Class : 12th Maths
Chapter- 2 : Inverse Trigonometric Functions

(i) $y = \sin^{-1}x$. Domain = [–1,1], Range = $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(ii) $y = \cos^{-1}x$. Domain = [–1,1] Range = $[0, \pi]$

(iii) $y = \text{cosec}^{-1}x$. Domain = R–(–1,1), Range = $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

(iv) $y = \sec^{-1}x$. Domain = R–(–1,1), Range = $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

(v) $y = \tan^{-1}x$. Domain = R, Range = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(vi) $y = \cot^{-1}x$. Domain = R, Range = $(0, \pi)$.

Domain and range of inverse trigonometric functions

(I) $\sin : R \to [-1,1]$

(ii) $\cos : R \to [-1,1]$

(iii) $\tan : R - \left\{x : x = (2n+1)\frac{\pi}{2} in \in Z\right\} \to R$

(iv) $\cot : R - \{x : x = h\pi, n \in Z\} \to R$

(v) $\sec : R - \left\{x : x = (2n+1)\frac{\pi}{2}, n \in Z\right\} \to R - (-1,1)$

(vi) $\text{cosec} : R - \{x : x = h\pi, n \in Z\} \to R - (-1,1)$

Trigonometric functions

The range of an inverse trigonometric function is the principal value branch and those values which lies in the principal value branch is called the principal value of that inverse trigonometric functions.

Some important relations

$\sin^{-1}x \neq (\sin x)^{-1}.(\sin x)^{-1}$

$= \frac{1}{\sin x}$ and same for other trigonometric functions.

Inverse Trigonometric Functions

Graphs of trigonometric functions and inverse trigonometric functions

Principal Value branch and principal value

(i) $y = \sin^{-1}x \Rightarrow x = \sin y$

(ii) $x = \sin y \Rightarrow y = \sin^{-1}x$

(iii) $\sin(\sin^{-1}x) = x$

(iv) $\sin^{-1}(\sin x) = x$

(v) $\sin^{-1}\frac{1}{x} = \text{cosec}^{-1}x$

(vi) $\cos^{-1}(-x) = \pi - \cos^{-1}x$

(vii) $\cos^{-1}\frac{1}{x} = \sec^{-1}x$

(viii) $\cot^{-1}(-x) = \pi - \cot^{-1}x$

(ix) $\tan^{-1}\frac{1}{x} = \cot^{-1}x$

(x) $\sec^{-1}(-x) = \pi - \sec^{-1}x$

(xi) $\sin^{-1}(-x) = -\sin^{-1}x$

(xii) $\tan^{-1}(-x) = -\tan^{-1}x$

(xiii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

(xiv) $\text{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$

(xv) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

(xvi) $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$

(xvii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$

(xviii) $2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1+x^2}{1-x^2}$

For *eg* : to find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$, let $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = y$,

$\Rightarrow \sin y = \frac{1}{\sqrt{2}}$ The range or the principal value branch of $\sin^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

and $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ So, the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is $\frac{\pi}{4}$