# Dpp. No.-13 TO 16

# Dpp. No.-13

## [SINGLE CORRECT CHOICE TYPE] [4 × 3 = 12]

Q.1 The domain of the function $f(x) = \frac{\sqrt{x+2}}{x^2-9}$ is

(A) $(-\infty, -3) \cup [2, \infty)$ (B) $[2, 3)$
(C) $[-2, 3) \cup (3, \infty)$ (D) $(-\infty, -3) \cup (3, \infty)$

Q.2 Consider two sets
$A = \{(x, y) \mid x^2 + y^2 = 4\}$ and
$B = \{(x, y) \mid (x-3)^2 + (y-4)^2 = r^2\}$
If the intersection of sets A and B has only one element, then the sum of all possible values of r is
(A) 10 (B) 7 (C) 4 (D) 3

Q.3 If $\cot^{-1}(x-1) - \cot^{-1}(x+1) = \frac{5\pi}{12}$, then the value of $\left(x^2 + \frac{4}{x^2}\right)$, is

(A) 4 (B) 6 (C) 8 (D) 10

Q.4 Number of integral values of 'a' for which the equation $\cos 2x + 3\sin^2 x + 4\sin x = a - 3$ has a real root is/are
(A) 10 (B) 9 (C) 8 (D) 7

## [MULTIPLE CORRECT CHOICE TYPE] [2 × 4 = 8]

Q.5 The value of $\sum_{r=1}^{\infty} \frac{r}{(r+1)!}$ is less than

(A) 2 (B) 4 (C) 6 (D) 8

Q.6 If the variable line $L \equiv y - mx - c = 0$ ($m, c \in R$ and $c \neq 0$) cuts the curve $y = x^2 - x$ at A and B such that $\angle AOB = \frac{\pi}{2}$ where O is origin, then

(A) $m + c - 1 = 0$ (B) $m - c + 1 = 0$
(C) line L always passes through (1, 1) (D) line L always passes through (– 1, – 1)

# Dpp. No.-14

## [SINGLE CORRECT CHOICE TYPE] [6 × 3 = 18]

Q.1 Number of cyphers after decimal before a significant figure starts in $\left(\frac{3}{2}\right)^{-100}$, is

[Given: $\log_{10}2 = 0.3010$ and $\log_{10}3 = 0.4771$]

(A) 16 (B) 17 (C) 18 (D) 19

Q.2 Number of solutions of the equation $2\sin(\{x\}) = 1$ in $x \in [0, 2\pi]$ is equal to

[**Note :** $\{k\}$ denotes fractional part of k.]

(A) 4 (B) 5 (C) 6 (D) 7

Q.3 The angle between a pair of tangents drawn from a point P to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is $2\alpha$. The equation of the locus of the point P is

(A) $x^2 + y^2 + 4x - 6y + 4 = 0$ (B) $x^2 + y^2 + 4x - 6y - 9 = 0$

(C) $x^2 + y^2 + 4x - 6y - 4 = 0$ (D) $x^2 + y^2 + 4x - 6y + 9 = 0$

Q.4 The sum of rational terms in the binomial expansion of $\left(\sqrt{2} + \sqrt[5]{3}\right)^{10}$ is

(A) 32 (B) 25 (C) 41 (D) 9

Q.5 In a triangle ABC, if the angle at B is twice the angle at C, then $(b^2 - c^2)$ is always equal to

[**Note :** All symbols used have usual meaning in triangle ABC.]

(A) $a^2$ (B) bc (C) ac (D) ab

Q.6 If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - .....\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - .....\right) = \frac{\pi}{2}$ for $0 < |x| < \sqrt{2}$ then x equals to

[JEE 2001 (screening)]

(A) 1/2 (B) 1 (C) – 1/2 (D) – 1

# Dpp. No.-15

## [SINGLE CORRECT CHOICE TYPE] [4 × 3 = 12]

Q.1 Let a, b, c $\in$ R such that a + b + c = 0 and a + b – c = 0, then the polynomial function $f(x) = ax^2 + bx + c; (a > 0)$ attains its least value at 'x' equal to

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) not possible

Q.2 If domain of f(x) is [– 1, 4], then number of integers in the domain of $g(x) = f(2-\sqrt{x}) + f\left(1-x^{\frac{1}{3}}\right)$ is

