Three dimensional Geometry

Shortest distance between

(i) two skew lines is the line segment perpendicular to both the lines

(ii) $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$ is $\left|\dfrac{(\vec{b}_1\times\vec{b}_2).(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|$

(iii) $\dfrac{x-x_1}{a_1}=\dfrac{y-y_1}{b_1}=\dfrac{z-z_1}{c_1}$ and $\dfrac{x-x_2}{a_2}=\dfrac{y-y_2}{b_2}=\dfrac{z-z_2}{c_2}$ is

$$\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}$$

(iv) Parallel line $\vec{r}=\overline{a_1}+\lambda\vec{b}$ and $\vec{r}=\overline{a_2}+\mu\vec{b}$ is $\dfrac{\vec{b}\times(\overline{a_2}-\overline{a_1})}{|\vec{b}|}$

Direction ratios and direction cosines of line

D. Cs of a line are the cosines of the angles made by the line with the positive direction of the co-ordinate axes. If l, m, n are the D. Cs of a line, then $l^2+m^2+n^2=1$. D. Cs of a line joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $\dfrac{x_2-x_1}{PQ}, \dfrac{y_2-y_1}{PQ}, \dfrac{z_2-z_1}{PQ}$,where $PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

D.Rs of a line are the no.s which are proportional to the D.Cs of the line if l, m, n are the D.Cs and a, b, c are D.Rs of a line, then

$l=\dfrac{a}{\sqrt{a^2+b^2+c^2}}$, $m=\dfrac{b}{\sqrt{a^2+b^2+c^2}}$, $n=\dfrac{c}{\sqrt{a^2+b^2+c^2}}$

Equation of a plane

(i) which is at distance 'd' from origin and D.C.s of the normal to the plane as l,m,n is $lx+my+nz=d$.

(ii) $\perp r$ to a given line with D.Rs. A,B,C and passing through (x_1,y_1,z_1) is $A(x-x_1)+B(y-y_1)+C(z-z_1)=0$

(iii) Passing through three non-collinear points (x_1,y_1,z_1) (x_2,y_2,z_2), (x_3,y_3,z_3) is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0.$

Vector equation of a plane

(i) which contains three non-collinear points having position vectors $\vec{a}, \vec{b}, \vec{c}$ is $(\vec{r}-\vec{a}).[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]=0$.

(ii) That passes through the intersection of planes $\vec{r}.\overline{n_1}=d_1$ & $\vec{r}.\overline{n_2}=d_2$ is $\vec{r}(\overline{n_1}+\lambda\overline{n_2})=d_1+\lambda d_2$, λ – non-zero constant.

Characteristics of planes

Two lines $\vec{r}=\overline{a_1}+\lambda\overline{b_1}$, $\vec{r}=\overline{a_2}+\mu\overline{b_2}$ are coplanar if $(\overline{a_2}-\overline{a_1}).(\overline{b_1}\times\overline{b_2})=0$. Equation of a plane that cuts co-ordinate axes at (a,0,0), (0,b,0), (0,0,c) is $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$.

The distance of a point with postion vector $\vec{a}$ from the plane $\vec{r}.\hat{n}=d$ is $|d-\vec{a}.\hat{n}|$. The distance from a point (x_1, y_1, z_1) to the plane $Ax+By+Cz+D=0$ is $\left|\dfrac{Ax_1+By_1+Cz_1+D}{\sqrt{A^2+B^2+C^2}}\right|$

Skew lines

These are the lines in space which are neither parallel nor intersecting. They lie in different planes. Angle between skew lines is the angle between two intersecting lines drawn from any point (origin) parallel to each of the skew lines.

Angle between the two lines

if l_1, m_1, n_1, l_2, m_2, n_2 are the D.Cs and $a_1, b_1, c_1, a_2, b_2, c_2$ are the D.Rs of the two lines and 'θ' is the acute angle between them, then

$\cos\theta=|l_1l_2+m_1m_2+n_1n_2|=\left|\dfrac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$

Line of vector equations in 3D

Vector equation of a line passing through the given point whose position vector is $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\vec{r}=\vec{a}+\lambda\vec{b}$

Vector equation of a line which passes through two points whose position vectors are $\vec{a}$ and $\vec{b}$ is $\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$

Equation of a line through point (x_1, y_1, z_1) and having D.Cs l, m, n is $\dfrac{x-x_1}{l}=\dfrac{y-y_1}{m}=\dfrac{z-z_1}{n}$ Also, equation of a line that passes through two points.

Angle between two lines

If 'θ' is the acute angle between $\vec{r}=\overline{a_1}+\lambda\overline{b_1}$, $\vec{r}=\overline{a_2}+\lambda\overline{b_2}$

then, $\cos\theta=\left|\dfrac{\overline{b_1}.\overline{b_2}}{|\overline{b_1}||\overline{b_2}|}\right|$

if $\dfrac{x-x_1}{l_1}=\dfrac{y-y_1}{m_1}=\dfrac{z-z_1}{n_1}$ and $\dfrac{x-x_2}{l_2}=\dfrac{y-y_2}{m_2}=\dfrac{z-z_2}{n_2}$ are the equations of two lines, then acute angle between them is $\cos\theta=|l_1.l_2+m_1.m_2+n_1.n_2|$