

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-17 TO 20

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-17

[SINGLE CORRECT CHOICE TYPE]

[4 × 3 = 12]

- Q.1 If $\lim_{x \rightarrow k} [\sin^{-1} x + \cos^{-1} x + \tan^{-1} x]$ does not exist, then sum of the all possible values of k is
[Note : $[y]$ denotes greatest integer function less than or equal to y .]
- (A) $\frac{-\sin 3}{\sin 1 \cdot \sin 2}$ (B) $\frac{-\sin 2}{\sin 1 \cdot \sin 3}$ (C) $\frac{-\sin 1}{\sin 2 \cdot \sin 3}$ (D) $\frac{-\sin 1 \cdot \sin 2}{\sin 3}$
- Q.2 A three digit number n is such that the last two digits of it are equal and differ from the first. The number of such n 's is
(A) 64 (B) 72 (C) 81 (D) 900
- Q.3 Total number of term, that are dependent on the value of x , in the expansion of $\left(x^2 - 2 + \frac{1}{x^2}\right)^n$ is equal to
(A) $2n + 1$ (B) $2n$ (C) n (D) $n + 1$
- Q.4 A monic quadratic polynomial $y = ax^2 + bx + c$ having roots of opposite sign. Then which of the following is always incorrect ?
(A) $\frac{-b}{2a} > 0$ (B) $\frac{-b}{2a} < 0$ (C) $ac - ab^2 < 0$ (D) $ab^2 - ac < 0$

[MULTIPLE CORRECT CHOICE TYPE]

[2 × 4 = 8]

- Q.5 Let $f: \mathbb{R} - \{0\} \rightarrow [0, \infty)$ be a function defined by $f(x) = \frac{|x-1|}{x^2}$. Then
(A) $f(x)$ is injective in $(2, \infty)$ (B) $f(x)$ is one-one in $(0, 1)$
(C) $f(x)$ is surjective (D) $f(x)$ is many-one in $(-\infty, 0)$
- Q.6 If all the roots of the equation $x^3 - 3x = 0$ satisfy the equation
 $(\alpha - \sin^{-1}(\sin 2))x^2 - (\beta - \tan^{-1}(\tan 1))x + \gamma^2 - 2\gamma + 1 = 0$ then which of the following vanishes?
(A) $\sin \alpha - \sin(\beta + \gamma)$ (B) $\cos \alpha + \cos(\beta + \gamma)$
(C) $\tan \alpha - \tan(\beta + \gamma)$ (D) $\cot \alpha + \cot(\beta + \gamma)$

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-18

[SINGLE CORRECT CHOICE TYPE]

[6 × 3 = 18]

- Q.1 $\lim_{x \rightarrow 0} \frac{(\sin x)^{10} - x^{10}}{x^9(\sin x - x)}$ is equal to
(A) 9 (B) 10 (C) 54 (D) 60
- Q.2 Let $f(x) = \sqrt{\frac{1}{x^2 + 2\sqrt{c}x + 1}}$. If domain of $f(x)$ is $(-\infty, \infty)$, then the number of integers in the range of c is
(A) 3 (B) 2 (C) 1 (D) 0
- Q.3 If $5 \sin^{-1}x + 3 \cos^{-1}y - 2 \cos^{-1}z = \frac{11\pi}{2}$, then the value of $x^{2017} + y^{2018} + z^{2016}$ equals
(A) 0 (B) 1 (C) 2 (D) 3
- Q.4 If α, β are roots of $2x^2 - 5x + p = 0$, such that $0 < \alpha < 1$ and $1 < \beta < 3$, then the number of integral values of p is
(A) 0 (B) 1 (C) 2 (D) 3
- Q.5 In the expansion of $(x^2 + 1)(x^3 + 1)(x + 1)^5$, the coefficient of x^3 is
(A) 29 (B) 35 (C) 16 (D) 9
- Q.6 The letters of the word EARLY are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word EARLY is
(A) 25 (B) 26 (C) 27 (D) 28

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-19

[SINGLE CORRECT CHOICE TYPE]

