Class : 12th Maths
Chapter- 1 : Relations and Functions

Relations and Functions

Composition of functions and Invertible functions

Invertible functions

A function $f : x \to y$ is invertible, if $\exists$ a function $g : y \to x$ such that $gof = I_x$ and $fog = I_y$. Then, g is the inverse of f. If f is invertible, then it is both one-one and onto and vice-versa. For *eg.* If $f(x) = x$ and $f : N \to N$, then f is invertible.
Theorem 1 : If $f : x \to y$, $g : y \to z$ and $h : z \to s$ are functions, then $ho(got) = (hog)of$.
Theorem 2 : Let $f : x \to y$ and $g : y \to z$ be two invertible functions, then gof is invertible and $(gof)^{-1} = f^{-1} o\, g^{-1}$.

Composition of functions

The composition of functions $f : A \to B$ and $g : B \to C$ is denoted by gof, and is defined as $gof : A \to C$ given by $gof(x) = g(f(x))\ \forall x \in A$. *e.g.* let $A = N$ and $f, g : N \to N$ such that $f(x) = x^2$ and $g(x) = x^3\ \forall\ x \in N$. Then $gof(2) = g(f(2)) = g(2^2) = 4^3 = 64$.

Binary Operations Definition and its types

A binary operation '*' on a set A is a function $* : A \times A \to A$ denoted by $a * b$ *i.e.* $\forall\ a,b, \in A$, $a * b \in A$. Commutative if $a * b = b * a\ \forall\ a,b \in A$. Associative if $(a * b) * c = a * (b * c)\ \forall a,b, \in A$. $e \in A$ is identity if $a * e = a = e * a\ \forall a \in A$. and $b \in A$ is the inverse of $a \in A$, if. $a * b = e = b * a$. Addition is a binary operation on the set of integers.

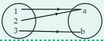
Types of functions

One - one (injective)

$f : x \to y$ is one-one if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2\ \forall\ x_1, x_2 \in x$. Other wise, f is many- one, f_1 is one-one.

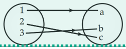
Onto (Surjective)

$f : x \to y$ is onto it for energy $y \in Y, \exists\ x \in X$ S.t. $f(x) = y$, f_2 is onto.



Bijective

$f : x \to y$ is both one-one and onto, then f is bijective. f_3 is bijective.



Types of Relations

Empty Relations Universal relation

A relation $R : A \to A$ is empty if $a \not R\, b\ \forall\ a, b \in A$. $R = \phi \subset A \times A$.
For *eg* : $R = \{(a,b) : a = b^2\}$, $A = \{1,5,10\}$

A relation $R : A \to A$ is universal if $a\ R\ b\ \forall, a,b \in A, R = A \times A$.
if $R = \phi$, then R is universal.

} Trivial Relations

Reflexive relation

A relation $R : A \to A$ is reflexive if $aRa\ \forall\ a \in A$

Symmetric Relation

A relation $R : A \to A$ is symmetric if $aRb \Rightarrow bRa\ \forall\ a, b \in A$

Transitive relation

A relation $R : A \times A$ is transitive if aRb, $bRc \Rightarrow aRc\ \forall\ a,b,c \in A$.

Equivalence relation
(reflexive, symmetric, transitive *e.g.*, Let T = the set of all triangles in a plane and $R : T \to T$ defined by $R = \{(T_1, T_2)\} : T_1$ is congruent to $T_2\}$.
Then, R is equivalence.