Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-21 to 22

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-21

[SINGLE CORRECT CHOICE TYPE] **[6 × 3 = 18]**

Q.1 $\lim\limits_{x\to 2} \dfrac{\sqrt{4-x^2}\,\sqrt{\cos\frac{\pi x}{4}}}{\sin(x-2)}$ is

(A) equal to $\sqrt{\pi}$ (B) equal to -2 (C) equal to $-\sqrt{\pi}$ (D) non existent

Q.2 Let $a_1, a_2, a_3, a_4, \ldots\ldots\ldots\, a_n$ be an A.P (all terms being distinct). If $a_1-\frac{a_2}{2},\ a_3-\frac{a_2}{2},\ a_4-\frac{a_2}{2}$ are in G.P. then a_8 is equal to

(A) 0 (B) -2 (C) 3 (D) 4

Q.3 $\lim\limits_{x\to\infty}\left(\sqrt{x^2+4x}-x\right)$ equals

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4

Q.4 If $\lim\limits_{x\to 0}\dfrac{\tan^3 x+\tan^3 2x+\tan^3 3x}{\tan^3 4x+\tan^3 5x+\tan^3 6x}=\dfrac{p}{q}$ (p, q ∈ N), then least value of (p + q) is

(A) 45 (B) 47 (C) 49 (D) 51

Q.5 If $g(x) = 1+\sqrt{x}$ and $f(g(x)) = 3+2\sqrt{x}+x$, then $f(x)$ is

(A) $1 + 2x^2$ (B) $2 + x^2$ (C) $1 + x$ (D) $2 + x$

Q.6 The value of $\sum\limits_{r=2}^{\infty}\tan^{-1}\left(\dfrac{1}{r^2-5r+7}\right)$, is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) $\frac{5\pi}{4}$

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

[SINGLE CORRECT CHOICE TYPE] [6 × 3 = 18]

Q.1 Let $f(x) = \begin{cases} \left(1+\ln(c^2+c+1)\tan^2(x-1)\right)^{\frac{1}{(\ln x)^2}} & , x \neq 1 \\ 3c & , x = 1 \end{cases}$, where $c \in R$

If $\lim\limits_{x\to1} f(x)$ exists but $f(x)$ is discontinuous at $x = 1$ then c can not take the value

(A) 1 (B) 2 (C) 3 (D) 4

Q.2 If $\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = -\frac{7\pi}{4}$ where $x \in (-1, 0)$ then x is equal to

(A) $2-\sqrt{3}$ (B) $-\frac{1}{\sqrt{3}}$ (C) $1-\sqrt{2}$ (D) $\sqrt{2}-1$

Q.3 The number of solution of the equation $\text{sgn}(\sin x) - \text{sgn}(\sin^2 x) = \sin^2 x + 2\sin x$ in $\left[\frac{-5\pi}{2}, \frac{7\pi}{2}\right]$ is

[**Note:** sgn(k) denotes signum function of k.]

(A) 10 (B) 6 (C) 13 (D) 9

Q.4 Three boys and three girls are to be seated around a table in a circle. Among them the boy X does not want any girl neighbour and the girl Y does not want any boy neighbour. The number of such arrangements possible is

(A) 6 (B) 4 (C) 8 (D) 9

Q.5 If the equation $x^2 + x - b^2 = 0$ and $x^2 - bx - 1 = 0$ have a common root, then number of values of b is/are

(A) 0 (B) 1 (C) 2 (D) 3

Q.6 Let $f : R \to R$, $g : R \to R$, $h : R \to R$ be derivable functions such that $f(x) = x^3 + x + 1$, $g(f(x)) = x$ and $h(g(g(x))) = 2x \ \forall\ x \in R$, then $h'(-1)$ is equal to

(A) 4 (B) 8 (C) 16 (D) 32

Dpp. No.-21 TO 22 (SOLUTIONS)

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-21

[SINGLE CORRECT CHOICE TYPE] **[6 × 3 = 18]**

Q.1 $\lim\limits_{x\to 2} \dfrac{\sqrt{4-x^2}\sqrt{\cos\dfrac{\pi x}{4}}}{\sin(x-2)}$ is

(A) equal to $\sqrt{\pi}$ (B) equal to -2 (C*) equal to $-\sqrt{\pi}$ (D) non existent

[Sol.$_{11/lcd/SC}$ $\lim\limits_{x\to 2} \dfrac{2\sqrt{2-x}\sqrt{\cos\dfrac{\pi x}{4}}}{(x-2)}$ put $x = 2 - h$

$$\lim_{h\to 0} \frac{2\sqrt{h}\sqrt{\cos\left(\dfrac{\pi}{2}-\dfrac{\pi h}{4}\right)}}{-h} = \lim_{h\to 0} \frac{2\sqrt{h}\sqrt{\sin\dfrac{\pi h}{4}}}{-h}$$

$$= \lim_{h\to 0} \frac{2h\sqrt{\dfrac{\pi}{4}}}{-h} = -\sqrt{\pi}$$. **Ans.**]