[6 × 3 = 18]

- Q.1 $\lim_{n \rightarrow \infty} (2 \log(3n) - \log(n^2 + 1))$ is equal to
 (A) $4 \log 6$ (B) $-\log 3$ (C) $\log 3$ (D) $2 \log 3$
- Q.2 If the function $f(x) = \lambda |\sin x| + \lambda^2 |\cos x| + g(\lambda)$, $\lambda \in \mathbb{R}$ (g is a function of λ) is periodic with fundamental period $\frac{\pi}{2}$, then
 (A) $\lambda = 0, 1$ (B) $\lambda = 1$ (C) $\lambda = 0$ (D) $\lambda = -1$
- Q.3 If there are 10 stations on a route and the train has to be stopped at 4 of them, then the number of ways in which the train can be stopped so that at least two stopping stations are consecutive is
 (A) 7C_4 (B) ${}^{10}C_4 - {}^8C_3$ (C) ${}^{10}C_4 - {}^7C_3$ (D) 8C_3
- Q.4 If $\sin \theta$ and $\cos \theta$ are two distinct roots of $ax^2 + bx + c = 0 \forall a, b, c \in \mathbb{R}, a \neq 0$ then which of the following is correct?
 (A) $a^2 - b^2 = 2ac$ (B) $b^2 - a^2 = 2ac$ (C) $c^2 - a^2 = 2ab$ (D) $a^2 - c^2 = 2ab$
- Q.5 Let $f(x) = \cos^{-1}(\cos 2) + \sin^{-1}(\sin 1) x - x^2$. If $\text{sgn}(f(x))$ is maximum for true set of values of $x \in (a, b)$, then $(a + b)$ is equal to
 (A) 0 (B) 1 (C) -1 (D) 3
- Q.6 The value of $\lim_{n \rightarrow \infty} \sum_{r=1}^n \left(\frac{2r-1}{n^2 + 2r+1} \right)$ is equal to
 (A) 2 (B) -1 (C) 1 (D) 2

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-20

[SINGLE CORRECT CHOICE TYPE]

[4 × 3 = 12]

- Q.1 If the range of the function $f(x) = \log_{\frac{\pi^2}{4}}(\tan^{-1}(x^2 - 4x + 3))$ is $(-\infty, \lambda)$, then the value of λ is
- (A) 0 (B) 1 (C) 2 (D) $\frac{1}{2}$
- Q.2 Let $f(x) = \frac{ax^2 + 2b}{x - 1}$ if $\lim_{x \rightarrow 1} f(x) = 2$, then the value of $[a - 2b]$, is
[Note: Where $[k]$ denotes the greatest integer less than or equal to k .]
(A) 3 (B) 2 (C) 1 (D) -1
- Q.3 25 small squares of a 5×5 chess board are coloured with five different colours available, such that each row contain all five colours and no two adjacent squares have same colour is equal to $5!(k)^4$. Then number of prime factor(s) of k is/are
(A) -2 (B) 0 (C) 2 (D) 3
- Q.4 $\lim_{x \rightarrow \infty} \left(\frac{3x^3 + 2x^2 + 1}{2x + 4 + 7x^3} \right)^{\left(\frac{x^2 - 1}{2x} \right)}$ is equal to
(A) 0 (B) 1 (C) $e^{-\frac{3}{7}}$ (D) $e^{\frac{5}{7}}$

[MULTIPLE CORRECT CHOICE TYPE]

[2 × 4 = 8]

- Q.5 If $f(x)$ is a monic polynomial function of degree 4 satisfying $f(i) = \frac{1}{i}$ for $i = 1, 2, 3, 4$ then
(A) number of zeroes at the end of $f(5)!$ is 4.
(B) number of divisors of $f(5)$ is 8.
(C) sum of even divisors of $f(5)$ is 56.
(D) sum of odd divisors of $f(5)$ is 18.
- Q.6 If the equations $x^2 - (p + q)x + 4 = 0$; $x^2 - 2qx + 8 = 0$ and $x^2 - (2p + q)x + 12 = 0$ have exactly one common root then which of the following is(are) **CORRECT**?
(A) $p = 2$ (B) $q = 3$
(C) Common root is 4 (D) Sum of all uncommon root is 6

