GGSRDN

Educational Services Private Limited

9th, 10th, NEET, JEE(Main/Advanced)

अभ्यास ही सबसे बड़ा गुरू है।

CLASS : XII (MATHS)

DPP

DAILY PRACTICE PROBLEM

DPP-1 to 10

Solutions

DPP 1 : Straight Line, Determinant, Fundamentals of Mathematics, Trigonometric Ratio
DPP 2 : Fundamentals of Mathematics, Circle, Quadratic Equation, Determinants
DPP 3 : Complex Number, Sequence & Progression, Permutation & Combination, Fundamentals of Mathematics, Quadratic Equation
DPP 4 : Fundamentals of Mathematics, Quadratic Equation, Function
DPP 5 : Inverse Trigonometric Function, Fundamentals of Mathematics, Quadratic Equation
DPP 6 : Inverse Trigonometric Function, Fundamentals of Mathematics, Quadratic Equation
DPP 7 : Fundamentals of Mathematics, Trigonometric Ratio, Inverse Trigonometric Function, Quadratic Equation
DPP 8 : Matrices, Fundamentals of Mathematics, Inverse Trigonometric Function
DPP 9 : Inverse Trigonometric Function, Matrices, Fundamentals of Mathematics
DPP 10 : Inverse Trigonometric Function, Matrices & Determinants, Function, Fundamentals of Mathematics

DPP NO. - 1

1. $(2\sin x - 1)(\cos x - 1) \le 0$

Case-1 : $\cos x = 1 \Rightarrow x = 0$

Case-2 : other wise $\cos x < 1$

$\Rightarrow \therefore\ \sin x \ge 1/2 \ \Rightarrow x \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

Hence from case-1 & case-2 $x \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right] \cup \{0\}$

2. Let $(x + 7y + A)(x - 5y + 8)$

$= x^2 + 2xy - 35y^2 - 4x + 44y - 12$

$\Rightarrow\ A + B = -4, AB = -12,\ 7B - 5A = 44$

$A = -6,\ B = 2,\ x + 7y - 6 = 0,\ x - 5y + 2 = 0$

on solving $12y - 8 = 0, y = \frac{2}{3},\ x = \frac{4}{3}$

and $\left(\frac{4}{3}, \frac{2}{3}\right)$

Line $5x + \lambda y - 8 = 0$ passes through

$\left(\frac{4}{3}, \frac{2}{3}\right)\ 5\left(\frac{4}{3}\right) + \left(\frac{2}{3}\right) - 8 = 0$

$20 + 2\lambda - 24 = 0 \qquad \lambda = 2$

3. $\begin{bmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{bmatrix}$

$= \frac{1}{abc}\begin{bmatrix} a(a^2+1) & a^2b & a^2c \\ b^2a & b(b^2+1) & b^2c \\ c^2a & c^2b & c(c^2+1) \end{bmatrix}$

$= \frac{abc}{abc}\begin{bmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{bmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3$

$(a^2+b^2+c^2+1)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix} = (a^2 + b^2 + c^2+1)$

Applying $C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$

$(a^2+b^2+c^2+1)\begin{vmatrix} 1 & 0 & 0 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix} = (a^2 + b^2 + c^2 + 1)$

4. $\frac{x^2+4x+4}{2x^2-x-1} > 0. \Rightarrow \frac{(x+2)^2}{(2x+1)(x-1)} > 0$

$\Rightarrow$ + + – +
-2 -½ 1

$\Rightarrow\ x \in (-\infty, -½) \cup (1, \infty) - \{-2\}$

5. $\frac{x^2 - 7|x| + 10}{x^2 - 6x + 9} < 0$

$\Rightarrow\ x \ne \{3\}$

$\Rightarrow\ \frac{x^2 - 7|x| + 10}{(x-3)^2}$

$\Rightarrow\ x^2 - 7|x| + 10 < 0$

case- 1
$x < 0$
$x^2 + 7x + 10 < 0$
$(x + 5)(x + 2) < 0$
$\Rightarrow x \in (-5, -2)$

case- 2
$x^2 - 7x + 10 < 0$
$\Rightarrow (x - 5)(x - 2) < 0$
$\Rightarrow x \in (2, 5)$
but $x \ne \{3\}$

$\therefore\ x \in (-5, -2) \cup (2, 3) \cup (3, 5)$

6. $\left|\frac{x^2 - 5x + 4}{x^2 - 4}\right| \le 1 \quad ;\ x \ne \{-2, 2\}$

$\Rightarrow\ -1 \le \frac{x^2 - 5x + 4}{x^2 - 4} \le 1$

$\therefore\ \frac{x^2 - 5x + 4}{x^2 - 4} \ge -1 \ \ \&\ \ \frac{x^2 - 5x + 4}{x^2 - 4} \le 1$

$\Rightarrow\ \frac{2x^2 - 5x}{x^2 - 4} \ge 0 \ \ \&\ \ \frac{5x - 8}{x^2 - 4} \ge 0$

+ – + – +
-2 0 2 5/2

&
– + – +
–2 8/5 2

$x \in (-\infty, -2) \cup [0, 2) \cup [5/2, \infty)$

& $x \in (-2, 8/5] \cup (2, \infty)$

Hence $x \in [0, 8/5] \cup [5/2, \infty)$

7. $\sin(\cos 1) - \cos(\sin 1) \qquad \sin(\cos 1) - \sin\left(\frac{\pi}{2} - \sin 1\right)$

Let $x_1 = \cos 1$ and $x_2 = \left(\frac{\pi}{2} - \sin 1\right)$

$x_1 - x_2 = (\cos 1 + \sin 1) - \frac{\pi}{2}$

$= \sqrt{2}\ \sin\left(\frac{\pi}{4} + 1\right) - \frac{\pi}{2}$ = Negative

$x_2 > x_1$

$\Rightarrow \sin x_2 > \sin x_1$

$\sin\left(\frac{\pi}{2} - \sin 1\right) > \sin(\cos 1)$

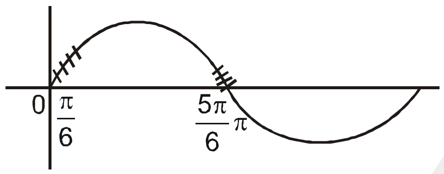
$\Rightarrow \cos(\sin 1) > \sin(\cos 1)$