(A) 6 (B) 7 (C) 8 (D) 9

Q.3 The number of real solutions of $\tan^{-1}\sqrt{x\,(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ is : [JEE '99]

(A) zero (B) one (C) two (D) infinite

Q.4 If number of arrangements of letters of word DEPARTMENT taken all at a time when no two T's are together and no two E's are together is $N\lfloor 7$, the N is equal to

(A) 108 (B) 116 (C) 120 (D) 144

## [MULTIPLE CORRECT CHOICE TYPE] [2 × 4 = 8]

Q.5 Let $f : R \to R$ be defined as $f(x) = \frac{\sin \pi\{x\}}{x^2 - x + 1}$ $\forall$ x $\in$ R, then

(A) f is neither even nor odd function (B) f is a zero function
(C) f is many-one and non-constant function (D) f is one-one function
[**Note :** {x} denotes fractional part of x.]

Q.6 The value of $\tan^2(\sec^{-1} 2) + \cot^2(\operatorname{cosec}^{-1} 3)$ is equal to

(A) 11 (B) 15 (C) $1 + \sec^2(\tan^{-1} 3)$ (D) $2 + \operatorname{cosec}^2(\cot^{-1} 3)$

# Dpp. No.-16

## [SINGLE CORRECT CHOICE TYPE] [5 × 3 = 15]

Q.1 Let $\left(5+2\sqrt{6}\right)^n = p + f$ where $n \in N$ and $p \in N$ and $0 < f < 1$ then the value of, $(f^2 - f + pf - p)$ is

(A) a natural number (B) a negative integer
(C) a prime number (D) an irrational number

Q.2 The number of six digit numbers that can be formed from the digits 1, 2, 3, 4, 5, 6 & 7 so that digits do not repeat and the terminal digits are even is :

(A) $\lfloor 3 \cdot \lfloor 4$ (B) $3 \cdot \lfloor 4$ (C) $\lfloor 2 \cdot \lfloor 3 \cdot \lfloor 4$ (D) $\lfloor 6$

Q.3 The value of $\sin\frac{\pi}{7} + \sin\frac{2\pi}{7} + \sin\frac{3\pi}{7}$ is equal to

(A) $\cot\frac{\pi}{14}$ (B) $\frac{1}{2}\cot\frac{\pi}{14}$ (C) $\tan\frac{\pi}{14}$ (D) $\frac{1}{2}\tan\frac{\pi}{14}$

Q.4 The first, third and eighth terms of an arithmetic sequence with common difference 9 (taken in that order) are in geometric progression. The seventh term of arithmetic sequence is

(A) – 42 (B) 66 (C) – 21 (D) 34

Q.5 $\tan^{-1}\left(\frac{2}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) + \tan^{-1}\left(\frac{2}{16}\right) + \tan^{-1}\left(\frac{2}{25}\right) + .....\infty$ equals

(A) $\pi - \tan^{-1}(1)$ (B) $\frac{\pi}{2} - \tan^{-1}(1)$ (C) $\pi - \tan^{-1}(3)$ (D) $\tan^{-1}(3)$

## [SUBJECTIVE] [1 × 5 = 5]

Q.6 Let A (1, 0) be a point on the circle $x^2 + y^2 = 1$. Through another point P (0, 2) chord is drawn to meet the circle at point B and C, then the locus of centroid of $\Delta ABC$ is $x^2 + y^2 + ax + by + c = 0$ then find the value of $(a + b + 18c)$.

# Dpp. No.-13 TO 16
# (SOLUTIONS)

**Fresher (For Class XII Appering) Target : JEE-(Mains / Advanced)**

# Dpp. No.-13

**[SINGLE CORRECT CHOICE TYPE]** **[4 × 3 = 12]**

Q.1 The domain of the function $f(x)=\frac{\sqrt{x+2}}{x^2-9}$ is

(A) $(-\infty,-3)\cup[2,\infty)$ (B) $[2,3)$

(C*) $[-2,3)\cup(3,\infty)$ (D) $(-\infty,-3)\cup(3,\infty)$

[Sol.$_{154/func/SC}$

$x+2\geq 0 \Rightarrow x\geq -2$

$x^2-9\neq 0 \Rightarrow x\neq \pm 3$

$[-2,3)\cup(3,\infty)$. **Ans.**]