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-17 TO 20
(SOLUTIONS)

Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)

Dpp. No.-17

[SINGLE CORRECT CHOICE TYPE]

[4 × 3 = 12]

Q.1 If $\lim_{x \rightarrow k} [\sin^{-1} x + \cos^{-1} x + \tan^{-1} x]$ does not exist, then sum of the all possible values of k is

[Note : $[y]$ denotes greatest integer function less than or equal to y .]

- (A*) $\frac{-\sin 3}{\sin 1 \cdot \sin 2}$ (B) $\frac{-\sin 2}{\sin 1 \cdot \sin 3}$ (C) $\frac{-\sin 1}{\sin 2 \cdot \sin 3}$ (D) $\frac{-\sin 1 \cdot \sin 2}{\sin 3}$

[Sol._{722/lcd/SC} Range of $\sin^{-1}x + \cos^{-1}x + \tan^{-1}x = \frac{\pi}{2} + \tan^{-1}x \forall x \in [-1, 1]$

$$\therefore R_f = \left[\frac{\pi}{4}, \frac{3\pi}{4} \right]$$

Range contains 2 integers 1, 2.

$$\therefore \frac{\pi}{2} + \tan^{-1}x = 1 \Rightarrow x = -\cot 1$$

$$\text{and } \frac{\pi}{2} + \tan^{-1}x = 2 \Rightarrow x = -\cot 2$$

$$\text{Sum} = -(\cot 1 + \cot 2) = -\left(\frac{\cos 1}{\sin 1} + \frac{\cos 2}{\sin 2} \right) = -\left(\frac{\sin 3}{\sin 1 \cdot \sin 2} \right). \text{ Ans.}]$$

Q.2 A three digit number n is such that the last two digits of it are equal and differ from the first. The number of such n 's is

- (A) 64 (B) 72 (C*) 81 (D) 900

[Sol._{352/perm/SC} If last two digits are 0, 0 the first digit can be filled in 9 ways.

If last two digit are non zero equal like 1 1 then first digit can not be 0 and 1.

$$\therefore \text{number of ways} = 9 \times 8 + 9 = 81. \quad]$$

Q.3 Total number of term, that are dependent on the value of x , in the expansion of $\left(x^2 - 2 + \frac{1}{x^2} \right)^n$ is equal

to

- (A) $2n + 1$ (B*) $2n$ (C) n (D) $n + 1$

$$[\text{Sol.}_{200/\text{bin}/\text{SC}} \Rightarrow \frac{(x^2 - 1)^{2n}}{x^{2n}}$$

$$T_{r+1} = \frac{{}^{2n}C_r \cdot (x^2)^{2n-r} (-1)^r}{x^{2n}}$$

At $r = n$, term is independent

Hence, total number of term = $2n + 1 - 1 = 2n$

independent of x]

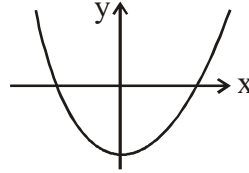
Q.4 A monic quadratic polynomial $y = ax^2 + bx + c$ having roots of opposite sign. Then which of the following is always incorrect ?

- (A) $\frac{-b}{2a} > 0$ (B) $\frac{-b}{2a} < 0$ (C) $ac - ab^2 < 0$ (D*) $ab^2 - ac < 0$

[Sol._{760/qe/SC} $a = 1$
 $y = x^2 + bx + c$

$\Rightarrow \frac{-b}{2a}$ can take any real value

$\Rightarrow c < 0$ **Ans.**]



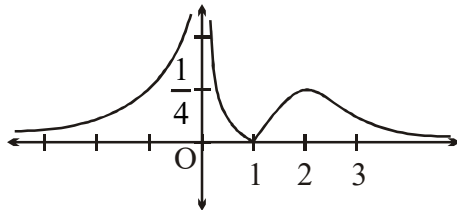
[MULTIPLE CORRECT CHOICE TYPE]

