Linear Programming

Fundamental Theorems

Theorem I : Let be the feasible region (convex polygon) for a R L.P. and let be the objective function. When has an Z = ax + by Z optimal value (max. or min.), where the variables are subject x ,y to the constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region, Theorem 2: Let be the feasible region for a L.P.P, and let R Z= ax + by be the objective function. If is bounded then the O.F has R Z both a max. and a min. value on and each of these occurs at a R corner point (vertex) of . If the feasible region is unbounded, R then a max. or a min. may not exist. If it exists, it must occur at a corner point of R.

Definition

A. L.P.P is one that is concerned with finding the optimal value (max. or min.) of a linear function of several variables (called objective function) subject to the conditions that the variables are non-negative and satisty a set of linear inequalities (called linear constraints). Variables are sometimes called decision variables and are non-negative.

Types of L.P.P

(i) Diet problems
(ii) Manufacturing Problem
(iii) Transportation Problems

Corner point method

(i)Find the feasible region of the L.P.P and determine its corner points (vertices). (ii) Evaluate the O.F. at each corner Z = ax+by point. Let and be the largest and smallest values respectively M m at these points. If the feasible region is unbounded, and are M m the maximum and minimum values of the O.F. If the feasible region is unbounded , then (i) ' ' is the max. value of the O.F., if the open M half plane determined by has no point in common with ax+by>M the feasible region. Otherwise, the O.F. has no maximum value. (ii) ' ' is the minimum value of the O.F., if the open half plane m determined by has no point in common with the feasible ax+by

Solution of L.P.P

The common region determined by all the constraints including the non-negative constraint of a L.P.P is called the feasible region (or solution region) for the problem. Points within and on the boundary of the feasible region represent feasible solutions of the constraints. Any point outside the feasible region is an infeasible solution. Any point in the feasible region that gives the optimal value (max. or min.) of the objective function is called an optimal solution.

For Eg : Max Z = 250x + 75y, subject to the
Constraints: $5x + y \le 100$
$x + y \le 60$
$x \ge y$ 0, $y \ge 0$ is an L.P.P