = 2 \quad \Rightarrow \quad f = \frac{3}{4} - \frac{1}{3} = \frac{5}{12}$

$\Rightarrow x = 2 + \frac{5}{12} = \frac{29}{12}$

$n = 3 \quad \Rightarrow \quad f = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

$\Rightarrow x = 3 + \frac{1}{6} = \frac{19}{6}$

$n = 4 \quad \Rightarrow \quad f = \frac{3}{8} - \frac{1}{3} = \frac{1}{24}$

$\Rightarrow x = 4 + \frac{1}{24} = \frac{97}{24}$

Case II : $\frac{1}{2} \le f < 1$, then [x] = n, [2x] = 2n + 1

$\Rightarrow \frac{1}{n} + \frac{1}{2n+1} = f + \frac{1}{3}$

$\Rightarrow \frac{5}{6} \le \frac{3n+1}{n(2n+1)} < \frac{4}{3}$ as $f \in \left[\frac{1}{2}, 1\right)$

$\Rightarrow$ Let $\frac{3n+1}{n(2n+1)} < \quad \Rightarrow \quad 9n + 3 < 8n^2 + 4n$

$\Rightarrow 8n^2 - 5n - 3 > 0 \quad \Rightarrow \quad n > 1, n < -\frac{3}{8}$

Let $\frac{3n+1}{n(2n+1)} \ge \frac{5}{6}$

$\Rightarrow 18n + 6 \ge 10n^2 + 5n \quad \Rightarrow \quad 10n^2 - 13n - 6 \le 0$

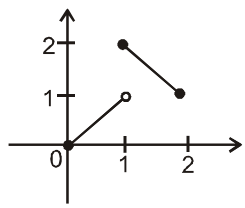
$\Rightarrow \frac{13 - \sqrt{409}}{20} \le n \le \frac{13 + \sqrt{409}}{20}$

$\Rightarrow 1 < n \le \frac{13 + \sqrt{409}}{20} \quad (n \in N)$

So, no solution in this case.

Hence, possible solutions are $\frac{29}{12}, \frac{19}{6}, \frac{97}{24}$

6. (a)



Function is one one onto

It is invertible

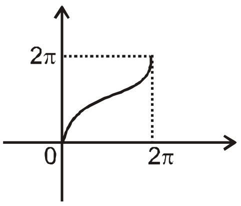
inverse is

$$x = f^{-1}(y) = \begin{cases} y, & 0 \le y < 1 \\ 3-y, & 1 \le y \le 2 \end{cases}$$

$$\Rightarrow f^{-1}(x) = \begin{cases} x, & 0 \le x < 1 \\ 3-x, & 1 \le x \le 2 \end{cases}$$

(b) $\pi \le \pi + x + \sin x < 3\pi$

$0 \le x + \sin x < 2\pi$



7. (i) $\lim\limits_{x\to\infty} \dfrac{x^{40}\left(\frac{2}{x}+1\right)^{40} x^5\left(\frac{4}{x}+1\right)^5}{x^{45}\left(\frac{2}{x}-1\right)^{45}} = \dfrac{(1)\,(1)}{(-1)} = -1.$

(ii) $\text{Lim}_{x\to 0} \dfrac{1-\cos^3 x}{x^2\left(\frac{\sin x}{x}\right)\cos x} = \lim\limits_{x\to 0} \dfrac{1-\cos^3 x}{x^2}$

$\left(\because \lim\limits_{x\to 0}\dfrac{\sin x}{x} = 1 \text{ and } \text{Lim}_{x\to 0} \cos x = 1\right)$

$\Rightarrow \text{Lim}_{x\to 0} \dfrac{+3\cos^2 x \sin x}{2x} = \dfrac{3}{2}$

(iii) $\text{Lim}_{x\to 0} \dfrac{\frac{1}{1+2x}.2 - \frac{2}{1+x}}{2x}$

$= \text{Lim}_{x\to 0} \dfrac{2+2x-2-4x}{2x(1+2x)(1+x)} = -1$

8. (i) $\lim\limits_{x\to 0} \dfrac{x^2.2^{2x} - x^2.2^{x+1} + x^2}{\cos 2x - 4\cos x + 3}$

$= \lim\limits_{x\to 0} \dfrac{x^2(2^x-1)^2}{(\cos 2x - 4\cos x + 3)}$

$= \lim\limits_{x\to 0} \dfrac{x^4\left(\frac{2^x-1}{x}\right)^2}{\cos 2x - 4\cos x + 3}$

$= \lim\limits_{x\to 0} \dfrac{4x^3 \quad (\ell n2)^2}{-2\sin 2x + 4\sin x}$

$= \lim\limits_{x\to 0} \dfrac{12x^2(\ell n2)^2}{-4\cos 2x + 4\cos x}$

$= \lim\limits_{x\to 0} \dfrac{24x(\ell n2)^2}{8\sin 2x - 4\sin x}$

$= \lim\limits_{x\to 0} \dfrac{24(\ell n2)^2}{16\cos 2x - 4\cos x}$

$= \dfrac{24}{12}(\ell n2)^2 = 2(\ell n2)^2$

(ii) $\lim\limits_{x\to\infty} \dfrac{x^4\left[\left(1+\frac{1}{x}\right)^4 - \left(1-\frac{1}{x}\right)^4\right]}{x^4\left[\left(1+\frac{1}{x}\right)^4 + \left(1-\frac{1}{x}\right)^4\right]} = \dfrac{1-1}{1+1} = 0$

DPP NO. - 22

1. $y = f(x) = \dfrac{\ell nx}{\sqrt{x}} \Rightarrow \quad f'(x) = \dfrac{\sqrt{x}\cdot\frac{1}{x} - \ell nx\cdot\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$

for defined x > 0

$f'(x) = \dfrac{2-\ell nx}{2x\sqrt{x}} = 0 \Rightarrow \ell nx = 2 \quad \Rightarrow x = e^2$

$\Rightarrow$ Range is $\left(-\infty, \dfrac{2}{e}\right]$

2. $\tan(2\pi|\sin\theta|) = \tan\left(\dfrac{\pi}{2} - 2\pi\,|\cos\theta|\right)$

$\Rightarrow 2\pi\,|\sin\theta| = \left(\dfrac{\pi}{2} - 2\pi\,|\cos\theta|\right) + n\pi$

$\Rightarrow |\sin\theta| + |\cos\theta| = \dfrac{3}{4}$

n = 1

n = 2 $\quad |\sin\theta| + |\cos\theta| = \dfrac{5}{4}$

n = 3 $\quad |\sin\theta| + |\cos\theta| = \dfrac{7}{4}$

Range cannot be one

3. $\sin^{-1}|\sin x| \ge \cos^{-1}|\cos x|$

4. $\lim\limits_{x\to 4} \dfrac{\frac{1}{2\sqrt{f(x)}}f'(x) - \frac{1}{2\sqrt{g(x)}}g'(x)}{\frac{1}{2\sqrt{x}}}$

$$= \frac{\frac{f'(4)}{2\sqrt{f(4)}} - \frac{g'(4)}{2\sqrt{g(4)}}}{\frac{1}{2\sqrt{4}}} = \frac{\frac{9}{\sqrt{2}} - \frac{6}{\sqrt{2}}}{\frac{1}{2}} = 3\sqrt{2}.$$

5. (i) When $x \to \infty$

$$\left(\frac{2x+1}{x-1}\right)^x \to \infty \qquad \Rightarrow \sec^{-1}(\infty) \to \frac{\pi}{2}$$

$$\tan^{-1}\left(\frac{1}{\sqrt{x+1}-\sqrt{x}}\right)$$

$$= \tan^{-1}(\sqrt{x+1} + \sqrt{x}) \to \frac{\pi}{2} \text{ (as } x \to \infty)$$

(ii) $$\lim_{n\to\pi} \sum_{r=1}^{n} \frac{r}{(r^2+1)^2-(r)^2}$$

$$= \frac{1}{2} \lim_{n\to\infty} \sum_{r=1}^{n} \frac{(r^2+r+1)-(r^2-r+1)}{(r^2+r+1)(r^2-r+1)}$$

$$= \frac{1}{2} \lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}\right)$$

$$= \frac{1}{2} \lim_{n\to\infty} \left(\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{13}\right)+ \ldots + \left(\frac{1}{n^2-n+1}-\frac{1}{n^2+n+1}\right)\right)$$

$$= \frac{1}{2} \lim_{n\to\infty} \left(\frac{1}{1} - \frac{1}{n^2+n+1}\right) = \frac{1}{2}(1-0) = \frac{1}{2}$$