[2 × 4 = 8]

Q.5 Let $f: \mathbb{R} - \{0\} \rightarrow [0, \infty)$ be a function defined by $f(x) = \frac{|x-1|}{x^2}$. Then

- (A*) $f(x)$ is injective in $(2, \infty)$ (B*) $f(x)$ is one-one in $(0, 1)$
(C*) $f(x)$ is surjective (D) $f(x)$ is many-one in $(-\infty, 0)$

[Hint: _{40046/func/MORE}



]

Q.6 If all the roots of the equation $x^3 - 3x = 0$ satisfy the equation

$(\alpha - \sin^{-1}(\sin 2))x^2 - (\beta - \tan^{-1}(\tan 1))x + \gamma^2 - 2\gamma + 1 = 0$ then which of the following vanishes?

- (A*) $\sin \alpha - \sin(\beta + \gamma)$ (B*) $\cos \alpha + \cos(\beta + \gamma)$
(C) $\tan \alpha - \tan(\beta + \gamma)$ (D*) $\cot \alpha + \cot(\beta + \gamma)$

[Sol._{40049/itf/MORE} $\because x = 0, \sqrt{3}, -\sqrt{3}$ are roots of quadratic

\therefore It is an identity.

$\therefore \alpha = \sin^{-1}(\sin 2) = \pi - 2, \beta = \tan^{-1}(\tan 1) = 1$ and $\gamma^2 - 2\gamma + 1 = 0 \Rightarrow \gamma = 1$

\therefore (A) $\sin \alpha - \sin(\beta + \gamma) = \sin(\pi - 2) - \sin 2 = 0$

(B) $\cos(\pi - 2) + \cos 2 = -\cos 2 + \cos 2 = 0$

(C) $\tan(\pi - 2) - \tan 2 = -2\tan 2 \neq 0$

(D) $\cot(\pi - 2) + \cot 2 = 0$]

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Dpp. No.-18

[SINGLE CORRECT CHOICE TYPE]

[6 × 3 = 18]

Q.1 $\lim_{x \rightarrow 0} \frac{(\sin x)^{10} - x^{10}}{x^9(\sin x - x)}$ is equal to

- (A) 9 (B*) 10 (C) 54 (D) 60

[Sol._{723/lcd(l)/SC} $\lim_{x \rightarrow 0} \frac{(\sin x)^{10} - x^{10}}{x^9(\sin x - x)}$

divide by x^{10}

$$\lim_{x \rightarrow 0} \frac{\left(\frac{\sin x}{x}\right)^{10} - 1}{\left(\frac{\sin x}{x} - 1\right)} \quad \text{Put } \frac{\sin x}{x} = t$$

$$\lim_{t \rightarrow 1} \frac{t^{10} - 1}{t - 1} = 10. \quad]$$

Q.2 Let $f(x) = \sqrt{\frac{1}{x^2 + 2\sqrt{c}x + 1}}$. If domain of $f(x)$ is $(-\infty, \infty)$, then the number of integers in the range of

c is

- (A) 3 (B) 2 (C*) 1 (D) 0

[Sol._{407/func/SC} Since domain is \mathbb{R}

$$\therefore x^2 + 2\sqrt{c}x + 1 > 0 \quad \forall x \in \mathbb{R}$$

$$\Rightarrow 4c - 4 < 0 \Rightarrow c < 1$$

$$\text{But } c \geq 0$$

$$\therefore c \in [0, 1). \quad]$$

Q.3 If $5 \sin^{-1}x + 3 \cos^{-1}y - 2 \cos^{-1}z = \frac{11\pi}{2}$, then the value of $x^{2017} + y^{2018} + z^{2016}$ equals

- (A) 0 (B) 1 (C) 2 (D*) 3

[Sol._{388/itf/SC} $5 \sin^{-1}x + 3 \cos^{-1}y - 2 \cos^{-1}z \Big|_{\max.} = \frac{11\pi}{2}$