8. $\sin\left(\frac{x}{2}\right) > 0$

$\Rightarrow \frac{x}{2} \in (2n\pi, (2n+1)\pi) \qquad x \in (4n\pi, (4n+2)\pi)$

and $\log_2\left(\sin\frac{x}{2}\right) < -1 \Rightarrow \sin\left(\frac{x}{2}\right) < 2^{-1}$

$\Rightarrow \sin\left(\frac{x}{2}\right) < \frac{1}{2}$



$\frac{x}{2} \in \left(0 + 2n\pi, \frac{\pi}{6} + 2n\pi\right) \cup \left(\frac{5\pi}{6} + 2n\pi, \pi + 2n\pi\right)$

$\Rightarrow x \in \left(4n\pi, \frac{\pi}{3} + 4n\pi\right) \cup \left(\frac{5\pi}{3} + 4n\pi, 2\pi + 4n\pi\right),$

$n \in I$

DPP NO. - 2

1. $x^2 + 7x + 13 = k^2$

$D = 49 - 4(13 - k^2) = 4k^2 - 3 = L^2$ (let)

$4k^2 - L^2 = 3$

$(2k + L)(2k - L) = 3 \times 1$

$\Rightarrow 2k + L = 3$

$2k - L = 1$

$\Rightarrow k = 1$

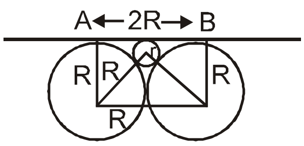
$x^2 + 7x + 13 = 1 \Rightarrow x^2 + 7x + 12 = 0$

$\Rightarrow x = -4, -3$

2. $(R - r)^2 + R^2 = (R + r)^2$

$\Rightarrow R^2 + r^2 - 2Rr + R^2 = R^2 + r^2 + 2Rr$

$\Rightarrow R^2 = 4Rr$



$\Rightarrow \frac{r}{R} = \frac{1}{4}$

3. $(\sin^2 x - (k + 3))(\sin^2 x + 1) = 0$

$\Rightarrow \sin^2 x = -1$ Not possible

$\sin^2 x = k + 3$

$\Rightarrow 0 \le k + 3 \le 1$

$-3 \le k \le -2$

4. $(x^2)^2 + (2)^2 + 4x^2 - 4x^2$

$= (x^2 + 2)^2 - (2x)^2 = (x^2 + 2x + 2)(x^2 - 2x + 2)$ is prime

if $x^2 + 2x + 2 = 1$ or $x^2 - 2x + 2 = 1$

$(x + 1)^2 = 0$ or $(x - 1)^2 = 0$

$x = -1$ or $x = 1$ only one value of N.

5. **Case-1 :** $x \in I$

$I^2 + I^2 = 13 \Rightarrow 2I^2 = 13 \Rightarrow I = \phi$

Case-2 : $x \notin I$

$x = I + f$ where $f \in (0, 1)$

$I^2 + (I + 1)^2 = 13 \Rightarrow I^2 + I - 6 = 0$

$\Rightarrow I = -3$ or $I = 2$

$\therefore x \in (-3, -2) \cup (2, 3)$

6. Let $A(\alpha, \beta)$, $B(2h - \alpha, 2k - \beta)$ and $\alpha^2 + \beta^2 = a^2$

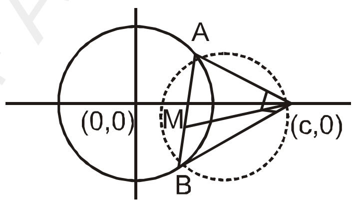
Also $(2h - \alpha)^2 + (2k - \beta)^2 = a^2$

$4h^2 - 4h\alpha + 4k^2 + \beta^2 - 4k\beta + \alpha^2 = a^2$

$4(h^2 - h\alpha - k\beta + k^2) = 0$

$h\alpha + k\beta = h^2 + k^2 \qquad \ldots (1)$

Also $m_1 m_2 = -1$



$\Rightarrow \left(\frac{c - \alpha}{-\beta}\right)\left(\frac{c - 2h + \alpha}{-2k + \beta}\right) = -1$

$\Rightarrow c^2 - 2ch + c\alpha - \alpha c + 2\alpha h - \alpha^2$

$= -2k\beta + \beta^2$

$\Rightarrow 2\alpha h + 2k\beta - 2ch + c^2 = \alpha^2 + \beta^2$

$\Rightarrow 2(h^2 + k^2) - 2ch + c^2 = a^2$

$\Rightarrow 2(x^2 + y^2) - 2cx + c^2 - a^2 = 0$

Aliter

$T = S_1 \Rightarrow xh + yk - a^2 = h^2 + k^2 - a^2$

equation of circle $S + \lambda L = 0$

$(x^2 + y^2 - a^2) + \lambda(xh + yk - h^2 - k^2) = 0$

pass (c, 0)

$c^2 - a^2 + \lambda(ch - h^2 - k^2) = 0$

$\lambda = \frac{c^2 - a^2}{h^2 + k^2 - ch}$

Centre $\left(\frac{-\lambda h}{2}, \frac{-\lambda k}{2}\right)$ lie on $xh + yk = h^2 + k^2$

$-\frac{\lambda h^2}{2} - \frac{\lambda k^2}{2} = h^2 + k^2$

$\frac{-\lambda}{2}(h^2 + k^2)^2 = h^2 + k^2$

$-\lambda = 2$

$\Rightarrow \dfrac{a^2-c^2}{h^2+k^2-ch} = 2 \quad \Rightarrow$ locus $2(x^2+y^2) - 2cx + c^2 - a^2 = 0$

7. $\Delta = \begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = \begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}^2$