6. $$\lim_{x\to 0} \left(\frac{\sin ax^2}{ax^2}\right).a = \lim_{x\to 0}\left(\frac{3}{4}+\frac{1}{4a}\right)$$

$$\Rightarrow a = \frac{3}{4} + \frac{1}{4a}$$

$\Rightarrow 4a^2 = 3a + 1 \qquad \Rightarrow 4a^2 - 3a - 1 = 0$

$\Rightarrow 4a^2 - 4a + a - 1 = 0 \Rightarrow (4a + 1)(a - 1) = 0$

$$a = 1,\ a = \frac{-1}{4}$$

7. $D = (-a)^2 - 4(1)(-a + 3) \geq 0 \Rightarrow a^2 + 4a - 12 \geq 0 \Rightarrow$

$(a + 6)(a - 2) \geq 0$

$a \in (-\infty, -6] \cup [2, \infty)$

$t_1 + t_2 = a$

$t_1 t_2 = 3 - a$

for $a \in (-\infty, -6]$, $\quad t_1 + t_2 < 0$, $t_1 t_2 > 0$ There for both roots are negative $\Rightarrow$ no. solution

for $a \in [2, \infty)$

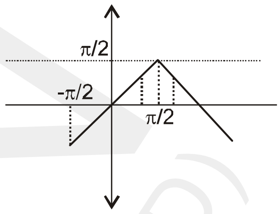
$t_1 + t_2 < 0$, and $t_1 t_2 < 0$ or $t_1 t_2 > 0$

at least one of the roots t_1 or t_2 is greater than zero.

$\Rightarrow$ $a \in [2, \infty)$ has at least one solution.

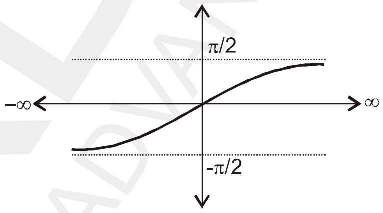
8. **(A)** Graph of $y = \sin^{-1}(\sin x)$

$$\text{LHL} = \lim_{x\to\pi/2^-} [\sin^{-1}(\sin x)] = 1$$



$$\text{RHL} = \lim_{x\to\pi/2^+} [\sin^{-1}(\sin x)] = 1$$

(B) $$\lim_{x\to-\infty} [\tan^{-1} x] = -2$$



(C) $$\lim_{x\to\pi/4} \frac{\sqrt{1-\sin 2x}}{(\pi-4x)\left(\sqrt{1+\sqrt{\sin 2x}}\right)}$$

$$= \lim_{h\to 0} \frac{\sqrt{1-\cos 2h}}{4h\left(\sqrt{1+\sqrt{\cos 2h}}\right)} = \lim_{h\to 0} \frac{\sqrt{2}\,|\sin h|}{4\sqrt{2}\,h}$$

$$\text{RHL} = \frac{1}{4} \lim_{h\to 0^+} \frac{\sin h}{h} = \frac{1}{4}$$

$$\text{LHL} = \frac{1}{4} \lim_{h\to 0^-} \frac{|-\sin h|}{-h} = \frac{-1}{4} \qquad \text{LHL} \neq \text{RHL}.$$

(D) $$\lim_{h\to 0} \left[\frac{\sin|0+h|}{0+h}\right] = \lim_{h\to 0}\left[\frac{\sin h}{h}\right] = 0$$

$$(\because \sin h < h \Rightarrow \frac{\sin h}{h} < 1 \text{ as } h \to 0)$$

DPP NO. - 23

1. $$\lim_{x\to 0} \frac{a\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots\right)-\left(1-\frac{b^2x^2}{2!}+\frac{b^4x^4}{4!}-\ldots\right)}{x^2} = 4$$

Limit exist if $(a - 1) = 0$ and $\frac{-a}{2} + \frac{b^2}{2} = 4$

$$a = 1 \quad \text{when} \quad \frac{b^2}{2} = 4 + \frac{1}{2}$$

$\Rightarrow$ $b^2 = 9$ $\Rightarrow$ $b = \pm 3$

2. f(9) = 3, f(10) = 5, f(11) = 11, f(12) = 3, f(13) = 13

$\therefore$ Number of elements in Range = 4

4. (i) $\lim_{x\to 0} \dfrac{\cos(xe^x) - \cos(xe^{-x})}{x^3}$.

$$= \lim_{x\to 0} \frac{-2.\sin\left(\frac{x.e^x + xe^{-x}}{2}\right)\sin\left(\frac{xe^x - xe^{-x}}{2}\right)}{\left(\frac{x.e^x + xe^{-x}}{2}\right)\left(\frac{xe^x - xe^{-x}}{2}\right)x^3}.\left(\frac{x.e^x + xe^{-x}}{2}\right)\left(\frac{xe^x - xe^{-x}}{2}\right)$$

$$= \lim_{x\to 0} \frac{-2\left(\frac{e^x + e^{-x}}{2}\right)\left(\frac{e^x + e^{-x}}{2}\right)}{x}$$

$$= \lim_{x\to 0} -2\left(\frac{e^{2x} - 1}{2x}\right)\frac{1}{e^x} = -2\,(1)\,\frac{1}{(1)} = -2$$

(ii) $e^{\lim_{x\to 0}\frac{\cos ax - 1}{\sin^2 bx}} = e^{\lim_{x\to 0}\frac{-a\sin ax}{2\sin bx\cos bx.b}}$

$$= e^{\frac{-a}{2b}\left(\frac{a}{b}\right)} = e^{\frac{-a^2}{2b^2}}$$

5. (i) $\lim_{x\to 2a} \dfrac{\dfrac{1}{2\sqrt{x-2a}} + \dfrac{1}{2\sqrt{x}}}{\dfrac{1}{2\sqrt{x^2 - 4a^2}}(2x)}$

$$= \lim_{x\to 2a} \frac{(\sqrt{x} + \sqrt{x-2a})(\sqrt{x^2 - 4a^2})}{2x\sqrt{x}(\sqrt{x-2a})}$$

$$= \lim_{x\to 2a} \frac{(\sqrt{x-2a} + \sqrt{x})(\sqrt{x+2a})}{2x\sqrt{x}}$$

$$= \frac{\sqrt{2a}(2\sqrt{a})}{2(2a)\sqrt{2}\sqrt{a}} = \frac{1}{2\sqrt{a}}$$

(ii) $\lim_{x\to 0^+} \left(\dfrac{(2^x - 1)^x - (2^x - 1)^x \sin x}{x^x}\right)^{\frac{1}{x}}$

$$= \lim_{x\to 0^+} \left(\frac{(2^x - 1)(1 - \sin x)^{1/x}}{x}\right)$$

$$= \lim_{x\to 0^+} \left(\frac{2^x - 1}{x}\right) \lim_{x\to 0^+} (1 - \sin x)^{1/x}$$

$$= \ell n2.\, e^{\lim_{x\to 0^+}\left(\frac{-\sin x}{x}\right)} = \frac{1}{e}\ell n2.$$

6. $a = \lim_{x\to \pi/6} \dfrac{\sin\left(x - \frac{\pi}{6}\right)}{\sqrt{3} - 2\cos x}$

$$a = \lim_{x\to \pi/6} \frac{\cos\left(x - \frac{\pi}{6}\right)}{2\sin x} = \frac{1}{2\left(\frac{1}{2}\right)} = 1$$

$$r = \lim_{x\to 0^+} \frac{\sin(x)^{1/3}\,\ell n(1+3x)}{(\tan^{-1}\sqrt{x})^2\left(e^{5x^{1/3}} - 1\right)}$$

$$r = \lim_{x\to 0^+} \frac{x^{1/3}\dfrac{\sin(x)^{1/3}}{x^{1/3}}.\dfrac{\ln(1+3x)}{3x}.3x}{x\left(\dfrac{\tan^{-1}\sqrt{x}}{\sqrt{x}}\right)^2\left(\dfrac{e^{5.x^{1/3}} - 1}{5.x^{1/3}}\right).5x^{1/3}}$$

$$r = \frac{3}{5} \qquad \text{so} = \frac{a}{1-r} = \frac{1}{1 - 3/5} = \frac{5}{2}.$$

