Class : 12th Maths
Chapter- 9 : Differential Equations

Differential Equations

Definition

An equation involving derivatives of the dependent variable with respect to independent variable (variables) is called a differential equation. If there is only one independent variation, then we call it as an ordinary differential equation. For eg: $2\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3=0.$

Variable Separation Method

It is used to solve such an equation in which variables can be separated completely. For eg: $y\,dx = x\,dy$ can be solved as $\frac{dx}{x}=\frac{dy}{y}$ Integrating both sides $\log x = \log y + \log c \Rightarrow \frac{x}{y}=c \Rightarrow x = cy$,is the solution.

Order of a Differential Equation

It is the order of the highest order derivative occurring in the Differential Equation For eg: the order of $\frac{dy}{dx}=e^x$ is one and order of $\frac{d^2y}{dx^2}+x=0$ is two.

The order of a Differential Equations representing a family of curves is same as the number of arbitrary constants present in the equation corresponding to the family of curves. For eg: Let the family of curves be $y = mx$, $m =$ constant, then, $y' = m$

$y = y'x \Rightarrow y = \frac{dy}{dx}x \Rightarrow x\frac{dy}{dx} - y = 0.$

Degree of a Differential Equation

It is defined if the Differential Equations is a polynomial equation in its derivatives, and is defined as the highest power (positive integer only) of the highest order derivative.

For eg: the degree of $\left(\frac{d^2y}{dx^2}\right)^3+\frac{dy}{dx}=0$ is three

Order and degree (if defined) of a D.E. are always positive integers.

Homogeneous Differential Equation

A Differential Equation which can be expressed in the form $\frac{dy}{dx}=f(x,y)$ or $\frac{dx}{dy}=g(x,y)$, where, $f(x, y)$ and $g(x, y)$ are homogeneous functions of degree zero is called a homogenous Differential Equation

For eg : $(x^2+xy)dy=(x^2+y^2)dx$

To solve this, we substitute $\frac{y}{x}=v \Rightarrow y=vx.$

Linear Differential Equation

A Differential Equation of the form $\frac{dy}{dx}+Py=Q$, where P, Q are constants or functions or 'x' only is called a first order linear Differential Equations its solution is $ye^{\int P.dx}=\int Q.e^{\int P.dx}dx+c$. For eg : $\frac{dy}{dx}+3y=2x$ has solution $ye^{\int 3.dx}=\int 2x.e^{\int 3.dx}dx+c \Rightarrow ye^{3x}=2\int xe^{3x}+c.$

Solution of a Differential Equation

A function which satisfies the given Differential Equation is called its solution. The solution which contains as many arbitrary constants as the order of the D.E. is called a general solution and the solution free from arbitrary constants is called particular solution.

For eg: $y=e^x+1$ is a solution of $y''-y'=0$. Since $y'=e^x$ and $y''=e^x \Rightarrow y''-y'=e^x-e^x=0.$

Formation of Differential Equation

To form a Differential Equation from a given function, we differentiate the function successively as many times as the no. of arbitrary constants in the given function, and then eliminate the arbitrary constants.

For eg: Let the function be $y=ax+b$, then we have to differentiate it two times, since there are 2 arbitrary constants a and b. $\therefore y'=a \Rightarrow y''=0$. Jhus $y''=0$ is the required Differential Equation.