$= (2abc)^2 = 4a^2b^2c^2$

8. $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$

$\Rightarrow \dfrac{1}{abc}\begin{vmatrix} -abc & ab^2+abc & ac^2+abc \\ a^2b+abc & -abc & bc^2+abc \\ a^2c+abc & b^2c+abc & -abc \end{vmatrix}$

$\Rightarrow \dfrac{abc}{abc}\begin{vmatrix} -bc & ab+ac & ac+ab \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3$

$(ab + bc + ca)\begin{vmatrix} 1 & 1 & 1 \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$\Rightarrow (ab + bc + ca)\begin{vmatrix} 1 & 0 & 0 \\ ab+bc & -(ab+bc+ac) & 0 \\ ac+bc & 0 & -(ab+bc+ca) \end{vmatrix}$

$= (ab + bc + ca)^3$

DPP NO. - 3

1. $\sum z_1 = 3\alpha$; $\sum z_1z_2 = 3\beta$; $z_1z_2z_3 = -\gamma$

for equilateral triangle $\sum z_1^2 = \sum z_1z_2$

$\Rightarrow \left(\sum z_1\right)^2 = 3\sum z_1z_2 \quad \Rightarrow \alpha^2 = \beta$

2. $d_1 = 4 \quad d_2 = 5$
$d = \text{lcm of } d_1$ & $d_2 = 20$
21, 41, 61,.....

$S = \dfrac{100}{2}[2 \times 21 + (99)20] = 100[21 + 990] = 101100$

3. $1 + 2 + ... + 9 = 45$
Seven digit has to be taken such that number is divisible by 9.
hence we can miss one pair of two digits out of (1, 8), (2, 7), (3, 6), (4, 5)

$\therefore$ Number of formed numbers $= {}^4C_1 \times 7!$

4. when $x \ge 1, (x^8 - x^5) + (x^2 - x) + 1 > 0$
when $x \le 0$, $x^8 - x^5 + x^2 - x + 1$ is positive
$0 \le x \le 1$, $x^8 + (x^2 - x^5) + (1 - x) > 0$
$\Rightarrow$ positive

5. Let $t = \cos x$
so $t^2 + t(1-a) - a^2 \le 0$ for $\forall\, t \in [-1, 1]$
so $f(-1) \le 0$ and $f(1) \le 0$

6. $a\alpha + b = \dfrac{-c}{\alpha}$, $a\beta + b = \dfrac{-c}{\beta}$

Sum of roots $= \dfrac{\alpha^2}{c^2} + \dfrac{\beta^2}{c^2}$

$= \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{c^2} = \dfrac{\dfrac{b^2}{a^2} - 2\dfrac{c}{a}}{c^2} = \dfrac{b^2 - 2ac}{a^2c^2}$

Product of roots $= \dfrac{\alpha^2\beta^2}{c^4} = \dfrac{c^2}{a^2c^4} = \dfrac{1}{a^2c^2}$

Equation is $a^2c^2x^2 - (b^2 - 2ac)x + 1 = 0$.

7. $\dfrac{2}{2}\log_3\left(2\left(\dfrac{1}{2}\right)^x - 1\right)$

$= \dfrac{3}{3}\log_3\left(\left(\dfrac{1}{4}\right)^x - 4\right)$

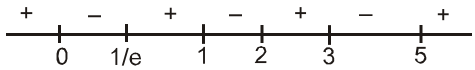
$\Rightarrow 2\left(\dfrac{1}{2}\right)^x - 1 = \left(\dfrac{1}{2}\right)^{2x} - 4$

Let $\left(\dfrac{1}{2}\right)^x = t$

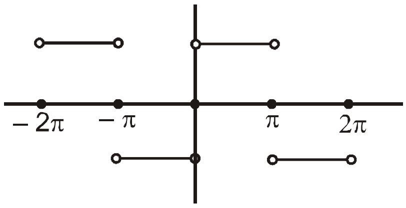
$\Rightarrow t > 0$
$\Rightarrow 2t - 1 = t^2 - 4$
$t^2 - 2t - 3 = 0$
$(t - 3)(t + 1) = 0$
$t = 3$, $t = -1$ (Not possible)

Now, $\left(\dfrac{1}{2}\right)^x = 3$

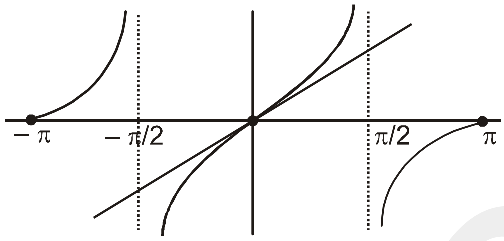
$\Rightarrow x = \log_{1/2} 3 = -\log_2 3$ irrational

8. (A) 

$x \in (0, 1/e) \cup [1, 2] \cup (3, 5)$
integer 1, 2, 4
(B) 7^{40+3} $\Rightarrow$ unit place digit is 3

(C) 

Graph of y = sgn(sgn sinx)

(D) 

DPP NO. - 4

2. $(x - a)(x - 5) = -2$
Since x & a are integral, we can have five combination.
(i) $x - a = 2$
and $x - 5 = -1 \Rightarrow a = 2$
(ii) $x - a = -2$
and $x - 5 = +1 \Rightarrow a = 8$
(iii) $x - a = 1$
and $x - 5 = -2 \Rightarrow a = 2$
(iv) $x - a = -1$
and $x - 5 = 2 \Rightarrow a = 8$

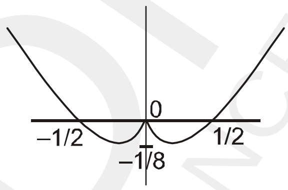
3. **Case I :** both roots are positive
(i) $D = (a - 3)^2 - 4a \geq 0$
$\Rightarrow a^2 - 6a + 9 - 4a \geq 0$
$\Rightarrow a^2 - 10a + 9 \geq 0$
$\Rightarrow (a - 9)(a - 1) \geq 0$
$a \leq 1$ or $a \geq 9$
(ii) Sum of roots > 0

$$\frac{(a-3)}{2} > 0 \Rightarrow a > 3 \Rightarrow a \in [9, \infty)$$

Case II : when one roots negative and one is positive
$f(0) < 0$ and $D \geq 0$
$\Rightarrow a < 0$ and $a \leq 1$ or $a \geq 9$
$\Rightarrow a \in (-\infty, 0)$
Ans. $a \in (-\infty, 0) \cup [9, \infty)$

4. $x + 2y > 0,\ x - 2y > 0$
$\Rightarrow x > 2|y| \geq 0$
and $(x + 2y)(x - 2y) = 4$
$\Rightarrow x^2 - 4y^2 = 4$ ...... (i)
Let $x - y = a$ or $x = a + y$
so, now (i) becomes $(a + y)^2 - 4y^2 = 4$
i.e. $3y^2 - 2ay + (4 - a^2) = 0$
for $y \in R$, $D \geq 0 \Rightarrow a \geq \sqrt{3}$
so, minimum value of $|x| - y = \sqrt{3}$.