7. $\dfrac{\tan 70^\circ - \tan 20^\circ - 2\tan 40^\circ}{\tan 10^\circ}$

$$= \frac{\cot 20^\circ - \tan 20^\circ - 2\tan 40^\circ}{\tan 10^\circ}$$

$$= \frac{2\dfrac{(\cos^2 20^\circ - \sin^2 20^\circ)}{2\sin 20^\circ \cos 20^\circ} - \dfrac{2\sin 40^\circ}{\cos 40^\circ}}{\tan 10^\circ}$$

$$= 4\,\frac{(\cos^2 40^\circ - \sin^2 40^\circ)}{2\sin 40^\circ \cos 40^\circ \tan 10^\circ}$$

$$= \frac{4\cos 80^\circ}{\sin 80^\circ \tan 10^\circ} = 4\,\frac{\tan 80^\circ}{\tan 80^\circ} = 4$$

8. $a - 3 > 0$ and $D < 0$

$\Rightarrow$ $a > 3$ ...(i)

and $4a^2 - 4(a - 3)(3a - 6) < 0$

$\Rightarrow$ $a^2 - (3a^2 - 15a + 18) < 0$

$\Rightarrow$ $-2a^2 + 15a - 18 < 0$

$\Rightarrow$ $2a^2 - 15a + 18 > 0$

$\Rightarrow$ $2a^2 - 12a - 3a + 18 > 0$

$\Rightarrow$ $2a(a - 6) - 3(a - 6) > 0$

$\Rightarrow$ $(a - 6)(2a - 3) > 0$

$a \in \left(-\infty, \frac{3}{2}\right) \cup (6, \infty)$ ...(ii)

from (i) and (ii)

$a \in (6, \infty)$.

DPP NO. - 24

1. $\lim_{x\to 0^+} \frac{\left(\frac{\pi}{2} - \cot^{-1}\{x\}\right)x}{\text{sgn}(x) - \cos x}$

as $x \to 0^+$ sgn (x) = 1 and $\{x\} \to x$

$= \lim_{x\to 0^+} \frac{\left(\frac{\pi}{2} - \cot^{-1} x\right)x}{1 - \cos x}$

$= \lim_{x\to 0^+} \frac{\tan^{-1} x . x}{1 - \cos x} = \lim_{x\to 0^+} \frac{\frac{\tan^{-1} x}{x}}{\frac{1-\cos x}{x^2}} = 2$

2. $\lim_{x\to 0} \frac{[x]^2}{x^2}$

RHL $= \lim_{h\to 0^+} - \frac{[h]^2}{h^2}$

$= \lim_{h\to 0^+} \frac{0(\text{exact})}{h^2} = 0$

LHL $= \lim_{h\to 0^-} \frac{[-h]^2}{(-h)^2} = \lim_{h\to 0^-} \frac{1}{h^2} = \infty$

$\Rightarrow$ ℓ does not exist

$\lim_{x\to 0} \frac{[x^2]}{x^2} = \lim_{x\to 0} \frac{0(\text{exact})}{h^2} = 0$

$\Rightarrow$ m exists

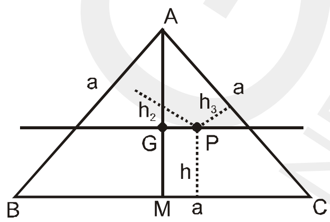
3. y = f(x) = $\frac{2\sec^2 x + 2\sec x + 1}{\sec^2 x + \sec x + 5}$

$y = 2 - \frac{9}{\sec^2 x + \sec x + 5}$

y is minimum when $\sec^2 x + \sec x + 5$ is minimum, y =

$2 - \frac{9}{5} = \frac{1}{5}$ ($\because$ sec x = –1)

4.



GM = h $\Rightarrow$ AM = 3h.

$\frac{1}{2}ah + \frac{1}{2}ah_1 + \frac{1}{2}ah_2 = \frac{1}{2}aH = \frac{1}{2}a(3h)$

$\Rightarrow$ $h + h_1 + h_2 = 3h$

$2h = h_1 + h_2$

5. $\left|\frac{\frac{c+7}{\sqrt{2}}}{\frac{c-3}{\sqrt{2}}}\right| = \frac{3}{2}$

$\frac{c+7}{c-3} = \pm\frac{3}{2}$

$\Rightarrow$ 2c + 14 = 3c – 9 or 2c + 14 = – 3c + 9 $\Rightarrow$

c = 23 or 5c = – 5

lines x – y – 1 = 0 or x – y – 23 = 0

and $\left|\frac{\frac{c+7}{\sqrt{2}}}{\frac{c-3}{\sqrt{2}}}\right| = \frac{2}{3}$

$\Rightarrow \frac{c+7}{c-3} = \pm\frac{2}{3}$ $\Rightarrow$ 3c + 21 = 2c – 6

c = – 27 or 3c + 21 = – 2c + 6

5c = – 15

Lines x – y – 3 = 0 or x – y – 27 = 0

6. For g(x) function

LHL = $\lim_{x\to 1^-}$ g(x) = 1

RHL = $\lim_{x\to 1^+}$ g(x) = 1

But g(1) = 0

$\Rightarrow$ $\lim_{x\to 1}$ g(x) exist but discontinuous at x = 1

for f(x) = [x] at x = 1

LHL = 0 , RHL = 1

f(x) = does not exist

Now gof(x) = g(f(x)) = g([x]) = 0

so it is continuous everywhere.

7. we know that cosec2θ + cot 2θ = cotθ

Now

g(x) = cosec2x + cosec2^2x + cosec2^3 x + ... + cosec2^nx + cot 2^nx

on calculating

= cotx

$$H(x) = \begin{cases} (\cos x)^{\cot x} + (\sec x)^{\operatorname{cosec} x}, & x > 0 \\ p, & x = 0 \\ \frac{e^x + e^{-x} - 2\operatorname{cosec} x}{x \sin x}, & x < 0 \end{cases}$$

H(x) is continuous at x = 0

$$p = \lim_{x\to 0^-} \frac{e^x + e^{-x} - 2\cos x}{x\sin x}$$

$$= \lim_{x\to 0^+} (\cos x)^{\cot x} + (\sec x)^{\text{cosec}\, x}$$

$$p = \lim_{x\to 0^-} \frac{e^x - e^{-x} + 2\sin x}{\sin x + x\cos x}$$

$$p = \lim_{x\to 0^-} \frac{e^x + e^{-x} + 2\cos x}{\cos x + \cos x - x\sin x} = \frac{1+1+2}{2} = 2$$

8. (i). $$\lim_{x\to 0} \frac{3^{6x} - 3^{5x} - 3^{4x} + 3^{2x} + 3^x - 1}{x^3}$$

$$= \lim_{x\to 0} \frac{(3^{6x}.6 - 5.3^{5x} - 4.3^{4x} + 2.3^{2x} + 3^x)}{3x^2} \ell n3$$

$$= \lim_{x\to 0} \frac{(36.3^{6x} - 25.3^{5x} - 16.3^{4x} + 4.3^{2x} + 3^x)}{6x} (\ell n3)^2$$

$$= \lim_{x\to 0} \left(\frac{216.3^{6x} - 125.3^{5x} - 64.3^{4x} + 8.3^{2x} + 3^x}{6}\right) (\ell n3)^3$$

$$= \left(\frac{216 - 125 - 64 + 8 + 1}{6}\right) (\ell n3)^3 = 6\ (\ell n3)^3$$

(ii) RHL = $$\lim_{x\to 0^+} \frac{(1+a^3) + 8e^{\frac{1}{x}}}{1 + (2+b+b^2)e^{\frac{1}{x}}} = 2$$

$$\Rightarrow \lim_{x\to 0^+} \frac{(1+a^3)e^{-1/x} + 8}{e^{-1/x} + (2+b+b^2)} = 2$$

$$\Rightarrow \frac{8}{2+b+b^2} = 2 \Rightarrow 4 = 2 + b + b^2$$

$\Rightarrow b^2 + b - 2 = 0$

$(b + 2)(b - 1) = 0$

$b = 1, b = -2$

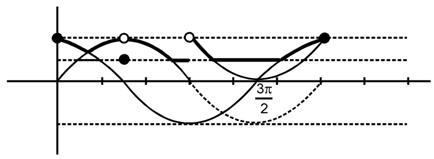
LHL = $$\lim_{x\to 0^-} \frac{(1+a^3) + 8e^{1/x}}{1 + (2+b+b^2)e^{1/x}} = 2$$

$$\Rightarrow \frac{(1+a^2) + 0}{1+0} = 2$$

$\Rightarrow a = 1$

DPP NO. - 25

2.