Q.2 Let $a_1, a_2, a_3, a_4, \ldots\ldots\ldots\ldots a_n$ be an A.P (all terms being distinct). If $a_1 - \dfrac{a_2}{2}$, $a_3 - \dfrac{a_2}{2}$, $a_4 - \dfrac{a_2}{2}$ are in G.P. then a_8 is equal to

(A*) 0 (B) -2 (C) 3 (D) 4

[Sol.$_{438/seq/SC}$ $a_2 = a_1 + d$

$a_3 = a_1 + 2d$

$\vdots$

$$a_1 - \frac{a_2}{2} = \frac{a_1 - d}{2}$$

$$a_3 - \frac{a_2}{2} = \frac{a_1 + 3d}{2}$$

$$a_4 - \frac{a_2}{2} = \frac{a_1 + 5d}{2}$$

$$\left(\frac{a_1 + 3d}{2}\right)^2 = \left(\frac{a_1 + 5d}{2}\right)\left(\frac{a_1 - d}{2}\right)$$

$a_1^2 + 9d^2 + 6a_1 d = a_1^2 - 5d^2 + 4a_1 d$

$14d^2 = -2a_1 d$

$\Rightarrow$ $d \neq 0$ (distinct)

Q.3 $\lim_{x\to\infty}\left(\sqrt{x^2+4x}-x\right)$ equals

(A) 1 (B) $\frac{3}{2}$ (C*) 2 (D) 4

[Sol.$_{693/lcd/SC}$ $\lim_{x\to\infty} x\left[\left(1+\frac{4}{x}\right)^{\frac{1}{2}}-1\right] = \lim_{x\to\infty} x\left(1+\frac{1}{2}\cdot\frac{4}{x}+\ldots\ldots-1\right) = 2$. **Ans.**]

Q.4 If $\lim_{x\to 0}\frac{\tan^3 x+\tan^3 2x+\tan^3 3x}{\tan^3 4x+\tan^3 5x+\tan^3 6x} = \frac{p}{q}$ (p, q $\in$ N), then least value of (p + q) is

(A) 45 (B) 47 (C*) 49 (D) 51

[Sol.$_{215/lcd/SC}$

$$\left[x^3\cdot\frac{\tan^3 x}{x^3}+8x^3\cdot\frac{\tan^3 2x}{(2x)^3}+27x^3\cdot\frac{\tan^3 3x}{(3x)^3}\right]\div\left[64x^3\cdot\frac{\tan^3 4x}{(4x)^3}+125x^3\cdot\frac{\tan^3 5x}{(5x)^3}+216x^3\cdot\frac{\tan^3 6x}{(6x)^3}\right]$$

$$= \frac{1+8+27}{64+125+216} = \frac{36}{405} = \frac{9\cdot 4}{9\cdot 45} = \frac{4}{45}$$ **Ans.**]

Q.5 If $g(x) = 1+\sqrt{x}$ and $f(g(x)) = 3+2\sqrt{x}+x$, then $f(x)$ is

(A) $1 + 2x^2$ (B*) $2 + x^2$ (C) 1 + x (D) 2 + x

[Sol.$_{200/func/SC}$ $f\left(1+\sqrt{x}\right) = 3+2\sqrt{x}+x$

Put $1+\sqrt{x} = y \Rightarrow f(y) = 2+y^2$.]

Q.6 The value of $\sum_{r=2}^{\infty}\tan^{-1}\left(\frac{1}{r^2-5r+7}\right)$, is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C*) $\frac{3\pi}{4}$ (D) $\frac{5\pi}{4}$

[Sol.$_{117/itf/SC}$ $\sum_{r=2}^{\infty}\tan^{-1}\left(\frac{(r-2)-(r-3)}{1+(r-3)(r-2)}\right) = \sum_{r=2}^{\infty}\left(\tan^{-1}(r-2)-\tan^{-1}(r-3)\right)$

$= \tan^{-1}0-\tan^{-1}(-1)$
$\tan^{-1}1-\tan^{-1}0$
$\tan^{-1}2-\tan^{-1}1$
:
:
$\tan^{-1}(n-2)-\tan^{-1}(n-3)$

$S_n = \tan^{-1}(n-2)+\frac{\pi}{4}$

$\therefore S_\infty = \frac{\pi}{2}+\frac{\pi}{4} = \frac{3\pi}{4}$. **Ans.**]

Dpp. No.-22

[SINGLE CORRECT CHOICE TYPE] **[6 × 3 = 18]**

Q.1 Let $f(x) = \begin{cases} \left(1+ln(c^2+c+1)\tan^2(x-1)\right)^{\frac{1}{(ln x)^2}} & , x \neq 1 \\ 3c & , x = 1 \end{cases}$, where $c \in R$

If $\lim\limits_{x\to1} f(x)$ exists but $f(x)$ is discontinuous at $x = 1$ then c can not take the value

(A*) 1 (B) 2 (C) 3 (D) 4

[Sol.$_{742/lcd(c)/SC}$ $\lim\limits_{x\to1} f(x) = e^{\lim\limits_{x\to1}\frac{ln(c^2+c+1)\tan^2(x-1)}{(ln\ x)^2}} = e^{ln(c^2+c+1)} = c^2 + c + 1$

$f(x)$ is discontinuous.