5. $2\log_3^2 x - |\log_3 x| = -a$
put $\log_3 x = t$
$2t^2 - |t| = -a$



for four solutions –a should belong to $\left(-\frac{1}{8}, 0\right)$

$\Rightarrow a \in \left(0, \frac{1}{8}\right)$

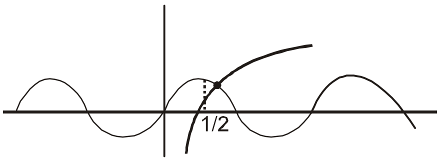
6. Let $x = \sqrt{a} + \sqrt{b}$
$x^2 = a + b + 2\sqrt{a}\sqrt{b}$
$\sqrt{ab} = \frac{x^2 - a - b}{2}$
$\because$ R.H.S is rational and L.H.S is irrational possible when

7. **Case I :** If $0 \leq r < \frac{1}{20}$, then $a + 5a + 10a + 20a$
$= 36K + 35 \Rightarrow 36a = 36K + 35$
which is not possible for any value of a and K as they belongs to integer. .....(a)

Case II : If $\frac{1}{20} \leq r < \frac{1}{10}$, then $a + 5a + 10a + 20a + 1 = 36K + 35$
$\Rightarrow 36a + 1 = 36K + 35 \Rightarrow 36a = 36K + 34$
which is again not possible $\forall$ a, K $\in$ integer
.....(b)

Case III : If $\frac{1}{10} \leq r < \frac{1}{5}$, then $a + 5a + 10a + 1 + 20a + 2 = 36K + 35 \Rightarrow 36a + 3 = 36K + 35$
which shows no value of a and K can be given.
.....(c)

Case IV : If $\frac{1}{5} \leq r < 1$, then $36a + 7 = 36K + 35$
Again, no solution is possible.
.....(d)
Hence, from (a), (b), (c) and (d) there exists no real value which possesses solutions.

8. (A) 

One solution

(B) $||x-2|-3| \le 0$

Possible when $|x-2|-3=0$

$|x-2|=3$

$x-2=\pm 3$

$x=5,\ -1$

(C) $x^3-3x+2=0$

$x=1$ satisfy

$\Rightarrow$ $x^2(x-1)+x(x-1)-2(x-1)=0$

$(x-1)(x^2+x-2)=0$

$(x-1)(x+2)(x-1)=0$

$x=1,\quad x=-2$

(D) x^5-4x^2+2x+1

$= x^4(x-1)+x^3(x-1)+x^2(x-1)$

$\quad -3x(x-1)-(x-1)$

$=(x-1)(x^4+x^3+x^2-3x-1)$

modulus of sum of cofficient

$=|1+1+1-3-1|=1$

DPP NO. - 5

1. $\cos\left(\cos^{-1}\dfrac{\sqrt{3}}{2}+\cos^{-1}\dfrac{1}{3}\right)$

$$= \cos\cos^{-1}\left(\frac{\sqrt{3}}{6}-\sqrt{1-\frac{3}{4}}\sqrt{1-\frac{1}{9}}\right)$$

$$= \frac{\sqrt{3}}{6}-\frac{1}{2}\cdot\frac{2\sqrt{2}}{3}=\frac{\sqrt{3}-2\sqrt{2}}{6}=\frac{\sqrt{3}-\sqrt{8}}{6}$$

2. Let $x=\sqrt{\log_a b} = \dfrac{\sqrt{\log b}}{\sqrt{\log a}}$

$\Rightarrow$ $x\log a=\sqrt{\log b}\ \ \sqrt{\log a}$ ... (i)

and $y=\sqrt{\log_b a} = \dfrac{\sqrt{\log a}}{\sqrt{\log b}}$

$\Rightarrow$ $y\log b=\sqrt{\log a}\ \ \sqrt{\log b}$ ... (ii)

(i) = (ii) $x\log_a = y\log_b$ $\Rightarrow$ $a^x=b^y$

$\Rightarrow$ $a^x-b^y=0$ Ans.

3. $\tan(\tan^{-1}5+\tan^{-1}3)$

$$=\tan\tan^{-1}\left(\frac{5+3}{1-5.3}\right)=\tan\tan^{-1}\left(\frac{8}{-14}\right)=\frac{-4}{7}$$

4. (i) $\tan^{-1}(1)=\dfrac{\pi}{4}$

(ii) $\cos^{-1}(1)=0$

(iii) $\sin^{-1}(1.57)$ = not defined $\sin^{-1}x$ defined when $-1\le x\le 1$

(iv) $\tan^{-1}x=\cot^{-1}(1/x)$ Not true for all x.

(v) Standard result.