Q.2 Consider two sets

$A=\{(x,y)\mid x^2+y^2=4\}$ and

$B=\{(x,y)\mid (x-3)^2+(y-4)^2=r^2\}$

If the intersection of sets A and B has only one element, then the sum of all possible values of r is

(A*) 10 (B) 7 (C) 4 (D) 3

[Sol.$_{236/cir/SC}$ $S_1: x^2+y^2=4$

$S_2: x^2+y^2-6x-8y+25-r^2=0$

$C_1: (0,0)$ and $r_1=2$

$C_2: (3,4)$ and $r_2=r$

and $C_1C_2=5$

If $r+2=5 \Rightarrow r=3$

If $5=|r-2| \Rightarrow r=2\pm 5 \Rightarrow r=7$

$\Rightarrow$ Sum = 10. **Ans.**]

Q.3 If $\cot^{-1}(x-1)-\cot^{-1}(x+1)=\frac{5\pi}{12}$, then the value of $\left(x^2+\frac{4}{x^2}\right)$, is

(A) 4 (B) 6 (C*) 8 (D) 10

[Sol.$_{290/itf/SC}$ We have $\cot^{-1}(x-1)-\cot^{-1}(x+1)=\frac{5\pi}{12}$

$\Rightarrow \left(\frac{\pi}{2}-\tan^{-1}(x-1)\right)-\left(\frac{\pi}{2}-\tan^{-1}(x+1)\right)=\frac{5\pi}{12}$

$\Rightarrow \tan^{-1}(x+1)-\tan^{-1}(x-1)=\frac{5\pi}{12} \Rightarrow \tan^{-1}\frac{2}{x^2}=\frac{5\pi}{12}$

$\Rightarrow x^2=\frac{2}{2+\sqrt{3}}=2\left(2-\sqrt{3}\right)$

Hence, $\left(x^2+\frac{4}{x^2}\right)=8$ **Ans.**]

Q.4 Number of integral values of 'a' for which the equation $\cos 2x + 3\sin^2 x + 4\sin x = a - 3$ has a real root is/are

(A) 10 (B*) 9 (C) 8 (D) 7

[Sol.$_{143/ph\text{-}2/SC}$ $1 - 2\sin^2 x + 3\sin^2 x + 4\sin x + 3 = a$

$\Rightarrow \sin^2 x + 4\sin x + 4 = a \quad \Rightarrow (\sin x + 2)^2 = a$

$\Rightarrow 1 \le a \le 9 \Rightarrow a = 1, 2, 3, 4 \ldots\ldots\ldots\ldots 9$ ]

## [MULTIPLE CORRECT CHOICE TYPE] [2 × 4 = 8]

Q.5 The value of $\sum_{r=1}^{\infty} \frac{r}{(r+1)!}$ is less than

(A*) 2 (B*) 4 (C*) 6 (D*) 8

[Sol.$_{40578/seq/MORE}$ $\sum_{r=1}^{n} \frac{r+1-1}{(r+1)!} = \sum_{r=1}^{n} \left( \frac{1}{r!} - \frac{1}{(r+1)!} \right) = 1 - \frac{1}{(n+1)!}$

$\Rightarrow \sum_{r=1}^{n} \frac{r}{(r+1)!} \to 1 \text{ as } n \to \infty$

$\therefore \sum_{r=1}^{\infty} \frac{r}{(r+1)!} = 1.$ ]

Q.6 If the variable line $L \equiv y - mx - c = 0$ ($m, c \in R$ and $c \neq 0$) cuts the curve $y = x^2 - x$ at A and B such that $\angle AOB = \frac{\pi}{2}$ where O is origin, then

(A*) $m + c - 1 = 0$ (B) $m - c + 1 = 0$

(C*) line L always passes through (1, 1) (D) line L always passes through (– 1, – 1)

[Sol.$_{40571/st.line/MORE}$ We have,

$x^2 - (x + y) \cdot \left( \frac{y - mx}{c} \right) = 0$ (using principle of homogenising)