3. Consider $f(x) = \cos x$ at $x = \frac{\pi}{2}$

4. $x^2 + y^2 = ax + by$, line $bx + ay = ab$

Homgeous equation

$$x^2 + y^2 = (ax + by)\left(\frac{bx + ay}{ab}\right)$$

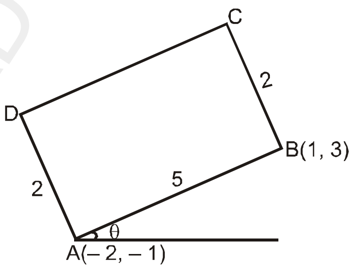
$\Rightarrow ab(x^2 + y^2) = abx^2 + aby^2 + (a^2 + b^2)xy$

$\Rightarrow xy = 0$

$x = 0$ and $y = 0$

Angle between lines = $\frac{\pi}{2}$

5. Area = 10



$\tan\theta = \text{slope} = \frac{4}{3}$

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$D(-2 \pm 2\cos(90 - \theta), -1 \pm 2\sin(90 - \theta))$

$$D\left(-2 \pm \frac{8}{5}, -1 \pm 2\left(\frac{3}{5}\right)\right)$$

$$D\left(\frac{-2}{5}, \frac{-11}{5}\right) \text{ and } \left(\frac{-18}{5}, \frac{1}{5}\right)$$

$C(1 \pm 2\cos(90 - \theta), 3 \mp 2\sin(90 - \theta)) = C(1 \pm 2\sin\theta, 3 \mp 2\cos\theta)$

$$= C\left(1 \pm 2\left(\frac{4}{5}\right), 3 \mp 2\left(\frac{3}{5}\right)\right) = C\left(\frac{13}{5}, \frac{9}{5}\right),$$

and $$\left(\frac{-3}{5}, \frac{21}{5}\right)$$

6. **True/False**

(A) $\because\ x > \ell nx$. for $x \to \infty$

and $[x] \to x$ as $x \to \infty$

So $\lim_{x\to\infty} \frac{\ell nx}{[x]} = 0$

as $x \to \infty$, then $\{x\} \in [0,1)$

and as $x \to \infty$, $\ell n\, x \to \infty$

So $\frac{\{x\}}{\ell nx} \to 0$

(B)

$$\lim_{x\to\infty}\left(\sqrt{x^4+ax^3+3x^2+bx+2}-\sqrt{x^4+2x^3-cx^2+3x-d}\right) = 4$$

$$\lim_{x\to\infty}\frac{(a-2)x^3+(3+c)x^2+(b-3)x+2+d}{\sqrt{x^4+ax^3+3x^2+bx+2}+\sqrt{x^4+2x^3-cx^2+3x-d}} = 4$$

$$\lim_{x\to\infty}\frac{(a-2)x+(3+c)+\frac{(b-3)}{x}+\frac{(2+d)}{x^2}}{\sqrt{1+\frac{a}{x}+\frac{3}{x^2}+\frac{b}{x^3}+\frac{2}{x^4}}+\sqrt{1+\frac{2}{x}-\frac{c}{x^2}+\frac{3}{x^3}-\frac{d}{x^4}}}$$

$= 4$

then $a - 2 = 0$ and $\frac{3+c}{2} = 4 \Rightarrow c = 5$

So $a - c = -3$

$|a - c| = 3$

(C) $\lim_{x\to 0}\left[\frac{\sin(\text{sgn}(x))}{\text{sgn}(x)}\right]$

L.H.L. $= \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(-h)$

$$= \lim_{h\to 0}\left[\frac{\sin\,\text{sgn}(-h)}{\text{sgn}(-h)}\right] = \left[\frac{-\sin 1}{-1}\right] = 0$$

R.H.L. $= \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(h)$

$$= \lim_{h\to 0}\left[\frac{\sin \text{sgn}(h)}{\text{sgn} h}\right] = [\sin\, 1] = 0.$$

(D) $\because$ as $x \to \infty$, then $\frac{x}{\sin x} \to \infty$

So $\lim_{x\to\infty} \sec^{-1}\left(\frac{x}{\sin x}\right) = \frac{\pi}{2}$

$$\frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} + \ldots.. < 1$$

$\sec^{-1}\left(\frac{\sin x}{x}\right)$ is not define

so, limit does not exist

7. LHL $= \lim_{x\to 0^-}\frac{1-a^x+xa^x\ell na}{a^x x^2}$

$$= \lim_{x\to 0^-}\frac{1-\left(1+\frac{x.\ell na}{1!}+\frac{x^2(\ell na)^2}{2!}\right)+x\ell na\left(1+\frac{x\ell na}{1!}\right)}{a^x x^2}$$

$$= \lim_{x\to 0^-}\frac{\frac{1}{2}(x\ell na)^2}{a^x.x^2}$$

LHL $= \frac{1}{2}(\ell na)^2$ ...(1)

RHL $= \lim_{x\to 0^+}\frac{(2a)^x - x(\ell n2+\ell na)-1}{x^2}$

$$= \lim_{x\to 0^+}\frac{1+\frac{x.\ell n2a}{1!}+\frac{(x\ell n2a)^2}{2!}-x\ell n2a-1}{x^2}$$

RHL $= \frac{1}{2}(\ell n2a)^2$ ...(2)

Now LHL = RHL

$\frac{1}{2}(\ell na)^2 = \frac{1}{2}(\ell n2a)^2$

$\Rightarrow a = \frac{1}{2}$

8. $f(x) = \begin{cases} x^2\cos\left(\frac{\pi}{2x}\right), & x < \frac{1}{3} \\ 0, & 0 \\ -x^2\cos\frac{\pi}{2x}, & x > \frac{1}{3}\end{cases}$

$$\text{RHD} = \lim_{h\to 0^+}\frac{-\left(\frac{1}{3}+h\right)^2\cos\left(\frac{\pi}{2\left(\frac{1}{3}+h\right)}\right)-0}{h}$$

$$= \lim_{x\to 0^+}\frac{-2\left(\frac{1}{3}+h\right)\cos\left(\frac{3\pi}{2+3h}\right)+\left(\frac{1}{3}+h\right)^2\sin\left(\frac{3\pi}{2+6h}\right)\frac{-3\pi(6)}{(2+6h)^2}}{1} =$$

$$\frac{\frac{-2}{3}(0)+\frac{1}{9}(-1)\left(\frac{-18\pi}{4}\right)}{1} = \frac{\pi}{2}$$

Similarly

$$\text{LHD} = \lim_{x\to 0^+}\frac{\left(\frac{1}{3}-h\right)^2\cos\left(\frac{3\pi}{2-6h}\right)-0}{-h}$$

$$= \lim_{x\to 0^+}\frac{2\left(\frac{1}{3}-h\right)(-1)\cos\left(\frac{3\pi}{2-6h}\right)+\left(\frac{1}{3}-h\right)^2\left(-\sin\left(\frac{3\pi}{2-6h}\right)\right)\left(\frac{-3\pi(-6)}{(2-6h)^2}\right)}{-1}$$

$$= \lim_{x\to 0^+}\frac{-\frac{2}{3}(0)+\frac{1}{9}\left(\frac{18\pi}{4}\right).(1)}{-1} = -\frac{\pi}{2}$$

DPP NO. - 26

1. $\lim\limits_{h\to 0}\dfrac{\dfrac{(x+h)^2}{\ell n^2(x+h)}-\dfrac{x^2}{(\ell nx)^2}}{h}$

$= \lim\limits_{h\to 0}\dfrac{(x+h)^2(\ell nx)^2 - x^2(\ell n(x+h))^2}{h.(\ell nx)^2.\ell n^2(x+h)}$

$= \lim\limits_{h\to 0}\dfrac{2(x+h)(\ell nx)^2 - x^2 2(\ell n(x+h)).\dfrac{1}{x+h}}{(\ell nx)^2.\ell n^2(x+h)+h.(\ell nx)^2.\dfrac{2\ell n(x+h)}{x+h}}$

$= \lim\limits_{h\to 0}\dfrac{2x(\ell nx)^2-2x\ell nx}{(\ell nx)^2\,(\ell nx)^2} = \dfrac{2x(\ell nx-1)}{(\ell nx)^3}$

2. $D^*g(x) = \lim\limits_{h\to 0}\dfrac{(x+h)^{2(x+h)}-x^{2x}}{h}$

$= \lim\limits_{h\to 0}\dfrac{(x+h)^{2(x+h)}.\left(2(x+h)\dfrac{1}{x+h}+2\ell n(x+h)\right)-0}{1}$

$= \lim\limits_{h\to 0} x^{2x}\,(2 + 2\ell nx)$

at x = 1, D*(g(x)) = 1(2 + 0) = 2

3. $([P + 1])^2 + [P]^2 - 2[P + 1] - 15 < 0$

$\because$ [P + 1] = [P] + 1

Let t = [P]