5. Equation whose roots are

$\dfrac{1}{x_1},\dfrac{1}{x_2}$ is $(a-1)x^2+x+a=0$

Sum of roots $\dfrac{1}{x_1}+\dfrac{1}{x_2}=\dfrac{-1}{a-1}=\dfrac{1}{1-a}$

Hence $a\ne 1$

Product of roots $=\dfrac{1}{x_1x_2}=\dfrac{a}{a-1}$

Now $\left|\dfrac{1}{x_1}-\dfrac{1}{x_2}\right|>1$

$$\Rightarrow \left(\frac{1}{x_1}+\frac{1}{x_2}\right)^2-4\left(\frac{1}{x_1x_2}\right)>1$$

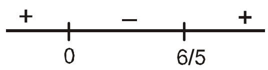
$$\left(\frac{1}{a-1}\right)^2-4\left(\frac{a}{a-1}\right)>1$$

$1-4a(a-1)>(a-1)^2$

$1-4a^2+4a>a^2-2a+1$

$5a^2-6a<0$

$a(5a-6)<0$ and $a\ne 1$



$a\in(0,1)\cup(1,6/5)$

6. $\alpha\beta=62,\quad \gamma,\ \delta$

$\alpha+\beta+\gamma+\delta=37$ ... (i)

and $\alpha\beta+\beta\gamma+\gamma\alpha+\alpha\delta+\beta\delta+\gamma\delta=k$ ...(ii)

$\alpha\beta\gamma+\beta\gamma\delta+\alpha\gamma\delta+\alpha\beta\delta=-808$

$62\gamma\delta-32\beta-32\alpha+62\delta=-808$ ... (iii)

$62(\gamma+\delta)-32(\alpha+\beta)=-808$

$\Rightarrow$ $62(37-(\alpha+\beta))-32(\alpha+\beta)=-808$

$\Rightarrow$ $\alpha+\beta=33,\ \gamma+\delta=4$ and $\alpha\beta\gamma\delta=-1984$

$\Rightarrow$ $62\gamma\delta=-1984$ $\Rightarrow$ $\gamma\delta=-32$

$\Rightarrow$ $62+\gamma(\alpha+\beta)+\delta(\alpha+\beta)-32=k$

$\Rightarrow$ $(\alpha+\beta)(\gamma+\delta)+30=k$ $\Rightarrow(33)(4)+30=K$

$\Rightarrow$ $K=162$

7. $x^2 + \left(\frac{x}{x-1}\right)^2 = 8 \Rightarrow \left(x + \frac{x}{x-1}\right)^2 - 2.x.\frac{x}{x-1}$

$= 8$® $\Rightarrow \left(\frac{x^2}{x-1}\right)^2 - 2\frac{x^2}{x-1} - 8 = 0$

$t^2 - 2t - 8 = 0$

$(t-4)(t+2) = 0$

$t = 4, t = -2$

$\frac{x^2}{x-1} = t = 4$ $\qquad \frac{x^2}{x-1} = -2$

$x^2 - 4x + 4 = 0$ $\qquad x^2 + 2x - 2 = 0$

$(x-2)^2 = 0$ $\qquad x = \frac{-2 \pm \sqrt{4-8}}{2}$

$x = 2$ $\qquad x = \frac{-2 \pm 2\sqrt{3}}{2}$

$x = -1 \pm \sqrt{3}$

Three solution

8. $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

$\Rightarrow \tan^{-1}(1.732) = \frac{\pi}{3}$

$\tan^{-1}\left(\frac{\pi}{3}\right) = \tan^{-1}\left(\frac{3.14}{3}\right)$

$= \tan^{-1}(1.04)$

DPP NO. - 6

1. $\sin^{-1}\left(\frac{a}{1+\frac{a}{3}}\right) + \cos^{-1}\left(\frac{1}{1-b}\right) = \frac{\pi}{2}$

$\Rightarrow \frac{3a}{3+a} = \frac{1}{1-b}$ Only (B) satisfy.

2. $\alpha + \beta = 12$

$ax^2 + bx + c = 0$

Roots α, β

Let $x = \alpha - 1 \Rightarrow \alpha = x + 1$

replace x by x + 1

$a(x+1)^2 + b(x+1) + c = 0$

$\Rightarrow$ Roots are $\alpha - 1, \beta - 1$

Now $\alpha - 1 + \beta - 1$

$= \alpha + \beta - 2 = 12 - 2 = 10$

3. $x^2 + (p + iq)x + 3i = 0$ Roots are α, β

$\alpha^2 + \beta^2 = 8$

$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 8 \Rightarrow (-(p+iq))^2 - 2(3i) = 8$

$\Rightarrow p^2 - q^2 + 2pqi - 6i = 8$

$\Rightarrow p^2 - q^2 + (2pq - 6)i = 8 + 0.i$

Comparing real and imaginary part

$p^2 - q^2 = 8, \quad 2pq - 6 = 0$

$\Rightarrow p^2 - q^2 = 8, \quad 2pq - 6 = 0, pq = 3$

$\Rightarrow p^2 - \left(\frac{3}{p}\right)^2 = 8$

$\Rightarrow p^4 - 8p^2 - 9 = 0$

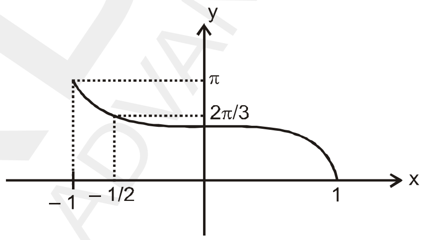
$(p^2 - 9)(p^2 + 1) = 0$

$p = \pm 3 \quad \because \quad p \in$ Real

when $p = 3, \quad q = 1$

$p = -3, \quad q = -1$

4.



$\cos^{-1}\left(\frac{n}{2\pi}\right) > \frac{2\pi}{3}$

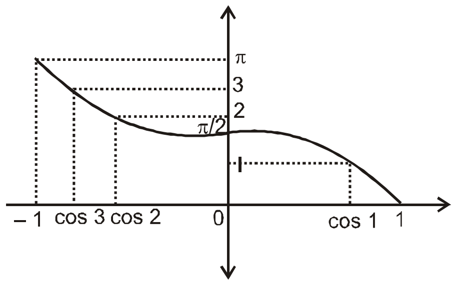
$-1 < \frac{n}{2\pi} < -\frac{1}{2}$

$-2\pi < n < -\pi$

$\Rightarrow -6.28 < n < -3.14$

$n = -4, -5, -6$

5.