↓ ↓ ↓

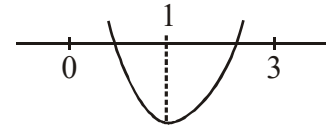
$$\leq \frac{5\pi}{2} \leq 3\pi \geq 0$$

$$\Rightarrow x = 1, y = -1, z = 1 \quad]$$

Q.4 If α, β are roots of $2x^2 - 5x + p = 0$, such that $0 < \alpha < 1$ and $1 < \beta < 3$, then the number of integral values of p is

- (A) 0 (B) 1 (C*) 2 (D) 3

[Sol._{763/qe/SC} $f(0) > 0 \Rightarrow p > 0 \dots(1)$
 $f(1) < 0 \Rightarrow 2 - 5 + p < 0 \Rightarrow p < 3 \dots(2)$
 $f(3) > 0 \Rightarrow 18 - 15 + p > 0 \Rightarrow p > -3 \dots(3)$
 Intersection of (1), (2) and (3)
 $p \in (0, 3)$
 Number of integral values = 2]



Q.5 In the expansion of $(x^2 + 1)(x^3 + 1)(x + 1)^5$, the coefficient of x^3 is

- (A) 29 (B) 35 (C*) 16 (D) 9

Q.6 The letters of the word EARLY are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word EARLY is

- (A) 25 (B*) 26 (C) 27 (D) 28

[Sol._{357/perm/SC} A E L R Y
 $A _ _ _ _ = 4! = 24$
 $E A L _ _ = 2! = 02$
 $E A R L Y = 1 = \underline{01}$
 $= \underline{27}.$

The number of words before EARLY = $27 - 1 = 26.$]

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Dpp. No.-19

[SINGLE CORRECT CHOICE TYPE]

[6 × 3 = 18]

Q.1 $\lim_{n \rightarrow \infty} (2 \log(3n) - \log(n^2 + 1))$ is equal to
(A) $4 \log 6$ (B) $-\log 3$ (C) $\log 3$ (D*) $2 \log 3$

[Sol._{726/lcd(l)/SC} $\lim_{n \rightarrow \infty} \log \left(\frac{9n^2}{n^2 + 1} \right) = \lim_{n \rightarrow \infty} \log \left(\frac{9}{1 + \frac{1}{n^2}} \right) = 2 \log 3$ Ans.]

Q.2 If the function $f(x) = \lambda |\sin x| + \lambda^2 |\cos x| + g(\lambda)$, $\lambda \in \mathbb{R}$ (g is a function of λ) is periodic with fundamental period $\frac{\pi}{2}$, then
(A) $\lambda = 0, 1$ (B*) $\lambda = 1$ (C) $\lambda = 0$ (D) $\lambda = -1$

[Sol._{422/func/SC} $f\left(\frac{\pi}{2} + x\right) = f(x) \quad \forall x \in \mathbb{R}$

$$\begin{aligned} \lambda |\cos x| + \lambda^2 |\sin x| + g(\lambda) &= \lambda |\sin x| + \lambda^2 |\cos x| + g(\lambda) \\ (\lambda - \lambda^2) |\cos x| + (\lambda^2 - \lambda) |\sin x| &= 0 \quad \forall x \in \mathbb{R} \\ \lambda - \lambda^2 = 0 &\Rightarrow \lambda = 0, 1 \text{ but } \lambda = 0 \text{ (rejected)} \\ \Rightarrow \lambda &= 1 \text{ Ans.} \end{aligned}$$

Q.3 If there are 10 stations on a route and the train has to be stopped at 4 of them, then the number of ways in which the train can be stopped so that at least two stopping stations are consecutive is
(A) 7C_4 (B) ${}^{10}C_4 - {}^8C_3$ (C*) ${}^{10}C_4 - {}^7C_3$ (D) 8C_3

[Sol._{362/perm/SC} Required number of ways = Total – number of ways in which no two stopping stations are consecutive.
 $= {}^{10}C_4 - {}^7C_3$.]