Now, put (coefficient of $x^2$) + (coefficient of $y^2$) = 0

$\Rightarrow \quad 1 = c + m$

Also, $(y - 1) + m(1 - x) = 0$

So, fixed point (1, 1) ]

# Dpp. No.-14

## [SINGLE CORRECT CHOICE TYPE] [6 × 3 = 18]

Q.1 Number of cyphers after decimal before a significant figure starts in $\left(\frac{3}{2}\right)^{-100}$, is

[Given: $\log_{10}2 = 0.3010$ and $\log_{10}3 = 0.4771$]

(A) 16 (B*) 17 (C) 18 (D) 19

[Sol.$_{273/log/SC}$ Let $N = \left(\frac{3}{2}\right)^{-100}$

$\therefore$ $\log N = 100\,[\log 3 - \log 2] = -100\,[0.4771 - 0.3010] = -100 \times 0.1761 = -17.61$

$\therefore$ Number of cyphers after decimal before a significant figure starts in $\left(\frac{3}{2}\right)^{-100}$, is $|-18|-1 = 17.$ ]

Q.2 Number of solutions of the equation $2\sin(\{x\}) = 1$ in $x \in [0, 2\pi]$ is equal to

[**Note :** $\{k\}$ denotes fractional part of k.]

(A) 4 (B) 5 (C*) 6 (D) 7

[Sol.$_{260/func/SC}$ $2\sin\{x\} = 1$

$\sin\{x\} = ½$

$\{x\} = \pi/6$ is the only value

$x = \frac{\pi}{6}, \frac{\pi}{6}+1, \frac{\pi}{6}+2, \frac{\pi}{6}+3, 4+\frac{\pi}{6}, 5+\frac{\pi}{6} \Rightarrow 6$ **Ans.** ]

Q.3 The angle between a pair of tangents drawn from a point P to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is $2\alpha$. The equation of the locus of the point P is

(A) $x^2 + y^2 + 4x - 6y + 4 = 0$ (B) $x^2 + y^2 + 4x - 6y - 9 = 0$

(C) $x^2 + y^2 + 4x - 6y - 4 = 0$ (D*) $x^2 + y^2 + 4x - 6y + 9 = 0$

[JEE '96, 1]

Q.4 The sum of rational terms in the binomial expansion of $\left(\sqrt{2} + \sqrt[5]{3}\right)^{10}$ is

(A) 32 (B) 25 (C*) 41 (D) 9

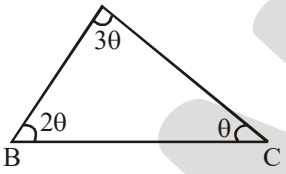
[Sol.$_{142/bin/SC}$ $T_{r+1} = {}^{10}C_r (2)^{\frac{r}{2}} \cdot \left(3^{\frac{1}{5}}\right)^{10-r} = {}^{10}C_r \cdot 2^{\frac{r}{2}} \cdot 3^{\frac{10-r}{5}}$;

$r = 0, 1, 2, \ldots\ldots, 9, 10$

$\therefore$ For rational terms, $r = 0, r = 10$

So, sum of rational terms $= 9 + 32 = 41$ **Ans.** ]

Q.5 In a triangle ABC, if the angle at B is twice the angle at C, then $(b^2 - c^2)$ is always equal to
[**Note :** All symbols used have usual meaning in triangle ABC.]
(A) $a^2$ (B) bc (C*) ac (D) ab




[Sol.$_{138/sot/SC}$ $b^2 - c^2$
$k^2(\sin^2 B - \sin^2 C)$
$k^2(\sin B + C) \cdot (\sin B - C)$
$k^2 \sin A \sin C \Rightarrow$ ac. **Ans.** ]

Q.6 If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - .....\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - .....\right) = \frac{\pi}{2}$ for $0 < |x| < \sqrt{2}$ then x equals to
[JEE 2001 (screening)]
(A) 1/2 (B*) 1 (C) – 1/2 (D) – 1