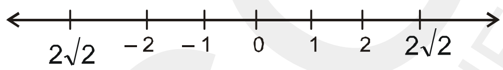
$(t + 1)^2 + t^2 - 2t - 2 - 15 < 0$

$\Rightarrow t^2 + 2t + 1 + t^2 - 2t - 17 < 0$

$\Rightarrow 2t^2 - 16 < 0$

$t^2 - 8 < 0$

$-2\sqrt{2} < t < 2\sqrt{2}$



t is integer

when t = – 2 = [P] $\Rightarrow$ P $\in$ [–2, – 1)

when t = – 1 $\Rightarrow$ P $\in$ [– 1, 0)

when t = 0 $\Rightarrow$ P $\in$ [0, 1)

when t = 1 $\Rightarrow$ P $\in$ [1, 2)

when t = 2 $\Rightarrow$ P $\in$ [2, 3)

$\Rightarrow$ P $\in$ [– 2, 3)

4. $\dfrac{x}{5} + \dfrac{y}{b} = 1$

$\dfrac{13}{5} + \dfrac{32}{b} = 1 \Rightarrow \dfrac{32}{b} = -\dfrac{8}{5} \Rightarrow b = -20$

$\dfrac{x}{5} - \dfrac{y}{20} = 1 \Rightarrow 4x - y = 20$

Line K has same slope $\Rightarrow -\dfrac{3}{c} = 4$

$c = -\dfrac{3}{4} \Rightarrow 4x - y = -3$

distance = $\dfrac{23}{\sqrt{17}}$

Hence correct option is (3)

5. Slope of normal at x = 0

$\dfrac{3}{1} = \dfrac{-1}{f'(0)}$

$\Rightarrow f'(0) = \dfrac{-1}{3}$

Now $\lim\limits_{x\to 0}\dfrac{x^2}{f(x^2)-5f(4x^2)+4f(7x^2)}$

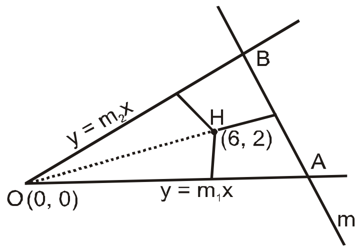
$= \lim\limits_{x\to 0}\dfrac{2x}{2xf'(x^2)-40xf'(4x^2)+56xf'(7x^2)}$

$= \dfrac{1}{9(-1/3)} = \dfrac{-1}{3}$

6. $m_1 + m_2 = -\dfrac{10}{3}$, $m_1m_2 = \dfrac{1}{3}$

Equation of line y = –3x + c

$\Rightarrow$ 3x + y – c = 0 ..... (i)



also point of intersection A is $\left(\dfrac{c}{3+m_1},\dfrac{cm_1}{3+m_1}\right)$

and (slope of AH) (slope of OB) = – 1

$\Rightarrow \dfrac{\left(\dfrac{cm_1}{3+m_1}-2\right)}{\left(\dfrac{c}{3+m_1}-6\right)} \times m_2 = -1$

$\Rightarrow (c - 2)m_1 m_2 = (-c + 18) + 6(m_1 + m_2)$

$(c - 2) = -3c + 54 - 60$

$\Rightarrow$ c = – 1

$\Rightarrow$ line 3x + y + 1 = 0 ans.

7. $f'(x) = \begin{cases} a(2x-1) & , \quad x<1 \\ 1 & , \quad 1\le x\le 3 \\ 2cx+d & , \quad x>3 \end{cases}$

(i) f(x) is continuous for all x.

$\therefore$ at x = 1

$\therefore$ b = 0 ...(1)

at x = 3 ;

LHL = Lt (3 – h – 1) = 2

RHL = $\underset{h\to 0}{\text{Lt}}$ c(3 + h)^2 + d(3 + h) + 2 = 9c + 3d + 2

$\therefore$ 9c + 3d + 2 = 2 or 3c = – d ...(2)

(ii) f '(1) does not exist means that the function f(x) is not differentiable at x = 1

$\therefore$ L' $\neq$ R' at x = 1 for f(x).

$$L' = \underset{h\to 0}{\text{Lt}} \frac{a(1-h)(-h)-0}{-h} = a$$

$$R' = \underset{h\to 0}{\text{Lt}} \frac{a(1-h)}{h} = 1 \therefore a \neq 1 \quad ...(3)$$

(iii) f '(x) is continuous at x = 3

$\therefore$ L = R = V at x = 3 for f '(x)

L = 1, R = $\underset{h\to 0}{\text{Lt}}$ 2c(3 + h) + d = 6c + d

$\therefore$ 6c + d = 1 or –2d + d = 1, by (2)

$\therefore$ d = –1 and hence c = 1/3

Hence a $\neq$ 1, b = 0, c = 1/3, d = –1 are the required values.

8. L.H.L = f(1 – h)

$$= \underset{h\to 0}{\text{Lim}} \frac{(1-h)^2-1}{(1-h)^2-2|(1-h)-1|-1}$$

$$= \frac{h(h-2)}{h(h-4)} = \frac{1}{2}$$

f(1) = 1/2

$$\underset{h\to 0}{\text{Lim}} f(1+h) = \underset{h\to 0}{\text{Lim}} \frac{(1+h)^2-1}{(1+h)^2-2|(1+h-1)|-1}$$

$$= \underset{h\to 0}{\lim} \frac{h^2+2h}{h^2}$$

$$= \underset{h\to 0}{\text{Lt}} \left(1+\frac{2}{h}\right) = \infty$$

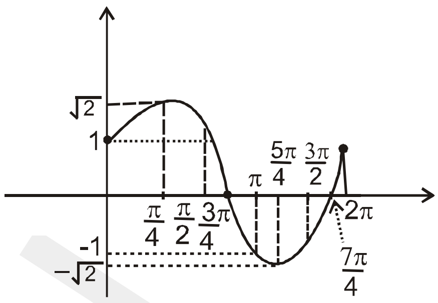
Since LHL $\neq$ RHL

$\therefore$ function is discontinuous at x = 1

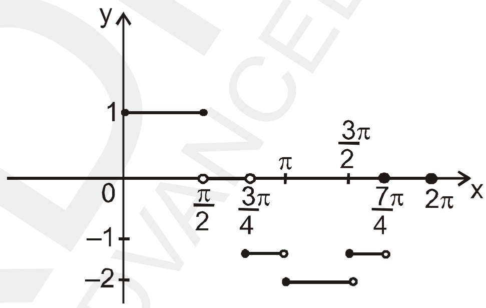
DPP NO. - 27

1. $f(x) = [\sin x + \cos x] = \left[\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)\right]$

graph of y = $\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$



graph of y = $\left[\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)\right]$



Discontinuous point at x = $\frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{3\pi}{2}, \frac{7\pi}{4}$ five points.

2. $\because \underset{x\to 0}{\text{Lim}} \frac{\sin[\cos x]}{1+[\cos x]}$, Let f(x) = $\frac{\sin[\cos x]}{1+[\cos x]}$

check at x = 0

L.H.L. = $\underset{x\to 0^-}{\text{Lim}} f(x) = \underset{h\to 0}{\text{Lim}} \frac{\sin[\cos(0-h)]}{1+[\cos(0-h)]}$

$= \underset{h\to 0}{\text{Lim}} \frac{\sin[\cos h]}{1+[\cos h]} = 0$

R.H.L. = $\underset{x\to 0^+}{\text{Lim}} f(x) = \underset{h\to 0}{\text{Lim}} f(0+h)$

$= \underset{h\to 0}{\text{Lim}} \frac{\sin[\cos h]}{1+[\cos h]} = 0$

$\therefore$ L.H.L. = R.H.L. $\therefore \underset{h\to 0}{\text{Lim}} \frac{\sin[\cos h]}{1+[\cos h]} = 0$

3. $\frac{dx}{d\theta} = -\sin\theta,\ \frac{dy}{d\theta} = 3\sin^2\theta\cos\theta$

So, that $\frac{dy}{dx} = -3\sin\theta\cos\theta$

$$\frac{d^2y}{dx^2} = -3\cos 2\theta \frac{d\theta}{dx} = \frac{3\cos 2\theta}{\sin\theta}$$

$$\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} = 9\sin^2\theta\cos^2\theta \cdot \frac{3\cos 2\theta}{\sin\theta}$$

$$= 9\sin^2\theta\cos^2\theta + 3\sin^2\theta\cos 2\theta$$

So, $$\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} + 3\Bigg|_{\theta=\pi/3} = \frac{57}{16}$$