Q.4 If $\sin \theta$ and $\cos \theta$ are two distinct roots of $ax^2 + bx + c = 0 \quad \forall a, b, c \in \mathbb{R}, a \neq 0$ then which of the following is correct?
(A) $a^2 - b^2 = 2ac$ (B*) $b^2 - a^2 = 2ac$ (C) $c^2 - a^2 = 2ab$ (D) $a^2 - c^2 = 2ab$

[Sol._{770/qe/SC} $\sin \theta + \cos \theta = \frac{-b}{a} \Rightarrow 1 + \sin 2\theta = \frac{b^2}{a^2} \quad \dots(1)$

$$\sin \theta \cos \theta = \frac{c}{a}, \quad \sin 2\theta = \frac{2c}{a} \quad \dots(2)$$

\Rightarrow From (1) and (2)

$$1 + \frac{2c}{a} = \frac{b^2}{a^2} \Rightarrow a^2 + 2ac = b^2$$

$$2ac = b^2 - a^2. \text{ Ans.}]$$

Q.5 Let $f(x) = \cos^{-1}(\cos 2) + \sin^{-1}(\sin 1) x - x^2$. If $\text{sgn}(f(x))$ is maximum for true set of values of $x \in (a, b)$, then $(a + b)$ is equal to

- (A) 0 (B*) 1 (C) -1 (D) 3

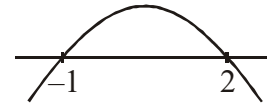
[Sol._{409/itf/SC} $f(x) = 2 + x - x^2 = -(x^2 - x - 2) = -(x - 2)(x + 1)$

$\therefore \text{sgn}(f(x)) = 1$

$\Rightarrow f(x) > 0$

$\therefore x \in (-1, 2)$

$\therefore \text{sum} = 1 \quad \text{Ans. }]$



Q.6 The value of $\lim_{n \rightarrow \infty} \sum_{r=1}^n \left(\frac{2r-1}{n^2 + 2r+1} \right)$ is equal to

- (A) 2 (B) -1 (C*) 1 (D) 2

[Sol._{729/lcd(l)/SC} $f(n) = \sum_{r=1}^n \left(\frac{2r-1}{n^2 + 2r+1} \right)$

$$g(n) = \sum_{r=1}^n \left(\frac{2r-1}{n^2 + 3} \right)$$

$$h(n) = \sum_{r=1}^n \left(\frac{2r-1}{n^2 + 2n+1} \right)$$

$$h(n) \leq f(n) \leq g(n)$$

$$\lim_{n \rightarrow \infty} (h(n)) \leq \lim_{n \rightarrow \infty} f(n) \leq \lim_{n \rightarrow \infty} g(n)$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{\sum (2r-1)}{n^2 + 2n+1} \leq \lim_{n \rightarrow \infty} f(n) \leq \lim_{n \rightarrow \infty} \frac{\sum (2r-1)}{n^3 + 3}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{n^2}{n^3 + 2n+1} \leq \lim_{n \rightarrow \infty} f(n) \leq \lim_{n \rightarrow \infty} \frac{n^2}{n^2 + 3}$$

$$\Rightarrow \lim_{n \rightarrow \infty} f(n) = 1. \text{ Ans.}]$$

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Dpp. No.-20

[SINGLE CORRECT CHOICE TYPE]

[4 × 3 = 12]

Q.1 If the range of the function $f(x) = \log_{\frac{\pi^2}{4}}(\tan^{-1}(x^2 - 4x + 3))$ is $(-\infty, \lambda)$, then the value of λ is

- (A) 0 (B) 1 (C) 2 (D*) $\frac{1}{2}$

[Sol._{433/itf/SC} $x^2 - 4x + 3 > 0$

$$\therefore \max = \log_{(\pi/2)^2} \left(\frac{\pi}{2} \right) = \frac{1}{2}. \quad]$$

Q.2 Let $f(x) = \frac{ax^2 + 2b}{x-1}$ if $\lim_{x \rightarrow 1} f(x) = 2$, then the value of $[a - 2b]$, is

[Note: Where $[k]$ denotes the greatest integer less than or equal to k .]