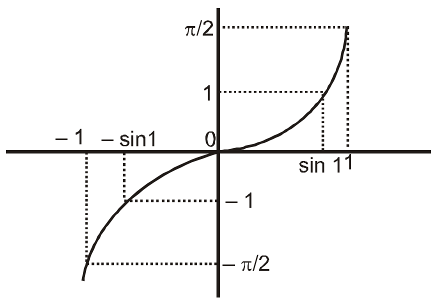


when $-1 < x \le \cos 3$, $[\cos^{-1}x] = 3$

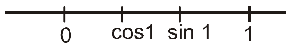
$\cos 3 < x \le \cos 2$, $[\cos^{-1}x] = 2$

$\cos 2 < x \le \cos 1$, $[\cos^{-1}x] = 1$

$\cos 1 < x \le 1$, $[\cos^{-1}x] = 0$



when $-1 \le x < -\sin 1$, $[\sin^{-1} x] = -2$
$-\sin 1 \le 1\ x < 0$, $[\sin^{-1} x] = -1$
$0 \le x < \sin 1$, $[\sin^{-1} x] = 0$
$\sin 1 \le x < 1$, $[\sin^{-1} x] = 1$
Only $[\sin^{-1}x] = [\cos^{-1} x] = 0$ holds
is $x \in (\cos 1, \sin 1)$



6. $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ domain is $[-1, 1]$

$\therefore\ -\frac{\pi}{4} \le \tan^{-1}x \le \frac{\pi}{4}$

7. Let $x^2 = t$
$(a + 1)\, t^2 - 3at\,(t + 1) + 4a(t + 1)^2 = 0$
$\Rightarrow\ t^2\,(a + 1 - 3a + 4a) + t\,(-3a + 8a) + 4a = 0$
$\Rightarrow\ t^2\,(2a + 1) + 5\,at + 4a = 0$
$\because\ t \ge 0$
(i) $D \ge 0$
$\Rightarrow\ 25a^2 - 4(4a)\,(2a + 1) \ge 0$
$\Rightarrow\ 25a^2 - 32a^2 - 16a \ge 0$
$\Rightarrow\ 7a^2 + 16a \le 0$
$\Rightarrow\ a(7a + 16) \le 0$

$a \in \left[\frac{-16}{7}, 0\right]$

(ii) $\frac{-5a}{2a+1} \ge 0 \Rightarrow \frac{a}{2a+1} \le 0$

$\Rightarrow\ a \in \left(-\frac{1}{2}, 0\right]$

8. (A) $f(x) = |x - 4| + |x - 6| + |x - 2|$

DPP NO. - 7

1 to 3.
$2^{|x+1|} - 2^x = |2^x - 1| + 1$
Case I
$x < -1$
$2^{-x-1} - 2^x = -(2^x - 1) + 1$
$\Rightarrow\ 2^{-x-1} = 2 \quad \Rightarrow\ -x - 1 = 1 \ \Rightarrow\ x = -2$
Case II
$-1 \le x < 0$
$2^{x+1} - 2^x = -(2^x - 1) + 1$
$2^{x+1} = 2^1 \quad \Rightarrow\ x = 0$ (Not accepted)
Case III
$x \ge 0$
$2^{x+1} - 2^x = 2^x - 1 + 1$
$2^{x+1} = 2^{x+1}$
True for all $x \ge 0$
least value of $x = -2$
number of integer less than 15 are $-2, 0, 1, 2, \ldots\ldots 14$
total 16 integers
and (9, 4) (4, 15) are two coprime numbers

4. $\sin(\theta + \phi - \phi) = 5\sin(\theta + \phi)$
$\sin(\theta + \phi)\cos\phi - \cos(\theta + \phi)\sin\phi = 5\sin(\theta + \phi)$
$\tan(\theta + \phi)\,(\cos\phi - 5) = \sin\phi$

$\tan(\theta + \phi) = \frac{\sin\phi}{\cos\phi - 5}$

5. Hold only $\sqrt{x^2 - x + 1} = 1$, $\sqrt{x^2 - x} = 0$
$x^2 - x = 0 \quad x = 0, x = 1$

6. $2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x + \pi - 2\tan^{-1}(x)$

defined for $|x| \ge 1$

7. $T_n = \tan^{-1}\left(\frac{2^{n+1} - 2^n}{1 + 2^{n+1}.\,2^n}\right)$

$\Rightarrow\ T_n = \tan^{-1}(2^{n+1}) - \tan^{-1}(2^n)$
$S_n = \Sigma T_n = (\tan^{-1}(2^2) - \tan^{-1}(2)) + (\tan^{-1}(2^3 - \tan^{-1}(2^2)$
$+ \ldots\ldots + \tan^{-1}(2^{n+1}) - \tan^{-1}(2^n)$
$= \tan^{-1}(2^{n+1}) - \tan^{-1}(2)$

$= \tan^{-1}\left(\frac{2^{n+1} - 2}{1 + 2.2^{n+1}}\right) = \cot^{-1}\left(\frac{1 + 4.2^n}{2(2^n - 1)}\right)$

8. Let $x^2 + x + 1 = t$
$(t + 1)^2 - (a - 3)\,(t + 1)t + (a - 4)t^2 = 0$
$\Rightarrow\ t^2\,(1 + a - 4 - a + 3) + t\,(2 - a + 3) + 1 = 0$

$t = \frac{-1}{5-a} = \frac{1}{a-5}$ But $t = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} \Rightarrow$

$\frac{1}{a-5} \ge \frac{3}{4} \Rightarrow \frac{4 - 3a + 15}{4(a-5)} \ge 0$

$\Rightarrow \frac{(3a - 19)}{(a-5)} \le 0 \quad \Rightarrow\ 5 < a \le \frac{19}{3}$

DPP NO. - 8

1. AB exists $\Rightarrow x + 5 = y$ ...(i)
BA exists $\Rightarrow 11 - y = x$ ...(ii)
On solving (1) & (2)
$y = 8$ & $x = 3$

2. $\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix} + 2x = \begin{bmatrix} 6+1+6 & 4+4-2 \\ 0+1-9 & 0+4+3 \end{bmatrix}$