4. $4a^2 + c^2 + 4ac = b^2$

$\Rightarrow$ $(2a + c)^2 = b^2$ $\Rightarrow$ $2a + c = \pm b$

$\Rightarrow$ $2a + c = b$, $2a + c = -b$

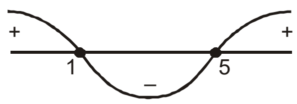
$2a - b + c = 0$, $2a + b + c = 0$

$a(2) + b(-1) + c = 0$, and $a(2) + b(1) + c = 0$

point (2, –1) and (2, 1) Ans. (B), (D)

5. $\because$ $f(x) = x^3 - 9x^2 + 15x + 6$

$\therefore$ $f'(x) = 3x^2 - 18x + 15 = 3(x-1)(x-5)$



$f'(x) < 0,\ x \in (1, 5)$

$f'(x) > 0,\ x \in (-\infty, 1) \cup (5, \infty)$ $(\because f(0) = 6)$

$\therefore$ $f(x) = 6 \Rightarrow x^3 - 9x^2 + 15x = 0$

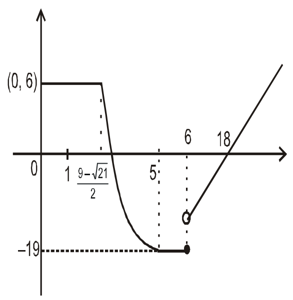
$\Rightarrow$ $x = 0,\ \frac{9 \pm \sqrt{21}}{2}$

Here $\frac{9+\sqrt{21}}{2} > 6$

$\therefore$ $x = \frac{9-\sqrt{21}}{2}$

Then

$$g(x) = \begin{cases} 6 & , \quad 0 \le x < \frac{9-\sqrt{21}}{2} \\ x^3 - 9x^2 + 15x + 6 & , \quad \frac{9-\sqrt{21}}{2} \le x \le 5 \\ -19 & , \quad 5 \le x \le 6 \\ x - 18 & , \quad x > 6 \end{cases}$$



6. $f(x) = \begin{cases} -x & , \ x \le 1 \\ 3 + x & , \ x > 1 \end{cases}$, $g(x) = \begin{cases} 3x & , \ x \le 1 \\ 2 + x & , \ x > 1 \end{cases}$

$$f(g(x)) = \begin{cases} -g, & g \le 1 \\ 3 + g, & g > 1 \end{cases}$$

$$= \begin{cases} -3x & , \ 3x \le 1 & , \ x \le 1 \\ 3 + 3x & , \ 3x > 1 & , \ x \le 1 \\ -2 - x & , \ 2 + x \le 1 & , \ x > 1 \\ 5 + x & , \ 2 + x > 1 & , \ x > 1 \end{cases}$$

$$= \begin{cases} -3x & , \ x \le 1/3 \\ 3 + 3x & , \ \frac{1}{3} < x \le 1 \\ -2 - x & , \ x = \phi \\ 5 + x & , \ x > 1 \end{cases}$$

7. $\cos^{-1}\left(\frac{y}{a}\right) = \log\left(\frac{x}{n}\right)^n$

$= n\log x - n\log n$

$$\frac{-1}{\sqrt{1 - \frac{y^2}{a^2}}}\left(\frac{y_1}{a}\right) = \frac{n}{x}$$

$\Rightarrow -xy_1 = n\sqrt{a^2 - y^2}$

$\Rightarrow x^2y_1^2 = n^2(a^2 - y^2)$

$\Rightarrow 2x^2y_1y_2 + y_1^2 2x = -2n^2yy_1$

$\Rightarrow 2x^2y_2 + 2xy_1 = -2n^2y$

$\Rightarrow 2x^2y_3 + 4xy + 2y_1 + 2xy_2 = -2n^2y_1$

$\Rightarrow 2x^2y_3 + 6xy_2 + (2 + 2n^2)y_1 = 0$

$\Rightarrow 2x^2y_4 + 4xy_3 + 6xy_3 + 6y_2 + (2 + 2n^2)y_2 = 0$

$\Rightarrow 2x^2y_4 + 10xy_3 + (8 + 2n^2)y_2 = 0$

$\Rightarrow x^2y_4 + 5xy_3 + (4 + n^2)y_2 = 0$

$\therefore$ $4 + n^2 = 8 \Rightarrow n^2 = 4$

$n = \pm 2$

8. $d = a\sin\theta$ .... (i)

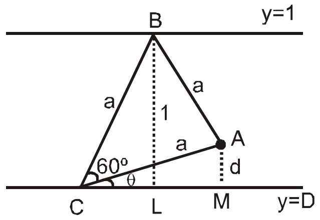
BL = $1 = QR\sin(\theta + 60^\circ)$

$1 = a\sin(\theta + 60^\circ)$

$\Rightarrow$ $1 = a\left(\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta\right)$ .... (ii)

$$\left(1 - \frac{a}{2}\sin\theta\right)^2 = \frac{3}{4}a^2\cos^2\theta = \frac{3}{4}a^2(1 - \sin^2\theta)$$

$$\left(1-\frac{d}{2}\right)^2 = \frac{3a^2}{4}\left(1-\frac{d^2}{a^2}\right)$$

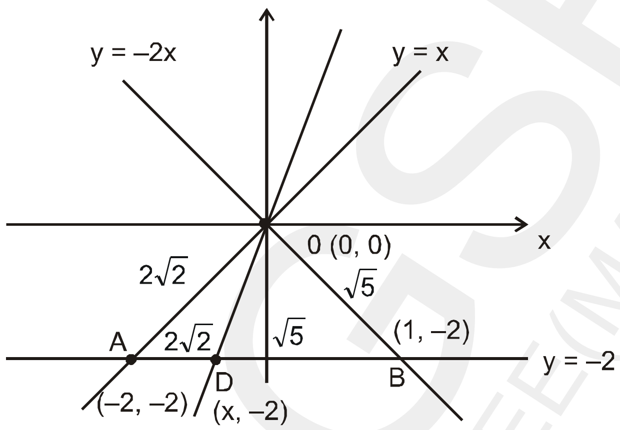


$\Rightarrow \; 4 - 4d + d^2 + 3d^2 = 3a^2$

$$a = 2\sqrt{\frac{d^2 - a + 1}{3}}$$

DPP NO. - 28

1. (C)



$\therefore$ AD : DB = $2\sqrt{2} : \sqrt{5}$

$\because$ OD is angle bisector of angle AOB

$\therefore$ St : 1 true

St. 2 false (obvious) **Ans.**

2. $5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$, $x + ky = 1$

Homogenous form

$\Rightarrow 5x^2 + 12xy - 6y^2 + 4x(x+ky) - 2y(x+ky) + 3(x+ky)^2 = 0$

Now for equally inclined to the coordinate axes cofficient of xy = 0

$\Rightarrow 12 + 4k - 2 + 6k = 0 \Rightarrow 10k = -10 \Rightarrow k = -1$

3. Let point p(t, t)

P, A′, B collinear

$m_{PA'} = m_{AB}$

$$\frac{t-3}{t-4} = \frac{13-3}{7-4} = \frac{10}{3}$$

$3t - 9 = 10t - 40$

$7t = 31$

$$t = \frac{31}{7}$$

$$\left(\frac{31}{7}, \frac{31}{7}\right)$$

4. Let Line be $ax + by + c = 0$

$P_1 + P_2 + P_3 = 0$

$$\Rightarrow \left|\frac{2a+c}{\sqrt{a^2+b^2}}\right| + \left|\frac{2b+c}{\sqrt{a^2+b^2}}\right| + \left|\frac{a+b+c}{\sqrt{a^2+b^2}}\right| = 0$$

$\Rightarrow 3a + 3b + 3c = 0$

$a + b + c = 0$

$a(1) + b(1) + c = 0$

Line passes through (1, 1)

5. line $x - y - 2 = 0$ put $y = 0$, $x = 2$

point A(2, 0)

equation of line

$$y - 0 = \frac{b}{a}(x-2) \quad \Rightarrow \quad bx - ay - 2b = 0$$

6. Let chord be $y = mx + c$

$y - mx = c$

Homogenous equation

$$4x^2 + y^2 - x\left(\frac{y-mx}{c}\right) + 4y\left(\frac{y-mx}{c}\right) = 0$$

for subtend right angle at origin, coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow \left(4+\frac{m}{c}\right) + \left(1+\frac{4}{c}\right) = 0 \quad \Rightarrow \quad 5 + \frac{m}{c} + \frac{4}{c} = 0 \quad \Rightarrow$$

$$\left(-\frac{4}{5}\right) = \frac{1}{5}(m) + c$$

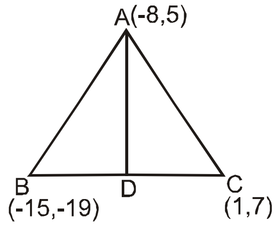
compare with $y = mx + c$

$$x = \frac{1}{5},\; y = \frac{-4}{5} \qquad \text{point } \left(\frac{1}{5}, -\frac{4}{5}\right)$$