- (A) 3 (B*) 2 (C) 1 (D) - 1

[Sol._{728/lcd(l)/SC} $\lim_{x \rightarrow 1} \frac{ax^2 + 2b}{x-1} \Rightarrow a + 2b = 0 \quad \dots(1)$

$$\lim_{x \rightarrow 1} \frac{(ax - 2b)(x-1)}{(x-1)} = \lim_{x \rightarrow 1} (ax - 2b) = 2$$

$$\Rightarrow a - 2b = 2 \quad \dots(2)$$

From equation (1) & (2), we get

$$a = 1, b = \frac{-1}{2}$$

$$\Rightarrow [a - 2b] = 2 \text{ Ans.}]$$

Q.3 25 small squares of a 5×5 chess board are coloured with five different colours available, such that each row contain all five colours and no two adjacent squares have same colour is equal to $5!(k)^4$. Then number of prime factor(s) of k is/are

- (A) - 2 (B) 0 (C*) 2 (D) 3

[Sol._{373/perm/SC} $5!(44)^4$.]

Q.4 $\lim_{x \rightarrow \infty} \left(\frac{3x^3 + 2x^2 + 1}{2x + 4 + 7x^3} \right)^{\left(\frac{x^2 - 1}{2x} \right)}$ is equal to

- (A*) 0 (B) 1 (C) $e^{-\frac{3}{7}}$ (D) $e^{\frac{5}{7}}$

[Sol._{730/lcd(1)/SC} $\lim_{x \rightarrow \infty} \underbrace{\left(\frac{3x^3 + 2x^2 + 1}{7x^3 + 2x + 4} \right)^{\left(\frac{x^2 - 1}{2x} \right)}_{\in(0,1)} = 0, \quad [(\text{fraction})^\infty] \quad]$

[MULTIPLE CORRECT CHOICE TYPE]

[2 × 4 = 8]

Q.5 If $f(x)$ is a monic polynomial function of degree 4 satisfying $f(i) = \frac{1}{i}$ for $i = 1, 2, 3, 4$ then

(A*) number of zeroes at the end of $f(5)!$ is 4.

(B*) number of divisors of $f(5)$ is 8.

(C*) sum of even divisors of $f(5)$ is 56.

(D) sum of odd divisors of $f(5)$ is 18.

[Sol._{40059/func/MORE} $x f(x) - 1 \equiv (x - 1)(x - 2)(x - 3)(x - 4)(x - \alpha)$

Put $x = 0 \Rightarrow \alpha = \frac{1}{24}$

$$\therefore f(x) = \frac{(x - 1)(x - 2)(x - 3)(x - 4) \left(x - \frac{1}{24} \right) + 1}{x}$$

$$\Rightarrow f(5) = \frac{4 \times 3 \times 2 \times 1 \times \frac{119}{24} + 1}{5} = 24$$

Now verify the options.]

Q.6 If the equations $x^2 - (p + q)x + 4 = 0$; $x^2 - 2qx + 8 = 0$ and $x^2 - (2p + q)x + 12 = 0$ have exactly one common root then which of the following is(are) **CORRECT**?

(A*) $p = 2$

(B*) $q = 3$

(C*) Common root is 4

(D*) Sum of all uncommon root is 6

[Sol._{40065/qe/MORE} Let $x^2 - (p + q)x + 4 = 0$ has roots $\alpha, \beta \quad \dots(1)$

Let $x^2 - 2qx + 8 = 0$ has roots $\alpha, \gamma \quad \dots(2)$

Let $x^2 - (2p + q)x + 12 = 0$ has roots $\alpha, \delta \quad \dots(3)$

From (1) and (2)

$$\alpha = \frac{4}{q - p}$$

Form (2) and (3)

$$\alpha = \frac{4}{2p - q}$$

From (1) and (3)

$$\alpha = \frac{8}{p}$$

$\Rightarrow p = 2, q = 3, \alpha = 4, \beta = 1, \gamma = 2, \delta = 3 \quad]$