$2X = \begin{bmatrix} 13 & 6 \\ -8 & 7 \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 12 & 8 \\ -12 & 4 \end{bmatrix}$

$\Rightarrow\ X = \begin{bmatrix} 6 & 4 \\ -6 & 2 \end{bmatrix}$

3. $A = \begin{bmatrix} x^2 & 1 & 0 \\ 2 & 2x & -1 \\ 4 & 5 & 24/x \end{bmatrix}$

$\Rightarrow\ f(x) = tr(A)$

$= x^2 + 2x + \frac{24}{x}$

Now AM ≥ GM

$\Rightarrow \frac{x^2+2x+\frac{8}{x}+\frac{8}{x}+\frac{8}{x}}{5} \geq \left(x^2.2x.\left(\frac{8}{x}\right)^3\right)^{1/5}$

$\Rightarrow x^2 + 2x + \frac{24}{x} \geq 5\,(2^{10})^{1/5} = 20$

4. $BC = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$

$BC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$(BC)^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A(BC) = A$

$A(BC)^2 = A$

$tr(A) + tr\left(\frac{ABC}{2}\right) + tr\left(\frac{A(BC)^2}{4}\right) + tr\left(\frac{A(BC)^3}{8}\right)$

$tr(A) + tr\left(\frac{A}{2}\right) + tr\left(\frac{A}{4}\right) + tr\left(\frac{A}{8}\right) + ...\infty$

$S = 3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + ...$

$S = \frac{3}{1-\frac{1}{2}} = 6$

5. $\tan^{-1} n + \tan^{-1}(n+1) + \tan^{-1}(n+2) = \pi$

$\Rightarrow \tan^{-1}\left(\frac{n+n+1}{1-n(n+1)}\right)$

$= \tan^{-1}(0) - \tan^{-1}(n+2) = \tan^{-1}\left(\frac{-n-2}{1}\right)$

$\Rightarrow \frac{2n+1}{1-n^2-n} = -n-2$

$\Rightarrow 2n + 1 = -n + n^3 + n^2 - 2 + 2n^2 + 2n$

$\Rightarrow n^3 + 3n^2 - n - 3 = 0 \Rightarrow n = 1$ satisfy

6. $x - \frac{1}{2} > 0 \Rightarrow x > \frac{1}{2}$ and $x + 1 > 0$

$\Rightarrow x > -1$

Let $\log_{x+1}(x - 0.5) = t \Rightarrow t = \frac{1}{t} \Rightarrow t = \pm 1$

when $\log_{(x+1)}\left(x - \frac{1}{2}\right) = 1$

$\Rightarrow x - \frac{1}{2} = x + 1$ Not possible

when $\log_{(x+1)}\left(x - \frac{1}{2}\right) = -1$

$\Rightarrow \left(x - \frac{1}{2}\right) = \frac{1}{(x+1)} \Rightarrow (2x-1)(x+1) = 2$

$\Rightarrow 2x^2 + x - 3 = 0 \Rightarrow 2x^2 + 3x - 2x - 3 = 0$

$\Rightarrow (2x+3)(x-1) = 0$

$x = -\frac{3}{2}, x = 1$

Only x = 1 possible

7. $\lim_{n\to\infty}\sum_{k=2}^{n}\cos^{-1}\left(\frac{1}{k}\cdot\frac{1}{k+1}+\sqrt{1-\frac{1}{k^2}}\cdot\sqrt{1-\frac{1}{(k+1)^2}}\right)$

$= \lim_{n\to\infty}\sum_{k=2}^{n}\left[\cos^{-1}\left(\frac{1}{k+1}\right)-\cos^{-1}\left(\frac{1}{k}\right)\right]$

$= \lim_{n\to\infty}\left(\cos^{-1}\left(\frac{1}{3}\right)-\cos^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{4}-\cos^{-1}\frac{1}{3}+\cos^{-1}\frac{1}{5} -\cos^{-1}\frac{1}{4}+....+\cos^{-1}\frac{1}{n+1}-\cos^{-1}\frac{1}{n}\right)$

$= \lim_{n\to\infty}\left(-\cos^{-1}\left(\frac{1}{2}\right)+\cos^{-1}\left(\frac{1}{n+1}\right)\right)$

$= -\frac{\pi}{3}+\cos^{-1}(0) = \frac{\pi}{2}-\frac{\pi}{3} = \frac{\pi}{6}$

Now $\lambda = \frac{120\pi}{\frac{\pi}{6}} = 720$

8. (A) $\log_{x^2}(2-x) < 0 \Rightarrow 2 - x < 1$

$\Rightarrow x > 1$ when $0 > x^2 > 1$ defined $2 - x > 0$

$\Rightarrow x < 2$ when $0 < x^2 < 1$ $x \in (-1, 0) \cup (0, 1)$

$2 - x > 1 \Rightarrow 1 > x$

(B) $(e^x - 1)(x^2(x-1) + 9(x-1)) < 0$

$\Rightarrow (e^x - 1)(x^2 + 9)(x-1) < 0$

$\Rightarrow x \in (0, 1)$

(C) $\frac{|x|(x-4)}{\log(x+2)} < 0$ defined for $x + 2 > 0$

$\Rightarrow x > -2 \quad x \neq -1$

when $\log(x+2) > 0 \Rightarrow x + 2 > 1$

$\Rightarrow x > -1$

and (i) when $-1 < x < 0$

$-x(x-4) < 0$

$x(x-4) > 0$

$x < 0$ or $x > 4$

Ans $x \in (-1, 0)$

No integral value

(ii) when $x \geq 0$

$x(x-4) < 0$

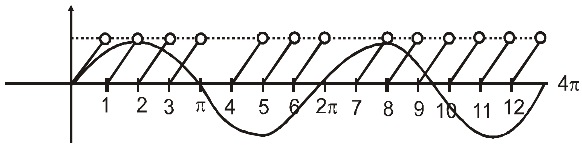
$x \in (0, 4)$

integers = 1, 2, 3

(iii) when $-2 < x < -1$

No integers

(D) $\sin x < \{x\}$



integer satisfy x = 4, 5, 6, 10, 11, 12

DPP NO. - 9

1. $(-12 + 4\pi) - (12 - 4\pi) = 8\pi - 24$

2. $\theta = \sin^{-1}\left(\sqrt{\frac{(\sqrt{3}-1)^2}{8}}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sec^{-1}(\sqrt{2})$

$\theta = \sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sec^{-1}\left(\sqrt{2}\right)$

$\theta = \frac{\pi}{12} + \frac{\pi}{6} + \frac{\pi}{4}$

$\theta = \frac{\pi + 2\pi + 3\pi}{12} = \frac{\pi}{2}$.