[Sol.$_{145/itf/SC}$ $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - .....\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - .....\right) = \frac{\pi}{2}$ for $0 < |x| < \sqrt{2}$

$$S_1 = \frac{x}{1 + \frac{x}{2}}$$

$$= \frac{2x}{x+2} \qquad 0 < |x| < \sqrt{2}$$

$$S_2 = \frac{\frac{x^2}{1 + \frac{x^2}{2}} = \frac{2x^2}{2 + x^2}}{\frac{2x}{x+2} = \frac{2x^2 x}{2 + x^2}}$$

$x^2 + 2x = 2 + x^2 \Rightarrow x = 1.$ ]

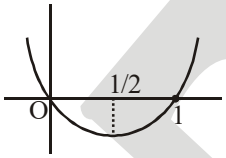
# Dpp. No.-15

**[SINGLE CORRECT CHOICE TYPE]** **[4 × 3 = 12]**

Q.1 Let a, b, c $\in$ R such that $a + b + c = 0$ and $a + b - c = 0$, then the polynomial function $f(x) = ax^2 + bx + c; (a > 0)$ attains its least value at 'x' equal to

(A) 0 (B) 1 (C*) $\frac{1}{2}$ (D) not possible

[Sol.$_{424/qe/SC}$ $a + b + c = 0$ ...(1)
$a + b - c = 0$ ...(2)
(1) – (2) $\Rightarrow$ $c = 0$
Hence, roots are 0 & 1
$\therefore$ $f(x) = ax(x - 1)$




minimum occurs at $x = \frac{1}{2}$ ]

Q.2 If domain of f(x) is [– 1, 4], then number of integers in the domain of $g(x) = f(2 - \sqrt{x}) + f\left(1 - x^{\frac{1}{3}}\right)$ is

(A) 6 (B) 7 (C) 8 (D*) 9

[Sol.$_{341/func/SC}$ $-1 \le 2 - \sqrt{x} \le 4 \Rightarrow -3 \le -\sqrt{x} \le 2 \Rightarrow -2 \le \sqrt{x} \le 3 \Rightarrow 0 \le x \le 9$ .....(1)

$-1 \le 1 - x^{\frac{1}{3}} \le 4 \Rightarrow -2 \le -x^{\frac{1}{3}} \le 3 \Rightarrow -3 \le x^{\frac{1}{3}} \le 2 \Rightarrow -27 \le x \le 8$ .....(2)
Intersection of (1) & (2) is $0 \le x \le 8$
$\therefore$ Number of integers = 9. **Ans.**]

Q.3 The number of real solutions of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2 + x + 1} = \frac{\pi}{2}$ is :

(A) zero (B) one (C*) two (D) infinite
[JEE '99, 2 (out of 200)]

[Sol.$_{144/itf/SC}$ $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2 + x + 1} = \frac{\pi}{2}$

$x(x + 1) \ge 0$ ....(1) $\Rightarrow$


Also $0 \le x^2 + x + 1 \le 1$
but $x^2 + x + 1$ is always $\ge 0$
$\therefore$ $x^2 + x + 1 \le 1$
$x(x - 1) \le 0$ ....(2)
sum (1) and (2) $x(x + 1) = 0$
$\Rightarrow x = 0$ or $x = -1$ ]

Q.4 If number of arrangements of letters of word DEPARTMENT taken all at a time when no two T's are together and no two E's are together is $N\lfloor 7$, the N is equal to

(A) 108 (B*) 116 (C) 120 (D) 144

[Sol.$_{239/perm/SC}$ D = 01; P = 01; R = 01; A = 01; M = 01; N = 01; T = 02; E = 02

$\therefore$ Number of words formed $= {}^9C_2 \times \frac{\lfloor 8}{\lfloor 2} - {}^8C_2 \times \lfloor 7$

$= 144\lfloor 7 - 28\lfloor 7 = (144-28)\lfloor 7 = 116\lfloor 7$ **Ans.** ]

## [MULTIPLE CORRECT CHOICE TYPE] [2 × 4 = 8]

Q.5 Let $f: R \to R$ be defined as $f(x) = \frac{\sin \pi\{x\}}{x^2 - x + 1}$ $\forall\, x \in R$, then

(A*) f is neither even nor odd function (B) f is a zero function

(C*) f is many-one and non-constant function (D) f is one-one function

[**Note :** {x} denotes fractional part of x.]

