Class : 12th Maths
Chapter- 13 : Probability

Probability

Random Variable

Real valued function whose domain is the sample space of a random experiment

The probability distribution of a random variable x is the system of numbers x: $x_1 \quad x_2 \quad . . . . , x_n$
$P(x)$: $p_1 \quad p_2 \quad . . . . p_n$ where , $p_i>0$, $\sum_{i=1}^{n} p_i=1, \quad i=1.2,...., \quad n.$

Conditional probability

The probability of the event E is called the conditional probability of E given that F has already occurred, and is denoted by $P(E/F)$. Also,

$$P(E/F)=\frac{P(E\cap F)}{P(F)}, P(F)\neq 0.$$

Properties

(i) $0\le P(E/F)\le 1$, $P(E'/F)=1-P(E/F)$

(ii) $P((E\cup F)/G)=P(E/G)+P(F/G)-P((E\cap F)/G)$

(iii) $P(E\cap F)=P(E)P(F/E), P(E)\neq 0$

(iv) $P(E\cap F)=P(F)P(E/F), P(F)\neq 0$

For eg : if $P(A)=\frac{7}{13}$, $P(B)=\frac{9}{13}$ and $P(A\cap B)=\frac{4}{13}$, then

$$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{4}{13}}{\frac{9}{13}}=\frac{4}{9}.$$

If E and F are independent, then P E ∩ F = P (E)P (F), P (E|F) = P (E), P (F) ≠ 0 and (F|F) = P(F), P(E) ≠ 0.

Variance and standard deviation

Let x be a R.V. whose possible values $x_1, x_2, ..., x_n$ occurs with probabilities $p(x_1), p(x_2), ... , p(x_n)$ respectively. Let, $\mu = E(x)$ be the mean of x. The variance of x, var (x) or $\sigma_x^2=\sum_{i=1}^{n}(x_i-\mu)^2 p(x_i)$ or $E(x-\mu)^2$

The non-negative number

$$6x=\sqrt{\text{var}(x)}=\sqrt{\sum_{i=1}^{n}(x_i-\mu)^2 p(x_i)}$$ is called the standard deviation of the R.V. 'X'. Also,

$\text{var}(x)=E(x^2)-[E(x)]^2$ For eg: $E(x) = 3$ and $E(x^2) = 10$, then var $x = 10-9 = 1$ and SD$=\sqrt{1}=1$.

Bernoulli's Trials

Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions :

(i)There should be a finite no. of trials.

(ii)The trials should be independent.

(iii)Each trial has exactly two outcomes: success or failure.

(iv)The probability of success remains the same in each trial.

For Binomial distribution,B (n, p), $P(X=x)=\left({}^nC_x q^{n-x} p^x, x=0,1,....,n\right)$ $(q=1-p)$

Theorem of total Probability

Let, $\{E_1, E_2 \ ....E_n\}$ be a partition of a sample space 'S' and suppose that each of $E_1, E_2, ...E_n$ has non-zero probability. Let 'A' be any event associated with S, then $P(A) = P(E_1)P(A|E_1) + P(E_2) P(A|E_2) + ... + P(E_n) P(A|E_n)$.

Bayes' Theorem

If $E_1, E_2, ..., E_n$ are events which constitute a partition of sample space S, *i.e.*, $E_1, E_2, . . . E_n$ are pair wise dis joint and $E_1\cup E_2\cup ....\cup E_n=S$ and A be any event with non-zero probability, then

$$P(E_i|A)=\frac{P(E_i)P(A|E_i)}{\sum_{j=1}^{n}P(E_j)P(A|E_j)}$$