7. AB = 25, AC = 15

$$\text{Ratio} = \frac{BD}{DC} = \frac{25}{15} = \frac{5}{3} \Rightarrow 5 : 3$$

Coordinate of D is $\left(\frac{5\times1+3}{5+3},\frac{-35-57}{5+3}\right)$



$= \left(-5, \frac{-92}{8}\right) = \left(-5, \frac{-23}{2}\right)$

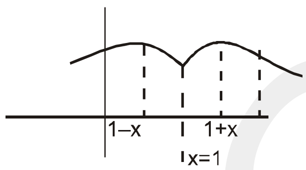
equation of AD is $y - 5 = \frac{\frac{-23}{2}-5}{-5+8}(x + 8) \Rightarrow 11x + 2y + 78 = 0$

8. f(x) is symmetric about x = 1

$\Rightarrow f(1 + x) = f(1 - x)$

replace x by x – 1

$f(x) = f(1 - (x - 1)) \Rightarrow f(x) = f(2 - x)$



9. Homogenous $px^2 + 2axy + qy^2 = r(ax + by)^2$

subtend right angle at origin

coff. of x^2 + coff. of $y^2 = 0 \Rightarrow (p - a^2r) + (q - b^2r) = 0 \Rightarrow r(a^2 + b^2) = p + q$.

10. (A) Let $ax^2 + 2hxy + by^2 + 2gx + 2fy + c \equiv (\ell_1x + m_1y + n_1)(\ell_2x + m_2y + n_2)$

comparing cofficients

$\ell_1\ell_2 = a$, $m_1m_2 = b$ $n_1n_2 = c$

$\ell_1 m_2 + \ell_2 m_1 = 2h$, $\ell_1n_2 + \ell_2n_1 = 2g$,

$m_1n_2 + m_2 n_1 = 2f$

(B) $af^2 + bg^2 + ch^2 = abc + 2fgh$ $(\because \Delta = 0)$

(C) $\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$ if lines are parallel

$\theta = 0^\circ$

$\Rightarrow h^2 = ab$

(D) Product of perpendiculars from the origin origin to the lines is

$$= \frac{|n_1|}{\sqrt{\ell_1^2+m_1^2}}\cdot\frac{|n_2|}{\sqrt{\ell_2^2+m_2^2}}$$

$$= \frac{|n_1n_2|}{\sqrt{\ell_1^2\ell_2^2+\ell_1^2m_2^2+\ell_2^2m_1^2+m_1^2m_2^2}}$$

$$= \frac{|n_1n_2|}{\sqrt{\ell_1^2\ell_2^2+(\ell_1m_2+\ell_2m_1)^2-2\ell_1\ell_2m_1m_2+m_1^2m_2^2}}$$

$$= \frac{|c|}{\sqrt{a^2+4h^2-2ab+b^2}} = \frac{|c|}{\sqrt{(a-c)^2+4h^2}}$$

DPP NO. - 29

1. We apply check for continuity at x = 0

$\text{LHS} = \lim_{x\to0^-} f(x) = \lim_{h\to0}(0 - h)$

$= \lim_{h\to0}(\cos h + \sin h)^{-\text{cosec } h}$ $(1^\infty$ from)

$= \exp\{\lim_{h\to0}(\cos h + \sin h - 1) \times -\text{cosec } h\}$

$$= \exp\left\{\lim_{h\to0}\left(-2\sin^2\frac{h}{2}+2\sin\frac{h}{2}\cos\frac{h}{2}\right)\times-\frac{1}{2\sin\frac{h}{2}\cos\frac{h}{2}}\right\} =$$

$$\exp\left\{\lim_{h\to0}\left(\frac{\sin\frac{h}{2}-\cos\frac{h}{2}}{\cos\frac{h}{2}}\right)\right\} = e^{-1}$$

$\text{RHL} = \lim_{x\to0^+} f(x) = \lim_{h\to0} f(0 + h)$

$$= \lim_{h\to0}\frac{e^{1/h}+e^{2/h}+e^{3/h}}{ae^{2/h}+be^{3/h}}$$

$$= \lim_{h\to0}\frac{e^{3/h}(e^{-2/h}+e^{-1/h}+1)}{(e^{3/h}\{ae^{-1/h}+b\})} \qquad [\because \lim_{h\to0} e^{-1/h} = 0]$$

$\therefore$ For continuity at x = 0

$e^{-1} = a = b^{-1} \Rightarrow a = \frac{1}{e}$, $b = e$

2. $ax^2 + bx + c$ = has imaginary roots and $f(-1) > 0 \Rightarrow ax^2 + bx + c > 0$

$|(ax^2 + bx + c) + (x + y)| = |ax^2 + bx + c| + |x + y|$

possible when $ax^2 + bx + c > 0$, $x + y > 0$

3. $y(y^2 - \cot^2x) = 0$ at $x = \frac{\pi}{4}$, $y = \pm 1$

$2yy' = -2\cot x \text{ cosec}^2x$ and y = 0

$y' = -\frac{2}{(\pm 1)} = \mp 2$

Tangent $\quad y - 1 = -2\left(x - \frac{\pi}{4}\right)$

$\Rightarrow \quad 4x + 2y = \pi + 2$

and $\; y + 1 = 2\left(x - \frac{\pi}{4}\right) \Rightarrow 4x - 2y = \pi + 2$

5. $\frac{dy}{dx} = -\sin(x + y).\left(1 + \frac{dy}{dx}\right)$

$-\frac{1}{2} = -(x + y) \cdot \frac{1}{2}$

$\sin(x + y) = 1$

$\Rightarrow \cos(x + y) = 0 \qquad \Rightarrow y = 0$

$\Rightarrow \cos x = 0 \quad \Rightarrow \quad x = -\frac{3\pi}{2}, \frac{\pi}{2}$

points are $\left(\frac{\pi}{2}, 0\right)$ and $\left(-\frac{3\pi}{2}, 0\right)$

$x + 2y = \frac{\pi}{2}, \qquad x + 2y = -\frac{3\pi}{2}$

6. $a\,\ell n\, y = \tan\left(\frac{x + b}{2a}\right)$

diff. w.r.t. x

$\frac{a}{y}.y' = \sec^2\left(\frac{x+b}{2a}\right).\frac{1}{2a}$

$\Rightarrow \frac{2a^2 y'}{y} = 1 + \tan^2\left(\frac{x+b}{2a}\right) = 1 + a^2(\ell n\, y)^2$

dividing both sides by a^2 we get

$\frac{2y}{y'} = \frac{1}{a^2} + (\ell n\, y)^2$

diff. w.r.t. x

$\frac{y(2y'') - 2(y')(y')}{y^2} = 0 + \frac{2\ell n y}{y} y'$

7. The given curve is

$y = (1 + x)^y + \sin^{-1}(\sin^2 x)$

at x = 0, y = 1

Point is (0, 1)

Rewritten the given curve (1) is

$y = e^{y\,\ell n(1+x)} + \sin^{-1}(\sin^2 x)$

$\frac{dy}{dx} = e^{y\,\ell n\,(1+x)}\left[\frac{y}{(1+x)} + \ell n(1+x)\frac{dy}{dx}\right]$

$+ \frac{1}{\sqrt{1 - \sin^4 x}}(2\sin x \cos x)$

$\Rightarrow \frac{dy}{dx}\left(1 - e^{y\,\ell n\,(1+x)}\,\ell n(1 + x)\right)$

$= \frac{y}{(1+x)} e^{y\,\ell n(1+x)} + \frac{\sin 2x}{\sqrt{1 - \sin^4 x}}$

$\Rightarrow \frac{dy}{dx} = \frac{\frac{y}{1+x}(1+x)^y + \frac{\sin 2x}{\sqrt{1-\sin^4 x}}}{1 - (1+x)^y\,\ell n(1+x)}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(0,1)} = \frac{1+0}{1-0} = 1$