[Sol.$_{40023/func/MORE}$ $f(x) = \frac{\sin(\pi(x-[x]))}{x^2 - x + 1}$ and $f(-x) = \frac{\sin(\pi(-x-[x]))}{x^2 + x + 1}$

Clearly f(x) is neither odd nor even function

at integer f(x) = 0

$\therefore$ f(x) is many-one function. ]

Q.6 The value of $\tan^2(\sec^{-1} 2) + \cot^2(\operatorname{cosec}^{-1} 3)$ is equal to

(A*) 11 (B) 15 (C*) $1 + \sec^2(\tan^{-1} 3)$ (D) $2 + \operatorname{cosec}^2(\cot^{-1} 3)$

[Sol.$_{40023/itf/MORE}$ $\because$ $\tan^2\theta = \sec^2\theta - 1$ and $\cot^2\theta = \operatorname{cosec}^2\theta - 1$

$\therefore$ Given expression $= \sec^2(\sec^{-1} 2) - 1 + \operatorname{cosec}^2(\operatorname{cosec}^{-1} 3) - 1$

$= 4 - 1 + 9 - 1 = 11$ **Ans.**]

# Dpp. No.-16

**[SINGLE CORRECT CHOICE TYPE]** **[5 × 3 = 15]**

Q.1 Let $\left(5+2\sqrt{6}\right)^n = p + f$ where $n \in N$ and $p \in N$ and $0 < f < 1$ then the value of, $(f^2 - f + pf - p)$ is

(A) a natural number (B*) a negative integer

(C) a prime number (D) an irrational number

[Sol.$_{57/bin/SC}$ $f^2 - f + pf - p = -f(1-f) - p(1-f) = -[(p+f)(1-f)]$

where $p + f = \left(5+2\sqrt{6}\right)^n$ and $1 - f = f' = \left(5-2\sqrt{6}\right)^n$

hence $f^2 - f + pf - p = -1$. **Ans.**]

Q.2 The number of six digit numbers that can be formed from the digits 1, 2, 3, 4, 5, 6 & 7 so that digits do not repeat and the terminal digits are even is :

(A) $\lfloor 3 \cdot \lfloor 4$ (B) $3 \cdot \lfloor 4$ (C) $\lfloor 2 \cdot \lfloor 3 \cdot \lfloor 4$ (D*) $\lfloor 6$

[Hint:$_{221/perm/SC}$ 1 . ② . 3 . ④ . 5 . ⑥ . 7 

${}^3C_2 \cdot 2! \cdot {}^5C_4 \cdot 4! = 6 \times 120 = 720$ ]

Q.3 The value of $\sin\frac{\pi}{7} + \sin\frac{2\pi}{7} + \sin\frac{3\pi}{7}$ is equal to

(A) $\cot\frac{\pi}{14}$ (B*) $\frac{1}{2}\cot\frac{\pi}{14}$ (C) $\tan\frac{\pi}{14}$ (D) $\frac{1}{2}\tan\frac{\pi}{14}$

[Sol.$_{398/ph-1/SC}$ Expression $= \underbrace{\sin\frac{\pi}{7} + \sin\frac{2\pi}{7} + \sin\frac{3\pi}{7}}_{\text{sine series}} = \frac{\sin\left(\frac{2\pi}{7}\right)\cdot\sin\left(\frac{3\pi}{14}\right)}{\sin\left(\frac{\pi}{14}\right)} = \frac{\sin\left(\frac{2\pi}{7}\right)\cdot\sin\left(\frac{\pi}{2}-\frac{4\pi}{14}\right)}{\sin\left(\frac{\pi}{14}\right)}$

$= \frac{2\sin\left(\frac{2\pi}{7}\right)\cdot\cos\left(\frac{2\pi}{7}\right)}{2\sin\left(\frac{\pi}{14}\right)} = \frac{\sin\left(\frac{4\pi}{7}\right)}{2\sin\left(\frac{\pi}{14}\right)} = \frac{\cos\left(\frac{\pi}{2}-\frac{4\pi}{7}\right)}{2\sin\left(\frac{\pi}{14}\right)} = \frac{1}{2}\cot\frac{\pi}{14}$ **Ans.** ]

Q.4 The first, third and eighth terms of an arithmetic sequence with common difference 9 (taken in that order) are in geometric progression. The seventh term of arithmetic sequence is

(A) – 42 (B*) 66 (C) – 21 (D) 34

[Sol.$_{270/seq/SC}$ a; a + 9; a + 18; .....

