Vector Algebra

Vectors

A quantity that has both magnitude and direction is called a vector. The distance between the initial and terminal points of a vector is called its magnitude. Magnitude of vector $\overrightarrow{AB}$ is $|AB|$.

Multiplication of vector by a scalar

If λ Multiplied to vector AB, then the magnitude is multiplied |λ| by and direction remain same (or opp.) according as λ is the + *ue* or, *ue*-

Unit Vectors

For a given vector a, the vector $\hat{a} = \frac{a}{|a|}$ gives the unit vector in the direction of a. for *eg* , if $a=5i$, then $\hat{a} = \frac{5i}{|5|} = i$,which is a unit vector.

Position Vectors

Position vector of a point $P\ (x, y, z)$ is $x\hat{i} + y\hat{j} + z\hat{k}$ and its magnitude is $OP(r) = \sqrt{x^2+y^2+z^2}$ .For eg: Position vector of $P\ (2,3,5)$ is $2\hat{i}+3\hat{j}+5\hat{k}$ and its magnitude is $\sqrt{2^2+3^2+5^2} = \sqrt{38}$.

Position of vectors

The Position vector of a point R dividing a line segment joining P,Q whose position vectors are a, b resp., in the ratio $m : n$
(i) internally is $\frac{na+mb}{m+n}$, (ii) externally is $\frac{mb-na}{m-n}$

Scalar Product of two vectors

If a, b are the vectors and 'θ', angle between them, then their scalar product $a.b = |a||b|\cos\theta$
$\Rightarrow \cos\theta = \frac{a.b}{|a||b|}$

Direction ration and direction cosines

The scalar components of a vector are its direction ratios, and represent its projections along the respective axes.
The magnitude (r) direction ratios (a,b,c) and direction cosines (l,m,n) of vector $a\hat{i}+b\hat{j}+c\hat{k}$ are related as:
$l = \frac{a}{r}, m = \frac{b}{r}, n = \frac{c}{r}$
For *eg* : If $\overrightarrow{AB} = \hat{i}+2\hat{j}+3\hat{k}$, then $r = \sqrt{1+4+9} = \sqrt{14}$
Direction ratios are (1,2,3)
and direction cosines are $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

Cross Product of two vectors

$a \times b = |a||b|\sin\theta\,\hat{n}$, $\hat{n}$ is a unit vector perpendicular to line joining a,b.

Types of Vectors

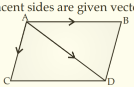
(i) Zero vector (initial and terminal points coincide)
(ii) Unit vector (magnitude is unity.
(iii) Coinitial vectors (same initial points)
(iv) Collinear vectors (parellel to the same line)
(v) Equal vectors (same magnitude and direction)
(vi) Negative of a vector (same magnitude, opp. direction)

Properties of Vectors

If we have two vectors
$a = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $b = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and λ is any scalar, then-

$$a+b = (a_1+b_1)\hat{i}+(a_2+b_2)\hat{j}+(a_3+b_3)\hat{k}$$
$$\lambda a = (\lambda a_1)\hat{i}+(\lambda a_2)\hat{j}+(\lambda a_3)\hat{k}$$
$$a.b = a_1b_1+a_2b_2+a_3b_3 \text{ and}$$
$$a\times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

The vector sum of two coinitial vectors is given by the diagonal of the parallelogram whose adjacent sides are given vectors.



if $\overrightarrow{AB}, \overrightarrow{AC}$ are the given vectors, then $\overrightarrow{AB}+\overrightarrow{AC} = \overrightarrow{AD}$

The vector sum of the three sides of a triangle taken in order is 0 .i.e

if ABC is given triangle, then $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA} = 0$.