$\therefore$ $c^2 + c + 1 \neq 3c$

$\Rightarrow$ $(c-1)^2 \neq 0 \Rightarrow c \neq 1$ **Ans.**]

Q.2 If $\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = -\frac{7\pi}{4}$ where $x \in (-1, 0)$ then x is equal to

(A) $2-\sqrt{3}$ (B) $-\frac{1}{\sqrt{3}}$ (C*) $1-\sqrt{2}$ (D) $\sqrt{2}-1$

[Sol.$_{434/itf/SC}$ $2\tan^{-1}x - 4\left(\frac{\pi}{2} + 2\tan^{-1}x\right) + 4\tan^{-1}x = \frac{-7\pi}{4}$

$2\pi + \theta = \frac{\pi}{4} \Rightarrow \theta = -\frac{7\pi}{4}$

$-2\tan^{-1}x = \frac{\pi}{4} \Rightarrow \tan^{-1}x = \frac{-\pi}{8}$

$x = -\left(\sqrt{2}-1\right)$.]

Q.3 The number of solution of the equation $\text{sgn}(\sin x) - \text{sgn}(\sin^2 x) = \sin^2 x + 2\sin x$ in $\left[\frac{-5\pi}{2}, \frac{7\pi}{2}\right]$ is

[**Note:** sgn(k) denotes signum function of k.]

(A) 10 (B*) 6 (C) 13 (D) 9

[Sol.$_{468/func/SC}$ **Case-I :** $\sin x = 0 \Rightarrow x = -2\pi, -\pi, 0, \pi, 2\pi, 3\pi$

$\text{sgn}(\sin x) - \text{sgn}(\sin^2 x) = \sin^2 x + 2\sin x$

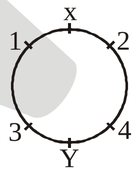
has 6 roots

Case-II : $\sin x \neq 0$

$\text{sgn}(\sin x) - 1 = \sin^2 x + 2\sin x$
$\text{sgn}(\sin x) = (1 + \sin x)^2$
no. real roots.]

Q.4 Three boys and three girls are to be seated around a table in a circle. Among them the boy X does not want any girl neighbour and the girl Y does not want any boy neighbour. The number of such arrangements possible is

(A) 6 (B*) 4 (C) 8 (D) 9



[Sol.$_{374/perm/SC}$ boys can be arranged in 2 ways and girls can be arranged in 2 ways.
boys will be together and girls will be together.
$\therefore$ total number of ways = 2 × 2 = 4 ways.]

Q.5 If the equation $x^2 + x - b^2 = 0$ and $x^2 - bx - 1 = 0$ have a common root, then number of values of b is/are

(A) 0 (B) 1 (C*) 2 (D) 3

Sol.$_{798/qe/SC}$ Let common root be α
$\alpha^2 + \alpha - b^2 = 0$ ....(1)
$\alpha^2 - b\alpha - 1 = 0$ ....(2)
Subtract (2) from (1), we get
$(b + 1)\alpha - b^2 + 1 = 0$
$\Rightarrow (b + 1)\alpha = b^2 - 1$
$\Rightarrow \alpha = b - 1$
Put in equation (1)
$(b - 1)^2 + (b - 1) - b^2 = 0$
$\Rightarrow b^2 - 2b + 1 + b - 1 - b^2 = 0$
$\Rightarrow b = 0.$
If both roots common then $b = -1$.]

Q.6 Let $f : R \to R$, $g : R \to R$, $h : R \to R$ be derivable functions such that $f(x) = x^3 + x + 1$, $g(f(x)) = x$ and $h(g(g(x))) = 2x \ \forall x \in R$, then $h'(-1)$ is equal to

(A) 4 (B) 8 (C) 16 (D*) 32

[Sol.$_{744/lcd(d)/SC}$ $\because h(g(g(x))) = 2x \Rightarrow h'(g(g(x))) \cdot g'(g(x)) \cdot g'(x) = 2$
Putting $g(g(x)) = -1 \Rightarrow g(x) = f(-1) = -1 \Rightarrow x = f(-1) = -1$
We get $h'(-1) \cdot g'(-1) \cdot g'(-1) = 2$

$\because g'(f(x)) = \dfrac{1}{f'(x)} \Rightarrow g'(-1) = \dfrac{1}{f'(-1)} = \dfrac{1}{4}$

$\therefore h'(-1) = 32$. **Ans.**]