3. $a_{ij} = -a_{ji} \Rightarrow$ skew symmetric matrix $\Rightarrow |A| = 0$

$b_{ij} = b_{ji} \Rightarrow$ symmetric matrix

Now, $|A^4 B^3| = |A^4||B^3| = (|A|)^4 (|B|)^3 = 0$

4. $|A| = 8$

$1 + \log_{\frac{1}{2}}\left(|A|^{|A|^{n-1}}\right) \qquad n = 3$

$1 + \log_{\frac{1}{2}}\left(8^{64}\right)$

$1 + \log_{\frac{1}{2}} 2^{192} = 1 - 192 = -191$

5. $|A| = \begin{vmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix} = 1(3) - 2(-1-4) = 3 + 10 = 13$

Now $|\text{adj adj}(A)| = |A|^{(n-1)^2} = (13)^{(3-1)^2} = (13)^4$

6. $p(x) = x^3 + ax^2 + bx + c$ Let roots are α, β, γ

$\frac{\alpha+\beta+\gamma}{3} = \frac{-a}{3}$, $\alpha\beta\gamma = -c$, $1 + a + b + c = \frac{-a}{3} = -c$

$\Rightarrow a = 3c$ and $1 + 3c + b + c = -c$

$b + 5c - 1 = 0$

also $p(0) = 2 \Rightarrow 0 + 0 + 0 + c = 2 \Rightarrow c = 2$

$b = -11$

7. $y = \frac{2x^2 + 2x + 3}{x^2 + x + 1}$

$\Rightarrow (y-2)x^2 + (y-2)x + y - 3 = 0$

$x \in R$, $D \geq 0$

$(y-2)^2 - 4(y-2)(y-3) \geq 0$

$\Rightarrow (y-2)(3y-10) \leq 0$

4 is good number and all good number are $(2, \infty)$

DPP NO. - 10

1. **(B)**

$\Delta = \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow 8 - k(k-2) - 2(2k-8) = 0$

$\Rightarrow 8 - k^2 + 2k - 4k + 16 = 0$

$\Rightarrow -k^2 - 2k + 24 = 0$

$\Rightarrow k^2 + 2k - 24 = 0$

$\Rightarrow (k+6)(k-4) = 0$

$\Rightarrow k = -6, 4$

Number of values of k is 2

2. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \sin^{-1} x$

$\Rightarrow \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}(x)$

$\Rightarrow \frac{2x}{1+x^2} = x$

$\Rightarrow 2 = 1 + x^2$

$\because x = \pm 1, \quad x = 0$

$-1 \leq \frac{1-x^2}{1+x^2} \leq 1 \Rightarrow -1 \leq \frac{1-x^2}{1+x^2} \leq 1$

$\Rightarrow -1-x^2 \leq 1 - x^2 \leq 1 + x^2$

$\Rightarrow 2x^2 \geq 0 \Rightarrow x \geq 0$

3. Let A = $\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$\Rightarrow$ det (A) = $a_1 (b_2c_3 - c_2b_3)$

$- a_2 (b_1c_3 - c_1b_3) + a_3 (b_1c_2 - c_1b_2)$

$= a_1b_2c_3 - a_1c_2b_3 + a_2c_1b_3 - a_2 b_1c_3 + a_3b_1c_2 - a_3 c_1b_2$

if any of the terms is non-zero, then det (A) will be non-zero and all the element of that term will be unity

Now there are 6 elements remaining out of which any one can be unity.

Hence number of non-singular matrices

$$= \underbrace{{}^6C_1}_{\text{choosing any one triplet}} \times \underbrace{{}^6C_1}_{\text{choosing any one element}}$$

Hence correct option is (3)

7. [x] [y] = x + y

(i) if x, y $\in$ I then xy = x + y

or $y = \dfrac{x}{x-1}$

$\Rightarrow$ (x, y) is (0, 0), (2, 2)

(ii) if x, y $\notin$ I

Let $x = I_1 + f_1$ and $y = I_2 + f_2$

then $I_1 + I_2 + f_1 + f_2 = I_1I_2 \Rightarrow f_1 + f_2 \in I$

$0 < f_1 + f_2 < 2 \Rightarrow f_1 + f_2 = 1.$

$I_1 + I_2 + 1 = I_1I_2 \Rightarrow I_1$

$= \dfrac{I_2+1}{I_2-1} = 1 + \dfrac{2}{I_2-1}.$

$I_2 - 1 = \pm 1, \pm 2, \quad I_2 = 2, 0, 3, -1$

$\Rightarrow \therefore I_1 = 3, -1, 2, 0$

$I_1 I_2 = 6, 0 \Rightarrow x + y = I_1I_2$

$\Rightarrow$ x + y = 0 or x + y = 6

8. $x + y + z = 12, x^2 + y^2 + z^2 = 96$

xy + yz + zx = 36 xyz

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2 (xy + yz + zx)$

$\Rightarrow 48 = 2 (36\ xyz) \Rightarrow xyz = \dfrac{2}{3}$

$\therefore$ xy + yz + zx = 24 ....(i)

also $x^3 + y^3 + z^3 - 3xyz = (x + y + z) (x^2 + y^2 + z^2 - xy - yz - zx)$

$\Rightarrow x^3 + y^3 + z^3 - 3\left(\dfrac{2}{3}\right) = 12 (96 - 24)$

$\Rightarrow x^3 + y^3 + z^3 = 12(72) + 2 = 866$ Ans.