Now, $a_1$, $a_3$ and $a_8$ in G.P. i.e. a, a + 18 and a + 63 in G.P.

$\therefore$ $(a + 18)^2 = a(a + 63) = a^2 + 63a \Rightarrow a = 12; d = 9$

So, $a_7 = a + 6d = 12 + 54 = 66$. **Ans.**]

Q.5 $\tan^{-1}\left(\frac{2}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) + \tan^{-1}\left(\frac{2}{16}\right) + \tan^{-1}\left(\frac{2}{25}\right) + .....\infty$ equals

(A) $\pi - \tan^{-1}(1)$ (B) $\frac{\pi}{2} - \tan^{-1}(1)$ (C) $\pi - \tan^{-1}(3)$ (D*) $\tan^{-1}(3)$

[Sol.$_{252/itf/SC}$ $T_1 = \tan^{-1}3 - \tan^{-1}1$

$T_2 = \tan^{-1}4 - \tan^{-1}2$

$T_3 = \tan^{-1}5 - \tan^{-1}3$

$\vdots \quad \vdots$

$T_{n-1} = \tan^{-1}(n+1) - \tan^{-1}(n-1)$

$T_n = \tan^{-1}(n+2) - \tan^{-1}(n)$

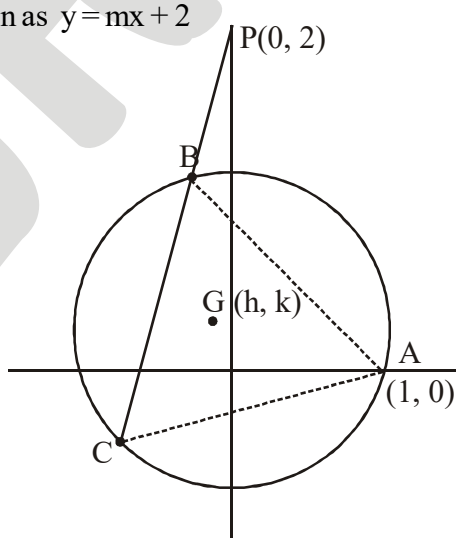
$\therefore \quad S_\infty = \pi - \left(\tan^{-1}(1) + \tan^{-1}(2)\right) = \tan^{-1}(3)$ **Ans.** ]

## [SUBJECTIVE] [1 × 5 = 5]

Q.6 Let A (1, 0) be a point on the circle $x^2 + y^2 = 1$. Through another point P (0, 2) chord is drawn to meet the circle at point B and C, then the locus of centroid of $\Delta ABC$ is $x^2 + y^2 + ax + by + c = 0$ then find the value of $(a + b + 18c)$.

[Ans. 0]

[Sol.$_{50004/cir/OMR}$ The equation BC can be written as $y = mx + 2$




solve $x^2 + y^2 = 1$ with $y = mx + 2$

$(m^2 + 1)x^2 + 4mx + 3 = 0.$

$x_1 + x_2 = \frac{-4m}{m^2+1}.$

$||^{ly} \quad \frac{(y-2)^2}{m^2} + y^2 = 1$

$(m^2 + 1)y^2 - 4y + (4 - m^2) = 0$

$y_1 + y_2 = \frac{4}{m^2+1}$

$3h = x_1 + x_2 + 1$ and $3k = y_1 + y_2$

$3h = \frac{-4m}{m^2+1} + 1$ and $3k = \frac{4}{m^2+1}$

$\frac{3h-1}{3k} = -m$

$3k(m^2 + 1) = 4 \Rightarrow 3k\left(\left(\frac{3h-1}{3k}\right)^2 + 1\right) = 4$

$\frac{(3h-1)^2}{3k} + 3k = 4$

$x^2 + y^2 - \frac{2}{3}x - \frac{4}{3}y + 1 = 0.$

$a = \frac{-2}{3}$ ; $b = \frac{-4}{3}$, $c = \frac{1}{9}$

$(a + b + 18c) = 0$. **Ans.**]