The slope of normal = –1

Thus the required equation of the normal is

$y - 1 = (-1)(x - 0)$

i.e., $y + x - 1 = 0$

8. $\frac{dx}{dt} = a(1 + \cos t)$

$\frac{dy}{dt} = a \sin t$

$\frac{dy}{dx} = \tan\frac{t}{2}$

$\frac{d^2y}{dx^2} = \sec^2\left(\frac{t}{2}\right)\frac{dt}{dx} = \sec^2\left(\frac{t}{2}\right).\frac{1}{a(1+\cos t)}$

$= \frac{1}{2a}\sec^4\left(\frac{t}{2}\right)$

$\frac{d^3y}{dx^2} = \frac{4}{2a}\sec^3\left(\frac{t}{2}\right).\left(\sec\frac{t}{2}\tan\frac{t}{2}\right).\frac{dt}{dx}$

$= \frac{2}{a}\sec^4\left(\frac{t}{2}\right)\tan\frac{t}{2}.\frac{1}{a(1+\cos t)}$

$= \frac{1}{a^2}\sec^6\left(\frac{t}{2}\right)\tan\left(\frac{t}{2}\right)$

DPP NO. - 30

1. $y = \left|x^2 - |x|\right|$

at x = –2, |x| = –x

$y = \left|x^2 + x\right|$

$y = x^2 + x \qquad (\because \text{ at } x = -2, \left(x^2 + x\right) > 0) \quad \Rightarrow$

$\frac{dy}{dx} = 2x + 1$

slope of normal = $\frac{-1}{2x+1} = \frac{-1}{-4+1} = \frac{1}{3}$.

2. Let tangent be drawn at A($3t^2$, $2t^3$) and it is normal at B($3t_1^2, 2t_1^3$).

Slope of tangent at A = $\frac{dy}{dt} + \frac{dx}{dt} = t$

Its equation is $(y - 2t^3) = t(x - 3t^2)$
or $y = tx - t^3$ ...(1)
Slope of normal at B = $-1/t_1$
But they represent the same line.

$\therefore\ t = -\frac{1}{t_1}$ or $tt_1 = -1$

Now tangent at A is $tx - y - t^3 = 0$

It passes through t_1 $\therefore\ t.3t_1^3 - 2t_1^3 - t^3 = 0$

or $3t_1^2(t - t_1) - (t^3 - t_1^3) = 0$

or $3t_1^2 - (t^2 + tt_1 + t_1^2) = 0$

or $2t_1^2 - tt_1 - t^2 = 0$ or $(t_1 - t)(2t_1 + t) = 0$

$\therefore\ t = -2t_1 = \frac{2}{t}$ by (2) $\therefore\ t^2 = 2$ or $t = \pm\sqrt{2}$

putting for t in (1), equation of tangent is

$y = \pm\sqrt{2}(x - 2)$

3. Any point P(h, k) on $x^2 - y^2 = a^2$ is ($a\sec\theta$, $a\tan\theta$)
This point will be nearest to y = 2x, if tangent at this point is parallel to y = 2x.

Now $x^2 - y^2 = a^2 \Rightarrow 2x - 2y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{x}{y} \Rightarrow \left(\frac{dy}{dx}\right)_{(a\sec\theta,\ a\tan\theta)} = \operatorname{cosec}\theta$

Slope of y = 2x is 2 $\Rightarrow \operatorname{cosec}\theta = 2$ or $\theta = \pi/6$

Thus, P(h, k) ≡ [a sec π/6, a tan π/6] ≡ $\left(\frac{2a}{\sqrt{3}}, \frac{a}{\sqrt{3}}\right)$

$\therefore\ h = \frac{2a}{\sqrt{3}}$, $k = \frac{a}{\sqrt{3}} \Rightarrow k = h/2$

So the required locus is 2y – x = 0

4. The curve intersect when $1 < x^2 < 3$ or $1 < x < \sqrt{3}$

or $-\sqrt{3} < x < -1$

Hence the given curves are $y = x^2 - 1$ ...(i)
and $y = 3 - x^2$ ...(ii)

The points of intersection are ($\pm\sqrt{2}$, 1)

$\therefore$ R ≡ ($\sqrt{2}$, 1) & S ≡ ($-\sqrt{2}$, 1)

At ($\sqrt{2}$, 1)

Slope of curve (i), $m_1 = 2x = 2\sqrt{2}$

Slope of curve (ii), $m_2 = -2x = -2\sqrt{2}$

$\therefore\ \tan\theta = \left|\frac{2\sqrt{2} + 2\sqrt{2}}{1 - 8}\right| = \frac{4\sqrt{2}}{7}$

5. $\frac{dx}{d\theta} = 2a\cos 2\theta(1 + \cos 2\theta) + a\sin 2\theta(-2\sin 2\theta)$

$= 2a\cos 2\theta + 2a\cos 4\theta = 4a\cos 3\theta\cos\theta$

$\frac{dy}{d\theta} = -2a\sin 2\theta(1 - \cos 2\theta) + a\cos 2\theta(2\sin 2\theta)$

$= -2a\sin 2\theta + 4a\sin 2\theta\cos 2\theta$

$= 2a(\sin 4\theta - \sin 2\theta)$

$= 4a(\cos 3\theta\sin\theta)$

$\frac{dy}{dx} = \tan\theta$

$\frac{d^2y}{dx^2} = \sec^2\theta\,\frac{d\theta}{dx} = \sec^2\theta\,\frac{1}{4a\cos 3\theta\cos\theta}$

$= \frac{1}{4a\cos 3\theta\cos\theta}$

Now $\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}} = \sec^3\theta$

$= \frac{1}{\cos^3\theta}$

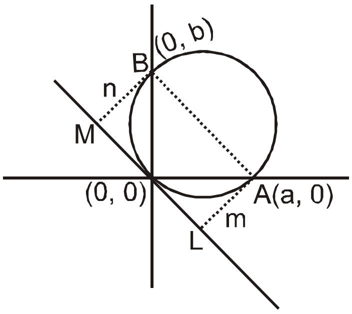
6. Any point on curve is $(h, 4h^3 - 2h^5)$
Slope of tangent = $12h^2 - 10h^4$

$\Rightarrow 12h^2 - 10h^4 = \frac{4h^3 - 2h^5 - 0}{h - 0}$

$h^2(1 - h^2) = 0$
$\Rightarrow$ h = 0, 0, –1, 1
Required points are (0, 0), (–1, –2), (1, 2)

7. Equation of circle $x(x - a) + y(y - b) = 0$
(AB as diameter)
$x^2 + y^2 - ax - by = 0$
Tangent at (0, 0)
ax + by = 0

$m = \frac{a^2}{\sqrt{a^2 + b^2}}$, $n = \frac{b^2}{\sqrt{a^2 + b^2}}$



$m + n = \sqrt{a^2 + b^2}$

$\Rightarrow \quad a = \pm \sqrt{m(m+n)}$, $b = \pm \sqrt{n(m+n)}$

$x^2 + y^2 \pm \sqrt{m(m+n)}\, x \pm \sqrt{n(n+m)}\, y = 0$

8. Equation of tangents $y = mx + \frac{4}{m}$ is tangent to circle $x^2 + y^2 = 8$

Now $\frac{16}{m^2} = 8(1 + m^2) \Rightarrow m^4 + m^2 - 2 = 0 \Rightarrow$

$(m^4 + 2)(m^2 - 1) = 0 \Rightarrow m = \pm 1$

tangents $y = x + 4$ and $y = -x - 4$

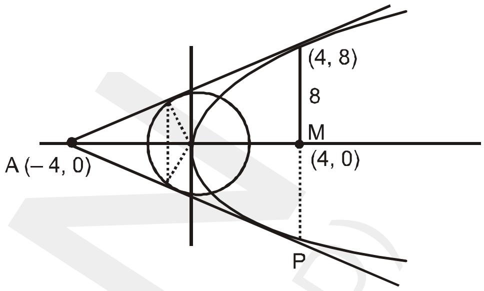
Any point of parabola $(4t^2, 8t)$ lie on $y = x + 4$

$8t = 4t^2 + 4 \Rightarrow t^2 - 2t + 1 = 0$

$(t - 1)^2 = 0$

$t = 1$

Point (4, 8) lie on parabola



AM = 8, PM = 8

Area of $\Delta AMP = \frac{1}{2}(8)(8) = 32$

Area of $APP' = 2(32) = 64$

$L = \sqrt{16 + 0 - 8} = 2\sqrt{2}$

$\text{Area} = \frac{RL^3}{R^2 + L^2} = \frac{(2\sqrt{2})(16\sqrt{2})}{8 + 8} = 4$

Required Area = 64 – 4 = 60 Ans.